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A HEURISTIC PROCEDURE FOR NATURAL DEDUCTION DERIVATIONS USING REDUCTIO AD ABSURDUM

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The beauty of a Fitch-style natural deduction system lies in the ease with which one constructs derivations. The student is taught that by examining the desired conclusion one can generally reconstruct (by working from both ends toward the middle) the steps necessary for the derivation. This procedure breaks down when the derivation requires the use of reductio ad absurdum (**RAA**). In this instance the difficulty involved in deciding what contradiction to derive from the hypothesis per absurdum (**HPA**) can be a considerable stumbling block to the completion of the derivation. What follows is a simple heuristic procedure which enables the student to select a natural contradiction for derivation. The assumption, other than that the system is a Fitch-style natural deduction system, is that the use of **RAA** is recommended only when no other standard strategy technique provides other direction.*

Suppose a set of hypotheses $\mathfrak{H} = \{h_1, \ldots, h_n\}$ entails a statement A. Then, if we assume $(\sim A)$ as the **HPA**, the set $\mathfrak{H} \cup \{\sim A\}$ is inconsistent. Assuming that $\sim A$ is not impossible, there is an assignment α on which $\sim A$ is true. On α some member of $\mathfrak{H}$ must be false (or else the set $\mathfrak{H} \cup \{\sim A\}$ would be consistent). Taking the values determined by α and applying them to the atomic components of the members of $\mathfrak{H}$ will yield (at least) one atomic variable that must be assigned both values: one value if the **HPA** and the remaining members of $\mathfrak{H}$ are to be true, and another value if the statement itself is to be true. This atomic variable and its negate is the contradiction to derive. Note that in determining values for the members of $\mathfrak{H}$ on the assignment α the most complex member of $\mathfrak{H}$ should be taken first.

One interesting fact is responsible for the procedure, and another is responsible for the way in which it works. The first fact is that, if all other techniques have been used before turning the **RAA**, then in most cases

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there will be only one assignment α on which the **HPA** is true. The second fact, obvious though it is, is that when the truth of a statement depends on a unique assignment then the truth-value of the atomic components of the statement can be derived. Presentation of these facts is often sufficient to facilitate **RAA** derivations. Since the **HPA** can only be true in one way, one simply locates that atomic component of a member of $\mathfrak{H}$ which must be false (true) if the **HPA** is to be true, and true (false) if the statement in which it occurs is to be true.

Three points in closing. The first is that the procedure highlights the fact that the **HPA** requires some member of $\mathfrak{H}$ to be false. This underscores the intuitive basis of the rule **RAA**. The second and third points are that the procedure is heuristic and may not always work, and that the procedure will not always yield the shortest derivation. In other words, common sense, intuition, and practice will not be replaced.

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