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CONCERNING AN ALLEGED SHEFFER FUNCTION

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Since Sheffer's discovery of functionally complete binary connectives for classical 2-valued logic over a half century ago, a number of results concerning Sheffer connectives have been obtained. Of these, we mention only Webb's *binary* Sheffer functions for classical n-valued logic and Martin's extensions of this result, Salomma's work on Sheffer connectives for infinitely-many-valued logics, McKinsey's indigenous binary Sheffer functions for Łukasiewicz-Tarski n-valued C-N logics, Massey's binary Sheffer connectives for S5, Hendry and Massey's positive solution to the Sheffer-spectrum problem as a corollary of a theorem of Post's, Hendry and Massey's simplified Sheffer functions for the Łukasiewicz-Tarski n-valued C-N logics when $n + 1$ is not divisible by 3, Massey's binary Sheffer functions for n-valued S5, and Massey's binary Sheffer connectives for S4. All but the last two of these results are mentioned, discussed, or reported in Hendry and Massey [1]; the penultimate result appeared in [2] while the last appeared in this journal [3].

To anyone familiar with the aforementioned literature on Sheffer functions, the recent claim of Wesselkamper [4] that the *ternary* function S is functionally complete for classical n-valued logic, $n \geqslant 2$, would seem trivial even if true. Unfortunately it is not even true. The semantics of Wesselkamper's connective runs as follows: for the values x, y, z of 'p', 'q', 'r' respectively, the value of '$S(p, q, r)$' is z or x according as $x = y$ or $x \neq y$. Let $\{1, \ldots, n\}$ be the set of truth values. We say that a truth-value i is a fixed point for a connective $\otimes$ just in case, where ϕ is any wff containing no connectives other than $\otimes$, the value of ϕ is i whenever i is assigned to each variable of ϕ. No connective that has even one fixed point can be a Sheffer function for classical n-valued logic, much less one that, like Wesselkamper's S, has n fixed points. Where, then, does Wesselkamper go wrong in his "proof" of the alleged functional completeness of S? In his *definientia*, he makes use of truth-value constants (such as '**T**' and '**F**' for the 2-valued case) without bothering to show that one can define these constants in terms

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of S alone—no such constants can be defined because of the fixed points of S. Nor can Łukasiewicz's equivalence connective 'E' be defined in terms of S, as Wesselkamper claims, because $E(2, 2) = 1$.

REFERENCES

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