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ON TWO IMMEDIATE INFERENCES BY LIMITATION

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In a situation where one is trying to determine the truth or falsity of a categorical proposition A in relation to a categorical proposition B which is given as true, it seems plausible to adjust A by means of conversion, obversion, and contraposition to some proposition C which has the same subject and predicate terms as B, and then to decide the truth or falsity of C by immediate inference according to the traditional square of opposition, and finally to decide A on the basis of the determination of C. Consider the following examples:

I. **A**: All S is P. **B**: Some S is non-P.
(1) All S is P.
(2) No S is non-P. (obverse of (1))

(2) is false if **B** is true; hence, (1), i.e., **A**, is false if **B** is true.

II. **A**: All S is P. **B**: Some S is non-P.
(1) All S is P.
(2) Some P is S. (converse (by limitation) of (1))
(3) Some S is P. (converse of (2))
(4) Some S is not non-P. (obverse of (3))

The truth-value of (4) is undetermined in relation to **B**, and it would seem that **A**'s truth-value is likewise undetermined.

III. **A**: No S is non-P. **B**: Some S is not P.
(1) No S is non-P.
(2) All S is P. (obverse of (1))

Given that **B** is true, (2) is false and so is **A**.

IV. **A**: No S is non-P. **B**: Some S is not P.
(1) No S is non-P.
(2) Some P is not non-S. (contrapositive (by limitation) of (1))
(3) Some P is S. (obverse of (2))
(4) Some S is P. (converse of (3))

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Given that **B** is true, the truth-value of (4) is undetermined and similarly for **A**.

These examples demonstrate that by the application of the rules of immediate inference of traditional logic, as presented in standard introductory texts,[1] one arrives at different answers as to the determinability of the truth-value of a statement **A** in relation to another statement **B**. It is no criticism of these examples to point out that the person who takes the longer routes in II and IV (as opposed to I and III) is dull and lacking in perspicacity, for different applications of the rules should give the same result. The person who divides 100 into 150 by long division should expect to get the same quotient as his friend who merely moves the decimal place of the dividend two places to the left.

Where **A**, **B**, and **C** represent standard form categorical propositions and where the truth-value of **B** is given, I have assumed the following:

(i) The truth-value of **A** is determinable in relation to **B** if and only if some proposition **C** derived from **A** on the basis of the traditional rules of immediate inference is determinable in relation to **B**.

Implicit throughout the discussion is the following rule:

(*) If the truth-value of **A** is determinable in relation to **B**, then **A** is true in relation to **B** if and only if **C** is true in relation to **B**.

With (i) and (*) at hand it is easy to see that by examples I and II (or in III and IV) one can derive a contradiction. The truth-value of **A** in relation to **B** is both determinable and undeterminable.

Perhaps we ought to forsake (i) for

(ii) The truth-value of **A** is determinable in relation to **B** if some proposition **C** derived from **A** on the basis of the traditional rules of immediate inference is determinable in relation to **B**.

This will save us from contradiction, for if, as in II and IV, some proposition **C** is undeterminable in relation to **B**, we cannot infer that **A** is also undeterminable. But (ii) raises another problem. Even if we were to know that all formulas **C** (other than **A** itself) derivable from **A** were undeterminable in relation to **B**, we would not be warranted in concluding that **A** is likewise undeterminable. Also, consider the following, where **B** is given as true:

V. **A**: Some S is non-P. **B**: Some non-P is not S.
(1) Some S is non-P.
(2) Some non-P is S. (converse of (1))

We would like to say that **A** is undeterminable in relation to **B** on the basis of this derivation alone, but on the basis of (ii) we are not so allowed.

1. See, for instance, Irving M. Copi, *Introduction to Logic* (4th ed.; New York: Macmillan Company, 1972), pp. 159-165.

But the problems of (ii) have just begun. Consider the following examples, where **B** is given as false:

VI. **A**: No *S* is non-*P*. **B**: No *S* is *P*.
(1) No *S* is non-*P*.
(2) All *S* is *P*. (obverse of (1))

The truth-value of (2) is undetermined in relation to **B**, and by (ii) we are unable to infer that **A** is undetermined too.

VII. **A**: No *S* is non-*P*. **B**: No *S* is *P*.
(1) No *S* is non-*P*.
(2) Some *P* is non-*S*. (contrapositive (by limitation) of (1))
(3) Some *P* is *S*. (obverse of (2))
(4) Some *S* is *P*. (converse of (3))

(4) is true if **B** is false, therefore, by (ii) and (*) **A** is true.

VIII. **A**: All *S* is non-*P*. **B**: All *S* is *P*.
(1) All *S* is non-*P*.
(2) No *S* is *P*. (obverse of (1))

The truth-value of (2) is undetermined in relation to **B**, yet we cannot conclude by (ii) that the truth-value of **A** is undetermined.

IX. **A**: All *S* is non-*P*. **B**: All *S* is *P*.
(1) All *S* is non-*P*.
(2) Some non-*P* is *S*. (converse (by limitation) of (1))
(3) Some *S* is non-*P*. (converse of (2))
(4) Some *S* is not *P*. (obverse of (3))

Since (4) is the contradictory of **B**, (4) is true and by (ii) **A** is determinable and is true.

On the basis of (ii) we concluded that in VII and IX **A** is determinable in relation to **B** and is in fact true. But clearly we wish to hold that in both cases **A** is undeterminable in relation to **B**. The problem now appears to be not the faulty formulation of (ii), but the application of conversion (by limitation) and contraposition (by limitation). Look what has happened in both VII and IX. Using immediate inferences by limitation, we have transformed statements which are logically equivalent to universal statements (which are undecidable in relation to **B**) into particular statements that are decidable. (In II and IV we transformed statements which are logically equivalent to universal statements (which are decidable in relation to **B**) into particular statements that are undecidable.)

But perhaps it is not the inferences by limitation that cause us the trouble. Our problem may be with the rule (*). If **C** is derived from **A** on the basis of one of the rules of inference by limitation, then the biconditional, '$\mathbf{A} \equiv \mathbf{C}$', is not valid, but only the conditional, '$\mathbf{A} \supset \mathbf{C}$'. But rule (*), in its assumption of a biconditional relationship between **A** and **C**, in effect leads us into the fallacy of affirming the consequent—as in IX, (4), i.e., **C** is true; therefore, (1), i.e., **A**, is true. Suppose we alter rule (*) to the following:

(**) If A is determinable in relation to B, then if C is false in relation to B, then A is false in relation to B.

But it is obvious that objections to (**) will be forthcoming, for (**) in conjunction with (ii) blocks certain desired inferences—namely, in those cases where '$A \equiv C$' is valid.

It now appears that the quest for some rule (or rules) that is applicable to all forms of immediate inference is doomed to failure. What we can provide, however, is a set of rules for all immediate inferences, certain cases of inferences by limitation excepted. By rule (iii) we restrict the application of inferences by limitation:

(iii) If either B is universal and false or B is particular and true, then the immediate inferences by limitation are prohibited in the derivation of C from A; and vice versa.

The inferences which are not prohibited are fully governed by (i) and (*). (We need not worry here about the problems surrounding (**), for in those cases where inferences by limitation are allowed the particular categorical proposition derived (i.e., C) is true just in case the universal proposition of same quality is true (i.e., a proposition logically equivalent to A).)

The inferences prohibited in (iii) do not occasionally vitiate this method of determining the truth-value of A in relation to B; they always do. For instance, suppose that B is any I-proposition and is given as true. Moreover, suppose A is logically equivalent to an E-proposition with the same subject and predicate terms as B. A is false in relation to B; but by applying the rules of inference by limitation to A we will derive ultimately an O-proposition C (with same subject and predicate as B) which is undeterminable in relation to B. Thus A which is determinable and false in relation to B gives rise to a formula C which is undeterminable.

The following chart indicates the relationships which exist when inferences by limitation are applied to prohibited cases:

B	A(≡)	C
I-p & T	A-p & U	I-p & T
I-p & T	E-p & F	O-p & U
O-p & T	A-p & F	I-p & U
O-p & T	E-p & U	O-p & T
A-p & F	A-p & F	I-p & U
A-p & F	E-p & U	O-p & T
E-p & F	A-p & U	I-p & T
E-p & F	E-p & F	O-p & U

The first column indicates the categorical form of B and B's truth-value. The middle column indicates what categorical form (with subject and predicate as in B) A is logically equivalent to and, thus, what the truth-value of A is in relation to B, if determinable; if not, 'U' for undeterminable. The third column indicates the categorical form (with subject and predicate as in **B**) and truth-value or undeterminability of the proposition **C** derived from **A** by means of the inferences by limitation.

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