Notre Dame Journal of Formal Logic
Volume IV, Number 2, April 1963

TOWARD A PRAGMATICAL EXPLICATION OF EPISTEMIC MODALITIES

ALBERT M. SWEET

G. H. von Wright[1] has formulated various systems of epistemic modalities, which clarify the relations of such predicates as "is known" and "is falsified" with truth functions and quantification. The question arises as to the nature of the interpretations which may be given to these systems. That is, one may ask under what conditions will a person be said to know a proposition (or sentence) to be true.

A pragmatical meta-language similar to that formulated by R. M. Martin[2] provides a means of interpreting empirically the non-iterated *de dicto* modalities of von Wright's system *V*. This presents the question of the truth of the epistemic principles so interpreted in a new light.

According to von Wright, the "epistemic modalities are said to be *de dicto* when they are about the mode or way in which a proposition is or is not known (to be true)."[3] By "proposition" we may understand "sentence," where we characterize the object-language under consideration.

The "epistemic modalities are said to be *de re* when they are about the mode or way in which an individual thing is known to possess or lack a certain property."[4] We shall not find it necessary to consider *de re* modalities, since the object-language with which we shall be concerned countenances no properties in its domain, with no apparent loss of descriptive power.

We shall consider only non-iterated modalities, since the effect of iterated modalities may be achieved by repeating the present analysis in a meta-meta-language.

1. Georg H. von Wright, *An Essay in Modal Logic* (North-Holland Publishing Company, Amsterdam: 1951).
2. Richard M. Martin, *Toward a Systematic Pragmatics* (North-Holland Publishing Company, Amsterdam: 1959).
3. von Wright, p. 29.
4. von Wright, p. 33.

Received August 8, 1962

First we shall formulate a pragmatical meta-language appropriate to explicating "is known" or "is verified" and related predicates, including the desired properties of these predicates. Then we shall compare the resulting system considered as an empirical theory, with von Wright's system V of epistemic modalities.

An explication of "is known" may be achieved in a pragmatical meta-language with the primitive predicates "**Acpt**", "**Pfm**", and "**B**", appearing in contexts:

"X **Acpt** a, t" ("The person X accepts the sentence a at time t"),

"X **Pfm** f, x, y, t" ("The person X performs an action of kind f upon an object of kind x, producing an object of kind y at time t"),

"t **B** t'" ("The time t is before the time t'").

"**Acpt**" and "**B**" are characterized in Martin's system of pragmatics[5] by obvious (and incomplete) axioms. For our epistemological purposes we depart somewhat from Martin's formulation of "**Pfm**", following rather A. Grzegorczyk[6] in having the variables "x", "y", "z", etc. range over classes of similar objects. We assume that "**Pfm**" may be characterized by suitable axioms, and that all the above predicates may be interpreted empirically.

Let us suppose that the syntactic variables "a", "b", "c", etc. range over the symbols of an ideal language which formalizes all discourse concerning whose empirical significance there is general agreement. (This may be made precise by the method of Grzegorczyk's paper.[7]) Although more restricted choices are possible, we specify that the object-language by type-theoretic, and call it "$\mathfrak{L}$", following Martin's usage.

The only variables of higher than first type with which we shall be concerned are argument expressions for the predicate "**Pfm**". These are translational variables of type 2 whose conventions are listed above, and for simplicity we omit superscripts indicating their types.

Let us now suppose that the users of $\mathfrak{L}$ possess a logic of induction, that is, a system of rules whereby from certain evidence statements they might decide which general statements to accept as descriptive axioms of $\mathfrak{L}$. We should still desire to be able to characterize, by behavioral considerations altogether, the ways in which evidence statements themselves are accepted by users of $\mathfrak{L}$. In particular, we should by such means wish to be able precisely to distinguish between acceptances of evidence statements based upon appropriate investigations actually performed under the proper conditions, and acceptances of evidence statements resulting from (using the familiar terminology of Peirce) tenacity, authority, or inclination.

To complete the suggestion from Peirce, we introduce the following axioms aimed at characterizing, among the users of $\mathfrak{L}$, behavior we wish to

---

5. Martin, pp. 37-38.
6. A. Grzegorczyk, "The Pragmatic Foundations of Semantics," *Synthèse*, VIII (1950-51): 300-324.
7. *Ibid.*

count as scientific investigation. We shall refer to the pragmatical metalanguage of $\mathfrak{L}$ containing these axioms as "*PMV*", since within this metalanguage we shall be able to define the predicate "*V*", as an explication of "is known" or "is verified".

It will be convenient to employ in the axioms and definitions to follow, the syntactic nomenclature:

"$\rightarrow$" names "$\supset$"
"$\leftrightarrow$" names "$\equiv$"
"$\wedge$" names "$.$"
"$-$" names "$\sim$"
"$\vee$" names "$\vee$".

We first introduce an expression which may be read: "The person *X* correlates acceptance of the sentence *a* with performing an action of kind *f* upon an object of kind *x*, producing an object (index) of kind *y*."

D1. "X **CorAcpt** a, f, x, y" for "(t) $(X$ **Pfm** $f, x, y, t \supset (\mathrm{E}t')$ $(t$ **U** t' . X **Acpt** $a, t')$: (z) $(X$ **Pfm** f, x, z, t . $z \neq y$: $\supset$: $(\mathrm{E}t')$ $(t$ **U** t' . X **Acpt** $(-a), t')))$"

"t **U** t'" means that the time t is *up to* the time t', and is defined by "**B**".[8]

We now introduce into *PMV the* following axioms.

S1. **S**(X) . **S**(Y) : $\supset$: X **CorAcpt** $a, f, x, y \supset Y$ **CorAcpt** a, f, x, y

S2. **S**(X) . X **Acpt** a, t : $\supset$: $(\mathrm{E}f, x, y)$ $(X$ **CorAcpt** a, f, x, y . $(\mathrm{E}Y, t')$ $($**S**(Y) . Y **Pfm** f, x, y, t' . t' **B** $t))$

S3. **S**(X) . X **CorAcpt** a, f, x, y : $\supset$: $(\mathrm{E}X, t)$ $($**S**(X) . X **Acpt** $(-a), t) \supset (\mathrm{E}X, t, z)$ $($**S**(X) . X **Pfm** f, x, z, t . $z \neq y)$

Perhaps sufficient empirical information would show that there is only one large class of persons satisfying *S1-S3*. It is for the plausibility of such an assertion that we required the object-language $\mathfrak{L}$ to be as comprehensive as possible. Let us suppose that there is such a unique class. In the theorems and definitions which follow, let us also understand "**S**" as designating the property of being in this unique large class of persons satisfying *S1-S3*.

From *S1* and *S2* it is provable that

T1. **S**(X) . X **Acpt** a, t : $\supset$: $(\mathrm{E}Y, f, x, y, t', t'')$ $($**S**(Y) . Y **Pfm** f, x, y, t' . t' **B** t . t' **U** t'' . Y **Acpt** a, t'' . $\sim t$ **B** t'' . (Z) $(Z$ **Pfm** f, x, y, t' . **S**(Z) : $\supset$: Z **Acpt** $a, t''))$

If *T1* expresses adequately the idea that in scientific acceptances in general, and in scientific acceptances of evidence sentences in particular, the appropriate investigations have been performed under the proper conditions,

8. Martin, p. 96.

at least an approach to an explication of one important sense of "knowing" is at hand.

D2. "Va" for "$(\mathrm{E}X, t)\ (\mathbf{S}(X)\ .\ X\ \mathbf{Acpt}\ a, t)$"

"Va" reads: "a is verified" or "a is known (to be true)". We employ "V" in this context, following von Wright's usage.

The definition *D2* seems in accord with any of the usual programs for constructing a logic of induction, in virtue of which the descriptive axioms of $\mathfrak{L}$ would be chosen. It is also intended to reflect Peirce's observation that scientific knowledge is based neither upon "first principles" (i.e. incorrigible theories) nor "first sensations".

We also introduce an expression "Fa", to be read: "a is falsified".

D3. "Fa" for "$V(-a)$"

S1-S3 do not suffice to prove the desired properties of "V" and "F", so that the resulting system may be considered to contain a portion of von Wright's system *V* of epistemic modalities. For this purpose we add to *PMV* the following axioms.

S4. $\mathbf{S}(X) \supset (t)\ (X\ \mathbf{AcptNor{-}}, G, t\ .\ X\ \mathbf{AcptNorv}, G, t)$

S5. $\mathbf{Thm}\ a \supset \sim (\mathrm{E}X, t)\ (X\ \mathbf{Acpt}\ (-a), t\ .\ \mathbf{S}(X))$

S6a. $\mathbf{Q}(f)\ .\supset .\ X\ \mathbf{Pfm}\ f, x, y, t \supset \sim (\mathrm{E}Y, z, t')\ (Y\ \mathbf{Pfm}\ f, x, z, t'\ .\ z \neq y)$

S6b. $\mathbf{S}(X)\ .\ X\ \mathbf{CorAcpt}\ a, f, x, y : \supset : Q(f)$

S4 stipulates that members of **S** accept the truth functional signs – and v in their normal sense, relative to any class G of sentences of $\mathfrak{L}$ under investigation. These predicates are defined by Martin in his system of pragmatics.[9]

S6a specifies a certain type of performance–kinds **Q**, which *S6b* attributes to the correlations of members of **S**. We assume that an empirical theory like *S6a* may be a theorem of *PMV*.

We may understand the following theorem, which follows from *S1-S3*, *S6*, to assert that $\mathfrak{L}$ is *pragmatically* consistent.

T2.11 $Va \supset \sim V(-a)$

If we should wish to limit the scope of *S6* to evidence sentences, then *S5* would be used in the proof of *T2.11*.

From *S4* we have the following meta-theorems, the first requiring our previous informal assumption that **S** is not empty.

MT2.21 If "a" is the structural description of a tautological sentence of $\mathfrak{L}$, then "Va" is a theorem of PMV.

MT2.31 If "$a \leftrightarrow b$" is the structural description of a tautological sentence of $\mathfrak{L}$, then "$Va \equiv Vb$" is a theorem of PMV.

---

9. Martin, pp. 40-41.

MT2.32 If "$a \leftrightarrow b$" is the structural description of a tautological sentence of $\mathfrak{L}$, then "$\sim Fa \equiv \sim Fb$" is a theorem of PMV.

Thus we now have:

T2.41 $F(-a) \equiv Va$

T2.42 $\sim Fa \vee \sim F(-a)$

From *S5* there follow immediately:

T2.51 **Thm** $a \supset \sim V(-a)$

T2.52 **Thm** $a \supset \sim Fa$

T2.53 $Va \supset \sim$ **Thm** $(-a)$

T2.54 $Fa \supset \sim$ **Thm** a

T2.61 (**Thm** $a \supset$ **Thm** $b) \supset$ (**Thm** $a \supset \sim Fb)$

From *S4* we also have:

T2.71 $V(a \wedge b) : \supset : Va \,.\, Vb$

T2.72 $\sim Fa \vee \sim Fb \,.\supset.\, F(a \vee b)$

We now consider an extensional version of von Wright's system *V*, formalized in the manner of his system *M*. This is simply the system *M* with the alethic modalities *M* and *N* replaced by the epistemic modalities $\sim F$ and *V*, respectively. All formulas of *V* without iterated modal operators may be interpreted as formulas of *PMV*, as exhibited in the following comparison of *V* and *PMV*. Our manner of interpreting meta-linguistically, non-modalized propositional letters of *V* is guided by empirical plausibility and deductive power in *PMV*, since these considerations guided our construction of *PMV*.

T2.42 and *T2.52* reflect von Wright's axiom *B1*. But whereas in von Wright's system, "$\sim Fa \vee \sim F \sim a$" is derivable from "$a \supset \sim Fa$", there is no analogous derivation in *PMV*. Hence we were required to introduce *T2.42* by other means.

T2.72 reflects half of von Wright's axiom *B2*. We shall return to this matter later.

MT2.32 reflects von Wright's rule *B1*. And *MT2.21* reflects von Wright's rule *B2*.

The addition to *PMV* of the axiom

S7. $Va \,.\, Vb : \supset : V(a \wedge b)$

suffices for *PMV* to contain most of the non-iterated portion of von Wright's system *V*, interpreted as above. Some theorems dependent on *S7* are:

T3.11 $V(a \wedge b) : \equiv : Va \,.\, Vb$

T3.12 $\sim F(a \vee b) \,.\equiv.\, \sim F \vee \sim Fb$

T3.21 $V(a \rightarrow b) \supset (Va \supset Vb)$

T3.22 $V(a \leftrightarrow b) \supset (Va \equiv Vb)$

T3.31 $V(a \rightarrow b) \,.\, V(b \rightarrow c) : \supset : V(a \rightarrow c)$

A difference between *PMV* and von Wright's system *V* results from the necessity of interpreting meta-linguistically in *PMV*, non-modalized propositional letters of *V*. Thus there are theorems in *V* with no analogues in *PMV*, such as "$(a \supset b) \supset (Va \supset b)$".

We leave open the question of the completeness of that portion of *PMV* corresponding to von Wright's system *V*, since a prior concern is axiom *S7*. Is *S7* empirically true? It is certainly doubtful, and it is an interesting result that such an intuitively plausible proposition becomes doubtful when interpreted in terms of acceptances.

We have introduced the axioms *S1*-*S7* in three groups, reflecting what appears to be their decreasing degree of empirical plausibility. On the empirical interpretation which we have formulated for *S7*, its truth would evidently involve an ideal communication or codification of the results of scientific investigation. This indicates a possible relationship between computing machines and epistemology.

One further question is whether one can find suitable means of supplementing *PMV*, in order to accommodate a complete epistemological theory similar to von Wright's system *VE*, which combines epistemic modalities with quantification. By the completeness of such theories is meant provability of (every truth of logic in the appropriate usual sense, involving epistemic modalities and) every "truth of logic . . . which depends on the specific nature of modal concepts."[10] The principles upon which these latter truths depend may, as we have seen, be considered to be empirical. This raises the question of their truth in a new perspective.

Georgia State College
Atlanta, Georgia

10. von Wright, p. 11.