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NOTE ON INDUCTIVE FINITENESS IN MEREOLOGY

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In this note we prove that for inductive finiteness,

$$[a]:\mathrm{Fin}\{a\}.\supset.\ \mathrm{Fin}\{\mathbf{st}(a)\}.$$

Sobociński proved this previously under the added hypothesis, $\mathrm{dscr}\{a\}$. Theorems quoted, but not stated in this note, refer to the Mereological Preliminaries in [1]. We shall also need the following well-known definitions and properties concerning inductive finiteness.

DF1. $[a\varphi]\therefore \varphi\{\Lambda\}:[Ab]:A\,\varepsilon\, a\,.\,\varphi\{b\}.\supset.\,\varphi\{b\cup A\}:\equiv.\,\mathrm{InR}\langle\varphi\rangle\{a\}$
DF2. $[a]\therefore[\varphi]:\mathrm{InR}\langle\varphi\rangle\{a\}.\supset.\,\varphi\{a\}:\equiv.\,\mathrm{Fin}\{a\}.$
F1. $[A]:\rightharpoondown\{A\}.\supset.\,\mathrm{Fin}\{A\}.$
F2. $[ab]:\mathrm{Fin}\{a\}.\,\mathrm{Fin}\{b\}.\supset.\,\mathrm{Fin}\{a\cup b\}.$
F3. $[ab]:a\overset{\infty}{\underset{\infty}{\circ}} b\,.\,\mathrm{Fin}\{b\}.\supset.\,\mathrm{Fin}\{a\}.$

We begin the proof with some auxiliary definitions.

D1. $[ACb]:C\,\varepsilon\,\varphi(bA).\equiv.\,C\,\varepsilon\, b\,.\,C\,\varepsilon\,\mathbf{el}(A).$
T1. $[bA].\,\varphi(bA)\subset b.$ [*D1*]
D2. $[ABCb]:C\,\varepsilon\,\Phi\langle bB\rangle(A).\equiv.\,C\,\varepsilon\,\mathbf{Kl}(\varphi(bA)).A\,\varepsilon\,\mathbf{Kl}(\varphi(bA)\cup B).$
$A\,\varepsilon\,\mathbf{st}(b\cup B)\backslash(\mathbf{st}(b)\cup B).$
D3. $[BCb]:C\,\varepsilon\,\Phi(bB).\equiv.\,C\,\varepsilon\, C\,.[\exists A].\,C\varepsilon\,\Phi\langle bB\rangle(A).$
T2. $[BCb]:C\,\varepsilon\,\Phi(bB).\equiv.[\exists A].\,C\,\varepsilon\,\Phi\langle bB\rangle(A).$ [*D3*]
T3. $[Bb].\,\Phi(bB)\subset\mathbf{st}(b).$ [*D3*; *D2*; *T1*; *M18*]
T4 $[BCb]:C\,\varepsilon\,\Phi(bB).\supset.[\exists A].\,C\,\varepsilon\,\Phi\langle bB\rangle(A).A\,\varepsilon\,\mathbf{st}(b\cup B)\backslash(\mathbf{st}(b)\cup B).$
[*D3*; *D2*]
T5. $[AB]:\rightharpoondown\{B\}.\,A\,\varepsilon\,\mathbf{Kl}(B).\supset.A\,\varepsilon\, B.$
$[AB]:\mathrm{Hyp}(2).\supset.$
$[\exists C].$
3) $C\,\varepsilon\, B.$ [*M10*; 2]
4) $B\,\varepsilon\, C.$ [3; 1]
5) $B=\mathbf{Kl}(B).$ [*M12*; 4]
$A\,\varepsilon\, B.$ [5, 2]
T6. $[ABb]:\rightharpoondown\{B\}.A\,\varepsilon\,\mathbf{st}(b\cup B)\backslash(\mathbf{st}(b)\cup B).\supset.[\exists C].\,C\,\varepsilon\,\Phi\langle bB\rangle(A).$
$C\,\varepsilon\,\Phi(bB).$

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$[AB]::\mathrm{Hyp}(2).\supset.\because$

$[\exists d].\because$

3) $d\subset b.$

4) $A\,\varepsilon\,\mathbf{Kl}(d\cup B):$ [M18; 2]

5) $d\circ\Lambda.\supset.A\,\varepsilon\,B:$ [T5; 1; 4]

6) $\sim(A\,\varepsilon\,B).$ [2]

7) $\sim(d\circ\Lambda):$ [5; 6]

8) $[D]:D\,\varepsilon\,d.\supset.D\,\varepsilon\,\mathbf{el}(A):$ [DM1; 4]

9) $d\subset\varphi(bA).$ [D1; 3; 8]

10) $d\cup B\subset\varphi(bA)\cup B.\because$ [9]

11) $A\,\varepsilon\,\mathbf{el}(\mathbf{Kl}(\varphi(bA)\cup B)):$ [M23; 4; 10]

12) $[D]:D\,\varepsilon\,B.\supset.D\,\varepsilon\,\mathbf{el}(A):$ [DM1; 4]

13) $\varphi(bA)\cup B\subset\mathbf{el}(A).$ [D1; 12]

14) $\mathbf{Kl}(\varphi(bA)\cup B)\,\varepsilon\,\mathbf{Kl}(\varphi(bA)\cup B).$ [M4; 11]

15) $\mathbf{Kl}(\varphi(bA)\cup B)\,\varepsilon\,\mathbf{el}(A).$ [M21; 13; 14]

16) $A\,\varepsilon\,\mathbf{Kl}(\varphi(bA)\cup B).$ [M2; 11; 15]

17) $[\exists D].D\,\varepsilon\,\varphi(bA).$ [7; 9]

18) $\mathbf{Kl}(\varphi(bA))\,\varepsilon\,\mathbf{Kl}(\varphi(bA)).$ [M9; 17]

19) $\mathbf{Kl}(\varphi(bA))\,\varepsilon\,\Phi\langle bB\rangle(A).$ [D2; 18; 16; 2]

$[\exists C].C\,\varepsilon\,\Phi\langle bB\rangle(A).C\,\varepsilon\,\Phi(bB).$ [19; T2]

T7. $[ABCDb]:C\,\varepsilon\,\Phi\langle bB\rangle(A).D\,\varepsilon\,\Phi\langle bB\rangle(A).\supset.C=D.$ [D2; M5]

The following thesis occurs in [2]

T8. $[ab].\mathbf{Kl}(a\cup b)\circ\mathbf{Kl}(\mathbf{Kl}(a)\cup b).$

T9. $[ABCDb]:C\,\varepsilon\,\Phi\langle bB\rangle(A).C\,\varepsilon\,\Phi\langle bB\rangle(D).\supset.A=D.$

$[ABCDb]:\mathrm{Hyp}(2).\supset.$

3) $C\,\varepsilon\,\mathbf{Kl}(\varphi(bA)).$

4) $A\,\varepsilon\,\mathbf{Kl}(\varphi(bA)\cup B).$ [D2; 1]

5) $C\,\varepsilon\,\mathbf{Kl}(\varphi(bD)).$

6) $D\,\varepsilon\,\mathbf{Kl}(\varphi(bD)\cup B).$ [D2; 2]

7) $\mathbf{Kl}(\varphi(bA))=\mathbf{Kl}(\varphi(bD)).$ [M5; 3; M5; 5]

8) $A\,\varepsilon\,\mathbf{Kl}(\mathbf{Kl}(\varphi(bA))\cup B).$ [T8; 4]

9) $D\,\varepsilon\,\mathbf{Kl}(\mathbf{Kl}(\varphi(bD))\cup B).$ [T8; 6]

10) $D\,\varepsilon\,\mathbf{Kl}(\mathbf{Kl}(\varphi(bA))\cup B).$ [9; 7]

$A=D.$ [M5; 8; 10]

T10. $[bB]:-\{B\}.\supset.\Phi(bB)\infty\,\mathbf{st}(b\cup B)\setminus(\mathbf{st}(b)\cup B).$ [T4; T6; T7; T9]

T11. $[bB]:-\{B\}.\supset.\mathbf{st}(b\cup B)\setminus(\mathbf{st}(b)\cup B)\overset{\propto}{\infty}\mathbf{st}(b).$ [T10; T3]

T12. $[bB]:-\{B\}.\mathrm{Fin}\{\mathbf{st}(b)\}.\supset.\mathrm{Fin}\{\mathbf{st}(b)\cup B\}.$

$[bB]:\mathrm{Hyp}(2).\supset.$

3) $\mathrm{Fin}\{\mathbf{st}(b\cup B)\setminus(\mathbf{st}(b)\cup B)\}.$ [F3; T11, 1; 2]

4) $\mathrm{Fin}\{B\}.$ [F1; 1]

5) $\mathrm{Fin}\{\mathbf{st}(b)\cup B\}.$ [F2; 2; 4]

$\mathrm{Fin}\{\mathbf{st}(b\cup B)\}.$ [F2; 3; 5; M14; M15]

D4. $[ab]:b\subset a.\mathrm{Fin}\{\mathbf{st}(b)\}.\equiv.\Phi\text{-}(a)\text{-}\{b\}.$

T13. $\mathbf{st}(\Lambda)\circ\Lambda.$ [M19; M10]

T14. $[a].\Phi\text{-}(a)\text{-}\{\Lambda\}.$ [D4; T13; F1]

T15. $[abB]: \Phi\{a\}\{b\}. B\, \varepsilon\, a .\supset. \Phi\{a\}\{b \cup B\}.$

$[abB]$: Hyp(2) $.\supset.$

3) $\mathrm{Fin}\{\mathbf{st}(b)\}.$ [D4; 1]

4) $b \subset a.$

5) $\mathrm{Fin}\{\mathbf{st}(b \cup B)\}.$ [T12; 2; 3]

6) $b \cup B \subset a.$ [4; 2]

$\Phi\{a\}\{b \cup B\}.$ [D4; 6; 5]

T16. $[a]. \mathrm{InR}\langle \Phi\{a\}\rangle \{a\}.$ [DF1; T14; T15]

T17. $[a]: \mathrm{Fin}\{a\} .\supset. \mathrm{Fin}\{\mathbf{st}(a)\}.$

$[a]$: Hyp . (1) $.\supset.$

2) $\Phi\{a\}\{a\}.$ [DF2; 1; T16]

$\mathrm{Fin}\{\mathbf{st}(a)\}.$ [D4; 2]

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