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THE LOGIC OF INTENDING AND BELIEVING

PETER A. FACIONE

The purpose here is to explore the conceptual relationships between a person's intending something and his believing something. In particular I wish to try to answer the question: "Is believing that something is possible a necessary condition for intending to do it?"

Before the subtle intracacies and ambiguities of our English mode of expression throw us too far off the track, let us adopt some alternative symbolism to express what we have to say about believing and intending. Let the triadic function,

$$\mathsf{I}^t(Ap)$$

abbreviate 'A intends at time t that p'. Let the triadic function,

$$\mathsf{T}^t(Ap)$$

abbreviate 'A believes at time t that p'.[1]

In a moment we shall see that the dating variable, $\ulcorner t \urcorner$, is indispensable. The variable $\ulcorner A \urcorner$ ranges over human beings and whatever else might be found to be capable of intending and believing. The $\ulcorner p \urcorner$ is a standard propositional variable.[2] If the values of $\ulcorner p \urcorner$ are non-temporal states of affairs, as for example, $6 + 2 = 8$ or *whoever sells books sells something*, then $\ulcorner \mathsf{I}^t(Ap) \urcorner$ would be false. $\ulcorner \mathsf{T}^t(Ap) \urcorner$ might, of course, be true. The

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1. This formalism brings with itself a host of ontological and epistemological puzzles. I adopt it for its clarity and because it shows the usefulness of certain formal techniques.

2. Propositional variables here range over propositions, not sentence-types that may be used to express statements. In this essay I am assuming that a proposition is a state of affairs. Inconsistent propositions are impossible states of affairs, logically true propositions are necessary states of affairs. The ontological hypotheses used here are, in my estimation, rather interesting. They contribute to the puzzles mentioned in note 1.

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propositions that might make $\ulcorner \mathsf{I}^t(Ap) \urcorner$ true involve either an explicit reference to some time, as in *I shall play golf tomorrow afternoon*, or a bound time variable, as in *I will own this place someday.* The question, then, is whether or not

(1) $(\mathsf{T}^t(A \Diamond p) \circledN \mathsf{I}^t(A\ p))$

is true. (1) asserts that A's believing at time t that p is possible is a necessary condition for his intending p at t. In modal system S5 the claim is equivalent to

$$\Box(\mathsf{I}^t(A\ p) \supset \mathsf{T}^t(A \Diamond p)).^3$$

There is a prima facie case for (1). When children try to do things that they cannot do, adults, intending to have the children stop, tell the children that they are not physically able to do what they intend to do. The adults are trying to make them aware that they are intending what is impossible for them. The assumption is that when the children realize this they will stop trying to do what they cannot do. Practical and reasonable folks just do not intend to do what they know they cannot do. When someone says that he intends to do something which others think is impossible for him they tell him that he is being unreasonable. If he says that he intends to do what is impossible for everyone, they would say that he is a fool. These rather commonplace practices indicate that there is a conceptual connection between what someone intends and what he does believe, or should believe, to be possible. In the usual course of life we accept the truth of (1).

These considerations do not, however, constitute a proof of (1). We might try to argue for (1) by reductio. Assume:

$$\sim\Box(\mathsf{I}^t(Ap) \supset \mathsf{T}^t(A \Diamond p)).$$

By truth-tabular manipulations it follows (in S5) that

(2) $\Diamond(\mathsf{I}^t(Ap)\ \&\ \sim\mathsf{T}^t(A \Diamond p))$.

Unfortunately it does not appear that (2) is inconsistent. Given the usual association of consistency with conceivability it would seem that someone could produce a case that satisfies (2). Cannot we, for example, say that many of us intend that we should never die but do not believe that this is possible?[4] Someone must have undertaken to do something which he did not believe was possible. Such a practice is surely not reserved of necessity for the insane. Think, for example, of people who leap from burning buildings in order to escape the flames. Do all of them believe that it is possible that they will not die from the fall?

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3. See my "A modal truth-tabular interpretation for necessary and sufficient conditions," *Notre Dame Journal of Formal Logic*, vol. XIII (1972), pp. 270-272. Because of the results there (1) entails, but is not equivalent to, $\ulcorner(\mathsf{I}^t(Ap) \supset \mathsf{T}^t(A \Diamond p))\urcorner$.

4. Cases like this are suggested in Carl G. Hedman's "Intending the impossible," *Philosophy*, vol. 45 (1970), pp. 33-38.

With these extreme cases in mind we should reject the prima facie case for (1). On the other hand, extreme cases should be interpreted with extreme care. Sometimes it is a mistake to make too much of the use of a specific word. Maybe we should have said "many of us do not *want* to die" or "many of us do not *wish* to die" instead of using 'intend'. Maybe people who leap from burning buildings do not intend to save their lives as much as they intend to avoid being consumed by the flames. But, such quibbling obscures the issue. (2), a not so very strong claim, is not obviously inconsistent. Thus, the reductio proof of (1) fails.

Perhaps we can argue directly for (1). Doing this will require the introduction of certain epistemological assumptions. I shall express these as laws or axioms in the system of epistemic logic that I develop out of the conceptual analysis of $\ulcorner \mathsf{I}^t(Ap) \urcorner$ and $\ulcorner \mathsf{T}^t(Ap) \urcorner$. (Of course, the system will not be fully-fledged; here it is only a vehicle for the consideration of the truth of (1). Given the chance, I would be inclined to include (1) as an axiom of a full system.)

Since $\ulcorner \mathsf{I}^t(Ap) \urcorner$ is false unless the values of $\ulcorner p \urcorner$ are temporal, we can introduce dated propositional variables into $\ulcorner \mathsf{I}^t(Ap) \urcorner$. Our first relevant epistemological truism is:

(3) $\mathsf{I}^t(Ap^{t'}) \supset t \leqslant t'$.

That is, any state of affairs that A intends does not temporally precede A's intending of it. Putting this in terms of more idiomatic infinitive constructions we have: if someone intends to do something, then the time of his intending is earlier or the same as the time at which he intends to act. Another epistemological axiom which we shall rely on is:

(4) $\mathsf{T}^t(A \sim p) \supset \sim \mathsf{T}^t(Ap)$.

That is, if A believes that it is not the case that p, then he does not, at the same time, believe that p. The converse of (4) is false, however, because it rules out ignorance, indifference, and uncertainty with respect to p at a given time.

To abbreviate 'A brings it about at time t that p' let us use the triadic function:

$$\mathsf{B}^t(Ap).$$

Now we have a better way of expressing A's intention to do something. Instead of $\ulcorner \mathsf{I}^t(Ap^{t'}) \urcorner$ we can now write the more subtle:

(5) $\mathsf{I}^t(A\mathsf{B}^{t'}(Aq))$,

where $\ulcorner p^{t'} \urcorner$ is replaced by $\ulcorner \mathsf{B}^{t'}(Aq) \urcorner$. By applying (5) to (3) we get an analogue of law (3).

$$\mathsf{I}^t(A\mathsf{B}^{t'}(Aq)) \supset t \leqslant t'.$$

Saying that $\ulcorner \mathsf{B}^t(Ap) \urcorner$ is true amounts to saying that A did something at time t which, in the normal course of things, would cause p. It would seem, then, that

(6) $\mathbf{B}^t(Ap) \supset p$

should be accepted as a law. But (6) is true only if the missing temporal element in $\ulcorner p \urcorner$ is identical to t. That is

(7) $\mathbf{B}^t(Ap^t) \supset p^t$

is the law. If we replace $\ulcorner p^t \urcorner$ in (7) with $\ulcorner p^{t'} \urcorner$, creating (7)′, (7)′ would be false. Events might occur "in the normal course of things" which A does not foresee, thus rendering his action at time t insufficient to bring about p at t'. Nevertheless, we can say that A did do something at time t and we can also say that A believed at that time that what he did would be sufficient to cause p at t' given the normal course of things. I believe that we can adopt the law:

(8) $\mathbf{B}^t(Ap^{t'}) \equiv (\exists r^t)(\mathbf{B}^t(Ar^t) \;\&\; \mathbf{T}^t(\mathbf{B}^t(Ar^t) \circledS p^{t'}))$.[5]

Thus armed let us see what follows from the assumptions involved in A's intending to do something, *viz.*

$$\mathbf{I}^t(Ap^{t'}) \text{ and } p^{t'} = \mathbf{B}^{t'}(Aq^{t''}).$$

We can express these assumptions as,

$$\mathbf{I}^t(A\mathbf{B}^{t'}(Aq^{t''})).$$

By applying (8) we can derive:

(9) $\mathbf{I}^t(A(\exists r^{t'})(\mathbf{B}^{t'}(Ar^{t'}) \;\&\; \mathbf{T}^{t'}(\mathbf{B}^{t'}(Ar^{t'}) \circledS q^{t''})))$.

This step reveals that I am making the further assumption:

$$(\mathbf{I}^t(Ap) \;\&\; \mathbf{T}^t(A(p \supset q))) \supset \mathbf{I}^t(Aq).$$

Perhaps this assumption is false, or perhaps A does not believe that (8) is true; in either case (9) does not follow. On the other hand, a reasonable A would accept my further assumption and would know something like (8) since neither of these are very imposing when taken from their formal expression back to the intuitions whence they sprang. Our worry should rather be that the assumption that A is a reasonable fellow may have brought in the truth of (1) by the back door. Let us see. Applying (7) to $\ulcorner \mathbf{B}^{t'}(Ar^{t'}) \urcorner$ in (9) we have:

(10) $\mathbf{I}^t(A(\exists r^{t'})(r^{t'} \;\&\; \mathbf{T}^{t'}(r^{t'} \circledS q^{t''})))$.

Thus, if A intends to do something, and if he is reasonable, then he intends that there be some state of affairs such that it occur and he believes that its occurrence is a sufficient condition for the occurrence of what he intended to accomplish.

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5. Although I have only argued for a conditional, I believe that the biconditional is acceptable. The converse of the conditional I supported seems to be obviously true. This discussion, indeed this whole essay, is in the spirit of the masterful work of the late Henry S. Leonard, "Authorship and purpose," *Philosophy of Science*, vol. 26 (1959), pp. 277-294.

This demonstration indicates that some conceptual relationship exists between intending and believing. The relationship discovered does not, however, satisfy (1). Moreover, it is not evident how one could deduce either

$$(\mathbf{I}^{t}(Ap^{t'}) \circledN \mathbf{T}^{t}(A \Diamond p^{t'}))$$

or

$$(\mathbf{I}^{t}(Ap^{t'}) \circledN \mathbf{T}^{t}(A \Diamond \mathbf{B}^{t'}(Aq^{t''})))$$

from the assumptions $\ulcorner \mathbf{I}^{t}(Ap^{t'})\urcorner$ and $\ulcorner p^{t'} = \mathbf{B}^{t'}(Aq^{t''})\urcorner$ without at least assuming that A is reasonable. Even then the deduction demands more than the simple, intuitive, epistemic system outlined here provides for.

Given that both direct and indirect proof have been inadequate we must step back from (1). It is premature to claim that (1) is false, for that has not been demonstrated either. Even if we cannot have (1) there must be something which justifies our practice of relating a person's beliefs and intentions as was done in the prima facie case above. Let us make one more attempt to elucidate it by considering:

(11) $(\mathbf{I}^{t}(Ap) \ \& \ \sim \mathbf{T}^{t}(A \sim \Diamond p))$.

Suppose that (11) were false. It would follow, then, by truth-tabular manipulations that

(12) $(\mathbf{I}^{t}(Ap) \supset \mathbf{T}^{t}(A \sim \Diamond p))$.

But (12) is surely false. We all can think of cases where someone intended to do something which he did not believe impossible. It follows by reductio, then, that (11) is true. From (11) by truth-tabular maniuplations we can derive:

(13) $(\mathbf{I}^{t}(Ap) \supset \sim \mathbf{T}^{t}(A \sim \Diamond p))$.

Applying law (4) to (13) we get:

(14) $(\mathbf{I}^{t}(Ap) \supset \mathbf{T}^{t}(A \Diamond p))$.

That is, if someone intends that p, then he believes that it is possible that p.

The logic developed here will not allow a decision about (1). We can, however, say that one intends something only if one believes it is possible. Whether or not this belief is a necessary condition for this intention is still an open question.

Bowling Green State University
Bowling Green, Ohio