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# NICE IMPLICATIONAL AXIOMS

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It might seem unlikely at this date that a new and interesting three-axiom set for classical implication would be found. However I do not remember in the literature the set {1, 2, 3} below. In number of axioms and basic implicational structure it is identical with Tarski's {1, 14, 7} which in some sense strengthens 2 and weakens 3; the variable occurrences are 7 *p*, 4 *q*, 4 *r*, against Tarski's 6 *p*, 4 *q*, 5 *r*. The conspicuous merit of {1, 2, 3} is the ease with which all the most famous and commonly named propositions can be developed; we have a minimum of material which is of merely local or contextual necessity and interest. For a discussion of axiomatics I know of no other set which assembles so much needed material in such short order. Witness the following:

1. *CpCqp* (Simp)
2. *CCqrCCpqCpr* (Weak Syll)
3. *CCCpqrCCrpp* (Roll)

D21 = 4. *CCqrCqCpr* (A Fortiori)
DD243 = 5. *CCCpqrCCrpCsp* (Łukasiewicz)
D23 = 6. *CCsCCpqrCsCCrpp*
D63 = 7. *CCCpqrCCprr* (Tarski)
D61 = 8. *CpCCpqq* (Aff or Pon)
DD228 = 9. *CqCCpCqrCpr* (Comm-Comm)
DD999 = 10. *CCpCqrCqCpr* (Comm)
DD10.1.ņ = 11. *Cpp* (Id)
D3.11 = 12. *CCCpqpp* (Peirce)
D3.12 = 13. *CCpCpqCpq* (Hilbert)
D10.2 = 14. *CCpqCCqrCpr* (Syll)
D10DD14.2.14 = 15. *CCCprsCCqrCCpqs*
D73 = 16. *CCpCqCprCqCpr*
DD15.16.9 = 17. *CCpqCCpCqrCpr* (Comm-Frege)
D10.17 = 18. *CCpCqrCCpqCpr* (Frege)
D14.1 = 19. *CCpqrCqr* (Syll-Simp)
D14.19 = 20. *CCCqrsCCCpqrs*

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D20.12 = 21. *CCCrCpqpp*
D4D20.17 = 22. *CCCpqrCsCCqCrtCqt* (Meredith)
D10.7 = 23. *CCpqCCCprqq* (Comm-Tarski)
DD2.23.19 = 24. *CCCpqrCCCqprr* (Dummett)

We have, among other possibilities, $Ax_C = \{1, 2, 3\} = \{1, 14, 7\}$ (Tarski) = $\{5\}$ (Łukasiewicz) = $\{1, 14, 12\}$ (Bernays) = $\{7, 19, 15s/r\}$ (Łukasiewicz) = $\{12, 14, CpC\alpha\beta\}$ (Łukasiewicz) = $\{3, 4\}$ (Meredith) = $\{2, 21\}$ (Meredith) = $\{14, 21\}$ (Meredith) = $\{2, 8, 12\}$ (Wajsberg), etc.

$Ax_{PosC} = \{22\}$ (Meredith) = $\{1, 17\}$ (Meredith) = $\{1, 18\}$ (Frege) = $\{1, 13, 14\}$ (Hilbert), etc.

$Ax_{LC} = \{PosC, 24\}$ (Dummett).

The Wajsberg set $\{2, 8, 12\}$ obviously requires comparison with $\{1, 2, 3\}$. Only one axiom uses three variables; while 3 is simplified, 1 is replaced with a more complex proposition; development along the foregoing lines is a little heavier.

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