A NOTE ON SCHEEFFER'S THEOREM

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This note deals exclusively with subsets of C, the linear continuum. We shall make use of some facts concerning Baire category, which can be found in [1, §19].

If S is a set and t is a real number, denote by S[t] the set obtained from S by translating it by the amount t:

$$S[t] = \{s + t : s \in S\}.$$

A theorem due to Scheeffer [2, p. 291] (quoted in [3, p. 55] and [4, p. 52]) can be expressed as follows:

If E *is at most enumerable, and* N *is nowhere dense, then there exists an everywhere dense set* D *such that, for every* $d \in D$, $E[d] \cap N$ *is empty.*

Our purpose is to show that, in this result, the hypothesis can be weakened and the conclusion strengthened:

THEOREM. *If* E *is at most enumerable, and* K *is of first category, then there exists a residual set* R *such that, for every* $r \in R$, $E[r] \cap K$ *is empty.*

Proof. Let the elements of E be

$$e_0, e_1, \cdots, e_n, \cdots \quad (n < \nu \leq \omega).$$

For every $n < \nu$, let R_n be the set of all real numbers r such that $e_n + r \notin K$; then

$$R_n = (C - K)[-e_n],$$

which is a residual set, because K is of first category. Let

$$R = \bigcap_{n<\nu} R_n.$$

Then R is a residual set and, for every $r \in R$, $E[r] \cap K$ is empty.

An analogous argument, involving measure instead of category, yields the following result:

If E *is at most enumerable, and* Z *is of measure zero, then there exists a set* T *such that* C - T *is of measure zero and, for every* $t \in T$, $E[t] \cap Z$ *is empty.*

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4. W. H. Young and G. C. Young, *The theory of sets of points*, Cambridge, 1906.

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