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UNUSUAL FEATURE OF S3*

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Zeman's [1] showed that if $L'\alpha$ is defined as $K\alpha L\alpha$ a wide variety of modal systems formulated in C-N-L contain their own bases in L' together with $L'CL'pp$. This is so generally the case with the so-called zero-systems S1°, S2°, etc. and applies also to Canty's R1*, R2* from [2], that any exception is remarkable. Sobociński's S3* from [3], which is classically based as R3* in [2], is such an exception. Relatively to S3* the L'-version of the S3 axiom (1) $LCLCpqLCLpLq$ is inferentially equivalent to (2) $LCKpqCLCpqCLpLq$. Taking the usual cube of the Boolean two-valued matrix for C and N together with L(*1*2345678) = (17787788) we have an S3*-matrix which gives (2) the value 7.

The idea of S3* is simple and elegant: just add (1) to S0.5. By further adding (2) we obtain a new system which properly contains S3*, is properly contained in S3, is classically based, and in which necessity is factorable in the sense of Zeman. We call this system S3**. That S3** is properly contained in S3 is shown by the four-valued matrix of [4], L(*1*234) = (1334) which verifies S3* and (2) but rejects $LCLpp$.

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