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SHORTER DEVELOPMENT OF AN AXIOM

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In [1] Łukasiewicz obtained the Bernays axioms for classical implication from his shortest sole axiom *CCCpqrCCrpCsp* with thirty-four detachments of which five were trivial. In [2] C. A. Meredith reduced those figures to thirty-three and four. If we take the sufficient Meredith pair, *CCCpqrCCrpp* and *CCqrCqCpr*, as the goal, they can be obtained with twenty-seven detachments of which five are trivial. The following theses are obtainable (see [1] or less explicitly [2], in any case Meredith's improvement is not here relevant) with twenty-two detachments of which four are trivial:

1. *CCCrpCpqCsCpq*
2. *CCCpqrCqr*
3. *CCCpqCrsCCpsCrs*
4. *CCCCCpqrtCspCCrpCsp*
5. *CCCCrpCspCCCpqrtCuCCCpqrt*
6. *CCCCpqrCsqCCCqtsCpq*

Thereafter we have:

7. *CCpCpqCrCpq* = **D41**
*8. *CCCpqrCCrpp* = **DD57n**
9. *CCCCprsCqrCqCpr* = **D63**
*10. *CCqrCqCpr* = **D29**

REFERENCES

[1] Łukasiewicz, J., "The shortest axiom of the implicational calculus of propositions," *Proceedings of the Royal Irish Academy*, Sect. A, 52 (1948), pp. 25-33.

[2] Meredith, C. A. and A. N. Prior, "Notes on the axiomatics of the propositional calculus," *Notre Dame Journal of Formal Logic*, vol. IV (1963), pp. 171-187.

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