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A FORMALIZATION OF "NOTHING"

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Unlike most contemporary analytic philosophers Anselm took seriously the idea that the word "nothing" may function, not only as a syncategorematic quantifier, but also as a *nominal* expression. Thus in *Monologion*[1] 19, he is concerned lest his assertion that nothing existed before God be taken to imply that some thing, namely that which is referred to by the word "nothing", existed before God. In *De Casu Diaboli*[2] 11 Anselm provides a comprehensive discussion of the question of whether "nothing" may function in this nominal way. He reaches the conclusion that in one of its senses the word "nothing" refers to a state of affairs—the state consisting of the absence of every thing.

Desmond P. Henry, in his book *The Logic of St. Anselm*,[3] gives an interesting interpretation of Anselm's remarks on the nominal "nothing" in *De Casu Diaboli* 11. I shall devote this paper to a discussion of Henry's view; first, expounding it; second, criticizing it; and third, suggesting a way to improve it.

Let us assume, with Henry, that there is a use of the word "nothing" which is not susceptible to analysis in terms of quantifiers, and furthermore that this use is exemplified in Anselm's statements: "Nothing is nothing", "It is not true that nothing is something", and "Nothing is not-something". According to Henry there is a counterpart to this nominal use of "nothing" in Leśniewski's Ontology. He holds that the symbol "$\wedge$" of Ontology is to be interpreted as "nothing" in just this nominal sense, and that this possibility of finding a logic in which there is a symbol which is interpreted as the nominal "nothing" provides a means of endowing Anselm's discourse with sense. We shall now examine these claims.

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1. Translated by S. N. Deane, Open Count Pub. Co., La Salle, Ill. (1903).

2. In *Truth, Freedom and Evil; Three Philosophical Dialogues*. Edited and translated by Jasper Hopkins and Herbert Richardson, Harper and Row, New York (1967).

3. D. P. Henry, *The Logic of St. Anselm*, Clarendon Press, Oxford (1967).

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The single axiom of Ontology is:

(1): $(a)(b)[a\,\varepsilon\, b \leftrightarrow. (\exists c)(c\,\varepsilon\, a)\ \&\ (c)(c\,\varepsilon\, a \rightarrow c\,\varepsilon\, b)\ \&\ (c)(d)(c\,\varepsilon\, a\ \&\ d\,\varepsilon\, a \rightarrow c\,\varepsilon\, d)]$

The variables range over all nominal expressions such as "Socrates", "the author of Waverly", "a human being", "a red thing", "Pegasus" and so on, whether they be singular, general, or empty. The symbol "ε", the only primitive of Ontology except for the universal quantifier, should be read in English as the word "is".

The formalizations in Ontology of Anselm's statements containing the nominal "nothing" involve symbols which are introduced in the following implicit definitions:

(2): $(a)[a\,\varepsilon\,\wedge . \leftrightarrow . (a\,\varepsilon\, a)\ \&\sim(a\,\varepsilon\, a)]$

(For all a, a is nothing iff a is a and it is false that a is a)

(3): $(a)[a\,\varepsilon\,\vee . \leftrightarrow . a\,\varepsilon\, a]$

(For all a, a is something iff a is a)

(4): $(a)(b)[a \subset b . \leftrightarrow . (c)(c\,\varepsilon\, a \rightarrow c\,\varepsilon\, b)]$

(For all a and b, all a is b iff, for all c, if c is a then c is b)

(5): $(a)(b)[a\,\varepsilon \sim b . \leftrightarrow . (a\,\varepsilon\, a)\ \&\ \sim(a\,\varepsilon\, b)]$

(For all a and b, a is non-b iff a is a and it is false that a is b)

Henry then proceeds to formalize Anselm's statements in terms of these symbols.

(6): $\wedge \subset \wedge$

((All) nothing is nothing)

(7): $\sim(\wedge\,\varepsilon\,\vee)$

(It is false that nothing is something)

(8): $\wedge \subset \sim \vee$

((All) nothing is not-something)

I feel that (6), (7) and (8) are inadequate as translations of Anselm's statements. I shall explain why.

(A) The formulae "$\wedge \subset \wedge$" and "$\wedge \subset \sim \vee$" should be read "All nothing is nothing" and "All nothing is not-something". Therefore they do not capture exactly Anselm's statements: "Nothing is nothing" and "Nothing is not-something". This discrepancy, although it is slight, is important concerning our evaluation of Henry's view, since he claims that it is the possibility of translating Anselm's statements into Ontology which saves us from rejecting them as nonsense. However, it seems that Anselm's statements *as they stand* are not accurately formalized in (6) and (8). Henry admits that there are differences between the "$\wedge$" defined above and the natural language nominal "nothing". But he does not realize that these differences apparently vitiate his intention to rescue Anselm's statements by formalizing them in terms of "$\wedge$".

(B) It seems to me intolerable that two different connectives, "ε" and "$\subset$" are used to represent the word "is" in Henry's formulae. There is no indication that the "is" in "Nothing is nothing" is meant differently by Anselm than the second "is" in "It is false that nothing is something". Why then should they be formalized in different ways? The answer, though not the justification, is simple. Whereas the formulae "$\wedge \subset \wedge$" and "$\sim(\wedge\, \varepsilon \vee)$" are both theses of Ontology, the alternate formalizations are not. Thus if we use "ε" to formalize "Nothing is nothing", we get "$\wedge\, \varepsilon \wedge$", which is easily shown to be logically false, as follows:

1.	$(a)[a\, \varepsilon \wedge . \leftrightarrow . (a\, \varepsilon\, a)\ \&\ \sim(a\, \varepsilon\, a)]$	By Def. (2)
2.	$\wedge\, \varepsilon \wedge \leftrightarrow . (\wedge\, \varepsilon \wedge)\ \&\ \sim(\wedge\, \varepsilon \wedge)$	Put "$\wedge$" for "a" in 1
3.	But $\sim(p\ \&\ \sim p)$	Theorem
4.	$\sim[(\wedge\, \varepsilon \wedge)\ \&\ \sim(\wedge\, \varepsilon \wedge)]$	From 3
5.	$\sim(\wedge\, \varepsilon \wedge)$	QED. From 2, 4, m.t.

Similarly, were we to formalize, "It is false that nothing is something" using "$\subset$" instead of "ε" we would get $\sim(\wedge \subset \vee)$. This, too, is logically false:

1.	$(a)(b)[a \subset b . \leftrightarrow . (c)(c\, \varepsilon\, a \rightarrow c\, \varepsilon\, b)]$	Def. (4)
2.	$\wedge \subset \vee . \leftrightarrow . (c)(c\, \varepsilon \wedge \rightarrow c\, \varepsilon \vee)$	Subt. "$\wedge$" and "$\vee$" for "a" and "b" respectively
3.	$(a)[a\, \varepsilon \wedge . \leftrightarrow . (a\, \varepsilon\, a)\ \&\ \sim(a\, \varepsilon\, a)]$	Def. (2)
4.	$s\, \varepsilon \wedge . \leftrightarrow . (s\, \varepsilon\, s)\ \&\ \sim(s\, \varepsilon\, s)$	Put "s" for "a"
5.	$\sim(s\, \varepsilon \wedge)$	From 4, since RHS is contradictory
6.	$s\, \varepsilon \wedge \rightarrow . s\, \varepsilon \vee$	From 5 (LHS is false)
7.	$(c)(c\, \varepsilon \wedge \rightarrow c\, \varepsilon \vee)$	Generalize 6
8.	$\wedge \subset \vee$ is a thesis	From 2, 7, m.p.
9.	Therefore $\sim(\wedge \subset \vee)$ is logically false.	From 8

We see, therefore, that if "Nothing is nothing" is to be rendered as a true formula of Ontology, the "is" must be represented as "$\subset$". And similarly, the "is" in "It is false that nothing is something" must be represented by "ε". I think that these facts indicate the "ad hoc" nature of Henry's formalizations. In order to commit Anselm's statements to the mold of Ontology, Henry is compelled to a multiple construal of the copula, which is not founded in Anselm's writings. Yet Henry claims that those statements find a natural expression in Ontology.

(C) In *Monologion* 19 Anselm distinguishes two senses of the word "nothing", one of which is explicable in terms of quantifiers, and the other of which is the nominal term presently under consideration. Thus, according to Anselm, the sentence

(9) Nothing taught me to fly.

can be taken either as

(10) Not- (something taught me to fly)

which in ordinary English is

(11) It is false that there is something which taught me to fly.

or alternatively, as

(12) Nothing, as an entity in itself, has taught me to fly.

in which the word "nothing" functions nominally to signify the absence of anything.

The propositions expressed in (11) and (12) can both be expressed in (9). Of these (11) is true as regards Anselm, since he has not been taught to fly; and (12) is false, since it implies that he has been taught how to fly.

It is unfortunate that Henry does not test his claim that the nominal "nothing" corresponds to the "$\wedge$" of Ontology by attempting to apply it to sentence (12). The results do not support his view.

For suppose we symbolize the nominal expression "a thing which taught me to fly" by "**t**". Then, by analogy with Henry's treatment above, we might formalize (12) in either of these two ways:

(13) $\wedge \subset \mathbf{t}$

or

(14) $\wedge\, \varepsilon\, \mathbf{t}$

Now (13) clearly will not do for Henry's purposes, since (13) is true (this is easily proved) whereas (12) is false. (14), on the other hand, is false; in this respect it is satisfactory. It is, however, not only false, but logically false, since it is a thesis of Ontology that $(a){\sim}(\wedge\, \varepsilon\, a)$. It seems to me that (12) is not logically false, but just false as a matter of fact. Therefore I feel that (14) is an inadequate translation of (12). In case there may be some doubt about whether (12) is just false as a matter of fact, I should like to suggest that there are sentences of the same kind as (12) which are more clearly merely contingent. For example:

(15) I want nothing.

As before this may mean:

(16) It is false that there is something which I want.

or

(17) I want nothing—as an entity itself—(i.e., the absence of everything).

I think that (17) is contingent. Most people do not want the absence of everything, but there does not appear to be any logical reason why they should not. Thus, even if (17) were false as regards some people, it would not be logically false. Therefore (17) is not accurately rendered in Ontology by

(18) $\wedge \varepsilon \omega$ (where "ω" stands for "a thing I want")

since (18) *is* logically false.

(D) Henry formalizes Anselm's statement "Nothing is nothing" as "$\wedge \subset \wedge$". This thesis of Ontology is an instance of the universal thesis:

$(a)(\wedge \subset a)$, which can be proved as follows:

1.	$(a)(b)[a \subset b . \leftrightarrow . (c)(c \varepsilon a \rightarrow c \varepsilon b)]$	Def. (4)
2.	$(b)[\wedge \subset b . \leftrightarrow . (c)(c \varepsilon \wedge \rightarrow c \varepsilon b)]$	Subst. $\wedge/a$ in 1
3.	$\wedge \subset s . \leftrightarrow . (c)(c \varepsilon \wedge \rightarrow c \varepsilon s)$	Inst. s/b in 2
4.	$\wedge \subset s . \leftrightarrow . (t \varepsilon \wedge \rightarrow t \varepsilon s)$	Inst. t/c in 3
5.	$(a)[(a \varepsilon \wedge) . \leftrightarrow . (a \varepsilon a) \ \& \sim(a \varepsilon a)]$	Def. (2)
6.	$t \varepsilon \wedge . \leftrightarrow . [t \varepsilon t \ \& \sim(t \varepsilon t)]$	Inst. t/a in 5
7.	$\sim(t \varepsilon \wedge)$	Principle of Contr. (From b, m.t.)
8.	$\wedge \subset s$	From 4 and 7
9.	$(a)(\wedge \subset a)$ Q.E.D.	U.G.

Thus, just as "$\wedge \subset \wedge$" is a thesis, being a particular case of "$(a)(\wedge \subset a)$", so is "$\wedge \subset \mathbf{h}$" a thesis, where "$\mathbf{h}$" stands for the nominal term "a human being". Therefore according to Ontology there is a sense in which both "Nothing is nothing" and "Nothing is a human being" can be formalized as theses. This, it seems to me, detracts from the significance of Anselm's claim that nothing is nothing. For that claim is represented in Ontology in such a way that, not only can nothing be said to be nothing, but also nothing can be said truly to be a human being. Thus, Henry's plan to give sense to Anselm's claim by formalizing it in Ontology, succeeds only at the cost of trivializing it. For it turns out that nothing is nothing only insofar as nothing is a human being, nothing is Aristotle, even nothing is something. I am not claiming that every acceptable formalization of the English sentence "Nothing is human" in Ontology produces a thesis. There are ways to make it come out false, as it should. The point I am making is that the second "$\wedge$" in "$\wedge \subset \wedge$" occurs trivially, since its replacement by any other nominal constant would preserve the logical truth of the formula.

The same objection applies to Henry's version of the third sentence, "Nothing is not something", namely "$\wedge \subset \sim\vee$". This is also merely an instance of the thesis "$(a)(\wedge \subset a)$". It does not significantly relate the terms "$\wedge$" and "$\sim\vee$", since "$\wedge$" is related in the same way to any nominal term.

I do not think that it is Henry's use of Ontology which creates the above difficulties; rather it seems that his particular ways of rendering Anselm's statements are mistaken. Alternate formalizations may be found which are not susceptible to these objections.

Two of the sentences in question appear in the following context:

"For it is not necessary that nothing be something simply because its name somehow signifies something; rather nothing must be nothing, because the word "nothing" signifies something only in the sense we've mentioned" [*my underlining*].[4]

It seems to me that Henry goes wrong when he treats both the words underlined twice and the words underlined brokenly as nominal expressions. One may grant Henry's assumption that Anselm is here attempting to say things about nothing in its nominal sense without supposing (as Henry does) that *every* occurrence of "nothing" and "something" must be treated in that way. I think that the context allows us to regard only the doubly underlined words as nominals, and the brokenly underlined words as quantifiers. Ontology would then give us the following rendering of Anselm's statements:

(6′) $(a) \sim (\wedge\, \varepsilon\, a)$

(Nothing is nothing)

(7′) $\sim (\exists a)(\wedge\, \varepsilon\, a)$

(It is false that nothing is something)

These formalizations are not open to any of the objections which count against Henry's account. Moreover, the logical equivalence of (6′) and (7′) brings out the close relation between Anselm's statements, while Henry's formulae, "$\wedge \subset \wedge$" and "$\sim(\wedge\, \varepsilon \vee)$", leave it something of a mystery.

As far as I can tell, Anselm does not explicitly assert the third sentence, "Nothing is not-something", what he does say is, "It is evident that this word 'nothing' does not at all differ from what I am calling 'not-something,' i.e., as far as its signification goes." In this passage Anselm is claiming that the expressions "nothing" and "not-something" signify the same things. We might render this claim as: "Nothing is equal to not-something" or, in Ontology: "$\wedge \circ \sim \vee$".[5] My objections (A), (B), and (D) would no longer apply to this corrected version.

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4. *De Casu Diaboli* 11.

5. I should like to thank Dr. Henry and Dr. Sobociński for warning me against the error of confusing this functor, "is equal to", defined in $(a)(b)(a \circ b . \leftrightarrow . a \subset b \;\&\; b \subset a)$, and the functor "is identical with", defined in $(a)(b)(a = b . \leftrightarrow . a\, \varepsilon\, b \;\&\; b\, \varepsilon\, a)$.