Algebra & Number Theory

Volume 9
2015
No. 2

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Atsushi Moriwaki

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ALGEBRA AND NUMBER THEORY 9:2 (2015)
dx.doi.org/10.2140/ant.2015.9.503

Semiample invertible sheaves with semipositive continuous hermitian metrics

Atsushi Moriwaki

Let (L, h) be a pair of a semiample invertible sheaf and a semipositive continuous hermitian metric on a proper algebraic variety over $\mathbb{C}$. In this paper, we prove that (L, h) is semiample metrized, answering a generalization of a question of S. Zhang.

Introduction

Let X be a proper algebraic variety over $\mathbb{C}$. Let L be an invertible sheaf on X, and let h be a continuous hermitian metric of L. We say that (L, h) is *semiample metrized* if, for any $\epsilon > 0$, there is $n > 0$ such that, for any $x \in X(\mathbb{C})$, we can find $l \in H^0(X, L^{\otimes n}) \setminus \{0\}$ with

$$\sup\{h^{\otimes n}(l, l)(w) \mid w \in X(\mathbb{C})\} \le e^{\epsilon n} h^{\otimes n}(l, l)(x).$$

Shouwu Zhang proposed the following question:

Question 0.1 [Zhang 1995, Question 3.6]. If L is ample and h is smooth and semipositive, does it follow that (L, h) is semiample metrized?

Theorem 3.5 of the same reference gives an affirmative answer in the case where X is smooth over $\mathbb{C}$. The purpose of this paper is to give an answer for a generalization of the above question. First of all, we fix some notation: We say that L is *semiample* if there is a positive integer n_0 such that $L^{\otimes n_0}$ is generated by global sections. Moreover, h is said to be *semipositive* (or we say that (L, h) is semipositive) if, for any point $x \in X(\mathbb{C})$ and a local basis s of L on a neighborhood of x, $-\log h(s, s)$ is plurisubharmonic around x (for the definition of plurisubharmonicity on a singular variety, see Section 1). Note that h is not necessarily smooth. By using the recent work of Coman, Guedj and Zeriahi [Coman et al. 2013], we have the following answer:

Theorem 0.2. *If L is semiample and h is continuous and semipositive, then (L, h) is semiample metrized.*

MSC2010: primary 14C20; secondary 32U05, 14G40.
Keywords: semiample metrized, semipositive.

1. Plurisubharmonic functions on singular complex analytic spaces

Let T be a reduced complex analytic space. An upper-semicontinuous function

$$\varphi : T \to \mathbb{R} \cup \{-\infty\}$$

is said to be *plurisubharmonic* if $\varphi \not\equiv -\infty$ and, for each $x \in T$, there is an analytic closed embedding $\iota_x : U_x \hookrightarrow W_x$ of an open neighborhood U_x of x into an open set W_x of $\mathbb{C}^{n_x}$ together with a plurisubharmonic function Φ_x on W_x such that $\varphi|_{U_x} = \iota_x^*(\Phi_x)$. For an analytic map $f : T' \to T$ of reduced complex analytic spaces and a plurisubharmonic function φ on T, it is easy to see that $\varphi \circ f$ is either identically $-\infty$ or plurisubharmonic on T'. By [Fornæss and Narasimhan 1980, Theorem 5.3.1], an upper-semicontinuous function $\varphi : T \to \mathbb{R} \cup \{-\infty\}$ is plurisubharmonic if and only if, for any analytic map $\varrho : \mathbb{D} \to T$, $\varphi \circ \varrho$ is either identically $-\infty$ or subharmonic on $\mathbb{D}$, where $\mathbb{D} := \{z \in \mathbb{C} \mid |z| < 1\}$. Moreover, if T is compact and φ is plurisubharmonic on T, then φ is locally constant.

Let ω be a smooth $(1, 1)$-form on T, that is, in the same way as in the definition of plurisubharmonic functions, ω is a smooth $(1, 1)$-form on the regular part of T and, for each $x \in T$, there is an analytic closed embedding $\iota_x : U_x \hookrightarrow W_x$ of an open neighborhood U_x of x into an open set W_x of $\mathbb{C}^{n_x}$ together with a smooth $(1, 1)$-form Ω_x on W_x such that $\omega|_{U_x} = \iota_x^*(\Omega_x)$. We assume that ω is locally given by $dd^c(u)$ for some smooth function u on a neighborhood of x. Let ϕ be a *quasiplurisubharmonic function* on T; that is, for each $x \in T$, ϕ can be locally written as the sum of a smooth function and a plurisubharmonic function around x. We say that ϕ is *ω-plurisubharmonic* if there is an open covering $T = \bigcup_\lambda U_\lambda$, together with a smooth function u_λ on U_λ for each λ, such that $\omega|_{U_\lambda} = dd^c(u_\lambda)$ and $\phi|_{U_\lambda} + u_\lambda$ is plurisubharmonic on U_λ. The condition for ω-plurisubharmonicity is often denoted by $dd^c([\phi]) + \omega \geq 0$.

Here we consider the following lemma:

Lemma 1.1. *Let $f : X \to Y$ be a surjective and proper morphism of algebraic varieties over $\mathbb{C}$. Let φ be a real-valued function on $Y(\mathbb{C})$.*

(1) *φ is continuous if and only if $\varphi \circ f$ is continuous.*

(2) *Assume that φ is continuous. Then φ is plurisubharmonic if and only if $\varphi \circ f$ is plurisubharmonic.*

Proof. (1) It is sufficient to see that if $\varphi \circ f$ is continuous, then φ is continuous. Otherwise, there are $y \in Y(\mathbb{C})$, $\epsilon_0 > 0$ and a sequence $\{y_n\}$ on $Y(\mathbb{C})$ such that $\lim_{n\to\infty} y_n = y$ and $|\varphi(y_n) - \varphi(y)| \geq \epsilon_0$ for all n. We choose $x_n \in X(\mathbb{C})$ such that $f(x_n) = y_n$. As $f : X \to Y$ is proper, we can find a subsequence $\{x_{n_i}\}$ of $\{x_n\}$ such that $x := \lim_{i\to\infty} x_{n_i}$ exists in $X(\mathbb{C})$. Note that

$$f(x) = \lim_{i\to\infty} f(x_{n_i}) = \lim_{i\to\infty} y_{n_i} = y,$$

so that, as $\varphi \circ f$ is continuous,

$$\varphi(y) = (\varphi \circ f)(x) = \lim_{i \to \infty} (\varphi \circ f)(x_{n_i}) = \lim_{i \to \infty} \varphi(f(x_{n_i})) = \lim_{i \to \infty} \varphi(y_{n_i}),$$

which is a contradiction, so that φ is continuous.

(2) We need to check that if $\varphi \circ f$ is plurisubharmonic, then φ is plurisubharmonic. By using Chow's lemma, we may assume that $f : X \to Y$ is projective. Moreover, since the assertion is local with respect to Y, we may further assume that there is a closed embedding $\iota : X \hookrightarrow Y \times \mathbb{P}^N$ such that $p \circ \iota = f$, where $p : Y \times \mathbb{P}^n \to Y$ is the projection to the first factor. The remaining proof is same as the last part of the proof of [Demailly 1985, Theorem 1.7]. Let $g : (\mathbb{D}, 0) \to (Y, y)$ be a germ of an analytic map. By the theorem of Fornæss and Narasimhan, it is sufficient to show that $\varphi \circ g$ is subharmonic. Clearly we may assume that g is given by the normalization of a 1-dimensional irreducible germ (C, y) in (Y, y). Using hyperplanes in $\mathbb{P}^N$, we can find $x \in X$ and a 1-dimensional irreducible germ (C', x) in (X, x) such that (C', x) lies over (C, y). Let $g' : (\mathbb{D}, 0) \to (X, x)$ be the germ of an analytic map given by the normalization of (C', x). Then we have an analytic map $\sigma : (\mathbb{D}, 0) \to (\mathbb{D}, 0)$ with $g \circ \sigma = f \circ g'$:

$$\begin{array}{ccc} (\mathbb{D}, 0) & \xrightarrow{g'} & (X, x) \\ {\scriptstyle\sigma}\big\downarrow & & \big\downarrow{\scriptstyle f} \\ (\mathbb{D}, 0) & \xrightarrow{g} & (Y, y) \end{array}$$

Changing a variable of $(\mathbb{D}, 0)$, we may assume that σ is given by $\sigma(z) = z^m$ for some positive integer m. Then $\varphi \circ g \circ \sigma$ is subharmonic because $\varphi \circ f$ is plurisubharmonic. Therefore, as σ is étale over the outside of 0, $\varphi \circ g$ is subharmonic on the outside of 0, and hence $\varphi \circ g$ is subharmonic on $(\mathbb{D}, 0)$ by the removability of singularities of subharmonic functions. □

2. Descent of a semipositive continuous hermitian metric

Here, we consider a descent problem of a semipositive continuous hermitian metric.

Theorem 2.1. *Let $f : X \to Y$ be a surjective and proper morphism of algebraic varieties over $\mathbb{C}$ with $f_*\mathcal{O}_X = \mathcal{O}_Y$. Let L be an invertible sheaf on Y. If h' is a semipositive continuous hermitian metric of $f^*(L)$, then there is a semipositive continuous hermitian metric h of L such that $h' = f^*(h)$.*

Proof. Let h_0 be a continuous hermitian metric of L on Y. There is a continuous function ϕ on $X(\mathbb{C})$ such that $h' = \exp(\phi) f^*(h_0)$. Let F be a subvariety of X such that F is an irreducible component of a fiber of $f : X \to Y$. Then, as

$$(f^*(L), h')|_F \simeq (\mathcal{O}_F, \exp(\phi|_F)),$$

we can see that $-\phi|_F$ is plurisubharmonic, so that $\phi|_F$ is constant. Therefore, for any point $y \in Y(\mathbb{C})$, $\phi|_{\mu^{-1}(y)}$ is constant because $\mu^{-1}(y)$ is connected, and hence there is a function ψ on $Y(\mathbb{C})$ such that $\psi \circ f = \phi$. By Lemma 1.1(1), ψ is continuous, so that, if we set $h := \exp(\psi)h_0$, then h is continuous on $Y(\mathbb{C})$ and $h' = f^*(h)$.

Finally, let us see that h is semipositive. As this is a local question on Y, we may assume that there is a local basis s of L over Y. If we set $\varphi = -\log h(s,s)$, then $\varphi \circ f$ is plurisubharmonic because h' is semipositive. Therefore, by Lemma 1.1(2), φ is plurisubharmonic, as required $\square$

3. The proof of Theorem 0.2

In the case where X is smooth over $\mathbb{C}$, L is ample and h is smooth, this theorem was proved by Zhang [1995, Theorem 3.5]. First we assume that L is ample. Then there are a positive integer n_0 and a closed embedding $X \hookrightarrow \mathbb{P}^N$ such that $\mathcal{O}_{\mathbb{P}^N}(1)|_X \simeq L^{\otimes n_0}$. Let h_{FS} be the Fubini–Study metric of $\mathcal{O}_{\mathbb{P}^n}(1)$. Let ϕ be the continuous function on $X(\mathbb{C})$ given by $h^{\otimes n_0} = \exp(-\phi)h_{\mathrm{FS}}|_X$. We set $\omega = c_1(\mathcal{O}_{\mathbb{P}^N}(1), h_{\mathrm{FS}})$. Then ϕ is $(\omega|_X)$-plurisubharmonic. Therefore, by [Coman et al. 2013, Corollary C], there is a sequence $\{\varphi_i\}$ of smooth functions on $\mathbb{P}^N(\mathbb{C})$ with the following properties:

(1) φ_i is ω-plurisubharmonic for all i.

(2) $\varphi_i \geq \varphi_{i+1}$ for all i.

(3) For $x \in X(\mathbb{C})$, $\lim_{i\to\infty} \varphi_i(x) = \phi(x)$.

Since X is compact and ϕ is continuous, (3) implies that the sequence $\{\varphi_i\}$ converges to ϕ uniformly on $X(\mathbb{C})$. We choose i such that $|\phi(x) - \varphi_i(x)| \leq \epsilon n_0/2$ for all $x \in X$. We set $h_i = \exp(-\varphi_i)h_{\mathrm{FS}}$. Then h_i is a semipositive smooth hermitian metric of $\mathcal{O}_{\mathbb{P}^N}(1)$. Therefore, there is a positive integer n_1 such that, for $x \in \mathbb{P}^N(\mathbb{C})$, we can find $l \in H^0(\mathbb{P}^N, \mathcal{O}_{\mathbb{P}^N}(n_1)) \setminus \{0\}$ with

$$\sup\{h_i^{\otimes n_1}(l,l)(w) \mid w \in \mathbb{P}^N(\mathbb{C})\} \leq e^{n_1(\epsilon n_0/2)} h_i^{\otimes n_1}(l,l)(x).$$

In particular, if $x \in X(\mathbb{C})$, then $l(x) \neq 0$ (so that $l|_X \neq 0$) and

$$\sup\{h_i^{\otimes n_1}(l,l)(w) \mid w \in X(\mathbb{C})\} \leq e^{\epsilon n_0 n_1/2} h_i^{\otimes n_1}(l,l)(x).$$

Note that

$$h^{\otimes n_0} e^{-\epsilon n_0/2} \leq h_i \leq h^{\otimes n_0} \tag{3-1}$$

on $X(\mathbb{C})$, because $h_i = h^{\otimes n_0}\exp(\phi - \varphi_i)$ and $-\epsilon n_0/2 \leq \phi - \varphi_i \leq 0$ on $X(\mathbb{C})$. Therefore,

$$\sup\{h^{\otimes n_0 n_1}(l,l)(w) \mid w \in X(\mathbb{C})\} e^{-n_0 n_1 \epsilon/2} \leq \sup\{h_i^{\otimes n_1}(l,l)(w) \mid w \in X(\mathbb{C})\}$$

and

$$h_i^{\otimes n_1}(l,l)(x) \le h^{\otimes n_0 n_1}(l,l)(x),$$

and hence

$$\sup\{h^{\otimes n_0 n_1}(l,l)(w) \mid w \in X(\mathbb{C})\} \le e^{n_1 n_0 \epsilon} h^{\otimes n_0 n_1}(l,l)(x),$$

as required.

In general, as L is semiample, there are a positive integer n_2, a projective algebraic variety Y over $\mathbb{C}$, a morphism $f : X \to Y$ and an ample invertible sheaf A on Y such that $f_* \mathcal{O}_X = \mathcal{O}_Y$ and $f^*(A) \simeq L^{\otimes n_2}$. Thus, by Theorem 2.1, there is a semipositive continuous hermitian metric k of A such that $(f^*(A), f^*(k)) \simeq (L^{\otimes n_2}, h^{\otimes n_2})$. Therefore, the assertion of the theorem follows from the previous observation.

4. A variant of Theorem 0.2

The following theorem is a consequence of Theorem 0.2 together with the arguments in [Zhang 1995, Theorem 3.3]. However, we can give a direct proof using ideas in the proof of Theorem 0.2.

Theorem 4.1. *Let X be a projective algebraic variety over $\mathbb{C}$. Let L be an ample invertible sheaf on X and let h be a semipositive continuous hermitian metric of L. Let us fix a reduced subscheme Y of X, $l \in H^0(Y, L|_Y)$ and a positive number ϵ. Then, for the given X, L, h, Y, l and ϵ, there is a positive integer n_1 such that, for all $n \ge n_1$, we can find $l' \in H^0(X, L^{\otimes n})$ with $l'|_Y = l^{\otimes n}$ and*

$$\sup\{h^{\otimes n}(l',l')(w) \mid w \in X(\mathbb{C})\} \le e^{n\epsilon} \sup\{h(l,l)(w) \mid w \in Y(\mathbb{C})\}^n.$$

Proof. In the case where X is smooth over $\mathbb{C}$ and h is smooth and positive, the assertion of the theorem follows from [Zhang 1995, Theorem 2.2], in which Y is actually assumed to be a subvariety of X. However, the proof works well under the assumption that Y is a reduced subscheme. First of all, let us see the theorem in the case where X is smooth over $\mathbb{C}$ and h is smooth and semipositive. As L is ample, there is a positive smooth hermitian metric t of L with $t \le h$. Let us choose a positive integer m such that $e^{-\epsilon/2} \le (t/h)^{1/m} \le 1$ on $X(\mathbb{C})$. If we set $t_m = h^{1-1/m} t^{1/m}$, then t_m is smooth and positive, so that, for a sufficiently large integer n, there is $l' \in H^0(X, L^{\otimes n})$ such that $l'|_Y = l^{\otimes n}$ and

$$\sup\{t_m^{\otimes n}(l',l')(w) \mid w \in X(\mathbb{C})\} \le e^{n\epsilon/2} \sup\{t_m(l,l)(w) \mid w \in Y(\mathbb{C})\}^n,$$

and hence the assertion follows because $e^{-\epsilon/2} h \le t_m \le h$ on $X(\mathbb{C})$.

For a general case, we use the same symbols n_0, $X \hookrightarrow \mathbb{P}^N$, h_{FS}, ϕ, ω and $\{\varphi_i\}$ as in the proof of Theorem 0.2. Clearly we may assume that $l \ne 0$. Since L is ample, if a_0 is a sufficiently large integer, then, for each $j = 0, \ldots, n_0 - 1$, there is

$l_j \in H^0(X, L^{\otimes n_0 a_0 + j})$ with $l_j|_Y = l^{\otimes n_0 a_0 + j}$. Let us fix a positive number A such that

$$\sup\{h^{\otimes n_0 a_0 + j}(l_j, l_j)(w) \mid w \in X(\mathbb{C})\} \le e^A \sup\{h(l, l)(w) \mid w \in Y(\mathbb{C})\}^{n_0 a_0 + j} \tag{4-1}$$

for $j = 0, \ldots, n_0 - 1$. We choose i with $|\phi(x) - \varphi_i(x)| \le \epsilon n_0/2$ for all $x \in X$, and we set $h_i = \exp(-\varphi_i) h_{\mathrm{FS}}$. As h_i is smooth and semipositive, for the given $\mathbb{P}^N$, $\mathcal{O}_{\mathbb{P}^N}(1)$, h_i, Y, $l^{\otimes n_0}$ (as an element of $H^0(Y, \mathcal{O}_{\mathbb{P}^N}(1)|_Y)$) and $n_0\epsilon/4$, there is a positive integer a_1 such that the assertion of the theorem holds for all $a \ge a_1$. We put

$$n_1 := n_0 \max\left\{a_1 + a_0 + 1,\ \frac{4A}{n_0\epsilon} - 3a_0 + 1\right\}.$$

Let n be an integer with $n \ge n_1$. If we set $n = n_0(a + a_0) + j$ $(0 \le j \le n_0 - 1)$, then

$$a \ge a_1 \quad \text{and} \quad a \ge \frac{4A}{n_0\epsilon} - 4a_0,$$

so that we can find $l'' \in H^0(\mathbb{P}^N, \mathcal{O}_{\mathbb{P}^N}(a))$ with $l''|_Y = l^{\otimes n_0 a}$ and

$$\sup\{h_i^{\otimes a}(l'', l'')(w) \mid w \in \mathbb{P}^N(\mathbb{C})\} \le e^{a(n_0\epsilon/4)} \sup\{h_i(l^{\otimes n_0}, l^{\otimes n_0})(w) \mid w \in Y(\mathbb{C})\}^a,$$

which implies that

$$\sup\{h^{\otimes n_0 a}(l'', l'')(w) \mid w \in X(\mathbb{C})\} \le e^{(3/4)n_0 a\epsilon} \sup\{h(l, l)(w) \mid w \in Y(\mathbb{C})\}^{n_0 a}, \tag{4-2}$$

because of (3-1). Here we set $l' = l'' \otimes l_j$. Then, $l'|_Y = l^{\otimes n}$ and, using (4-1) and (4-2), we have

$$\begin{aligned}
&\sup\{h^{\otimes n}(l', l')(w) \mid w \in X(\mathbb{C})\} \\
&\qquad \le \sup\{h^{\otimes n_0 a}(l'', l'')(w) \mid w \in X(\mathbb{C})\} \sup\{h^{\otimes n_0 a_0 + j}(l_j, l_j)(w) \mid w \in X(\mathbb{C})\} \\
&\qquad \le e^{(3/4)n_0 a\epsilon + A} \sup\{h(l, l)(w) \mid w \in Y(\mathbb{C})\}^n,
\end{aligned}$$

which implies the assertion because $(3/4)n_0 a\epsilon + A \le \epsilon n$. □

5. Arithmetic application

As an application of Theorem 0.2, we have the following generalization of the arithmetic Nakai–Moishezon criterion (see [Zhang 1995, Corollary 4.8]).

Corollary 5.1. *Let $\mathscr{X}$ be a projective and flat integral scheme over $\mathbb{Z}$. Let $\mathscr{L}$ be an invertible sheaf on $\mathscr{X}$ such that $\mathscr{L}$ is nef on every fiber of $\mathscr{X} \to \mathbb{Z}$. Let h be an F_∞-invariant semipositive continuous hermitian metric of $\mathscr{L}$, where F_∞ is the complex conjugation map $\mathscr{X}(\mathbb{C}) \to \mathscr{X}(\mathbb{C})$. If $\widehat{\deg}(\hat{c}_1((\mathscr{L}, h)|_{\mathscr{Y}})^{\dim \mathscr{Y}}) > 0$ for all horizontal integral subschemes $\mathscr{Y}$ of $\mathscr{X}$, then, for an F_∞-invariant continuous hermitian invertible sheaf $(\mathscr{M}, k)$ on $\mathscr{X}$, $H^0(\mathscr{X}, \mathscr{L}^{\otimes n} \otimes \mathscr{M})$ has a basis consisting of strictly small sections for a sufficiently large integer n.*

Proof. Let X be the generic fiber of $\mathscr{X} \to \operatorname{Spec}(\mathbb{Z})$ and let Y be a subvariety of X. Let $\mathscr{Y}$ be the Zariski closure of Y in $\mathscr{X}$. As

$$\widehat{\deg}(\hat{c}_1((\mathscr{L}, h)|_{\mathscr{Y}})^{\dim \mathscr{Y}}) > 0,$$

$(\mathscr{L}, h)|_{\mathscr{Y}}$ is big by [Moriwaki 2012, Theorem 6.6.1], so that $H^0(\mathscr{Y}, \mathscr{L}^{\otimes n_0}|_{\mathscr{Y}}) \setminus \{0\}$ has a strictly small section for a sufficiently large integer n_0. Moreover, if we set $L = \mathscr{L}|_X$, then $L|_Y$ is big, and hence $\deg(L^{\dim Y} \cdot Y) > 0$ because L is nef. Therefore, L is ample by the Nakai–Moishezon criterion for ampleness. In particular, by Theorem 0.2, h is semiample metrized. Thus the assertion follows from the arguments in [Zhang 1995, Theorem 4.2]. $\square$

Acknowledgements

I would like to thank Professor Zhang and the referee for their comments and suggestions.

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Communicated by Shou-Wu Zhang
Received 2014-11-09 Revised 2015-01-01 Accepted 2015-02-16

moriwaki@math.kyoto-u.ac.jp *Department of Mathematics, Faculty of Science, Kyoto University, Kyoto 606-8502, Japan*

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Algebra & Number Theory (ISSN 1944-7833 electronic, 1937-0652 printed) at Mathematical Sciences Publishers, 798 Evans Hall #3840, c/o University of California, Berkeley, CA 94720-3840 is published continuously online. Periodical rate postage paid at Berkeley, CA 94704, and additional mailing offices.

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Algebra & Number Theory

Volume 9 No. 2 2015