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DEFINITIONS OF SEMANTICAL REFERENCE AND SELF-REFERENCE

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Consider a language, $\mathcal{L}$, which contains T, as its only semantical predicate; $F_1^1 \ldots F_n^1 \ldots F_1^m \ldots F_n^m$ as syntactical predicates; variables and quantifiers ranging over the sentences of $\mathcal{L}$.*

D-1: For any sentence p, p^* is a sentence just like p except that in p^* each occurrence of T in p is replaced by the first monadic syntactical predicate not occurring in p (call it '*').

D-2: An S-*-variant of $\mathfrak{M}_i$ is a model, $\mathfrak{M}_j$, which is just like $\mathfrak{M}_i$ except that the interpretation of * may vary *outside* S. (where S is some subset of the domain of $\mathfrak{M}_i$).

D-3: A subset, S, of D_i is *determinative* in $\mathfrak{M}_i$ for p iff p^* is true in all S-*-variants of $\mathfrak{M}_i$ or false in all S-*-variants of $\mathfrak{M}_i$.

D-4: The intersection of the sets determinative in $\mathfrak{M}_i$ of p is the set of sentences that p *directly semantically refers* to in $\mathfrak{M}_i$.

D-5: A sequence of sentences, such that each member (excepting a last member) directly semantically refers (in $\mathfrak{M}_i$) to its successor is a *sequence of semantical reference* (in $\mathfrak{M}_i$).

D-6: If A precedes B in a sequence of semantical reference (in $\mathfrak{M}_i$) then A *semantically refers* to B (in $\mathfrak{M}_i$).

D-7: If A semantically refers to A (in $\mathfrak{M}_i$), A is *semantically self-referential* (in $\mathfrak{M}_i$).

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*These definitions were circulated to some people working on self-reference in 1970. Their appearance here is occasioned by Mr. Paul Vincent Spade's interesting and sympathetic article, "An alternative to Brian Skyrms' approach to the Liar," *Notre Dame Journal of Formal Logic*, vol. XVII (1976), pp. 137-146.

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D-8: A is *grounded* in $\mathfrak{M}_i$ iff every sequence of semantical reference in $\mathfrak{M}_i$ in which A occurs has a last member.

D-9: A is *founded* in $\mathfrak{M}_i$ iff every sequence of semantical reference in $\mathfrak{M}_i$ in which A occurs either has a last member, or is a sequence which cycles (i.e., repeats itself) after A but within which A does not occur more than once.

Comments: Mr. Spade and I agree that founded as well as grounded sentences should be guaranteed bivalence. We differ in setting up the relation of *direct semantical reference* (D-1 - D-4). My relation is a kind of *essential* reference. For instance '$a_2 = a_2 \vee \mathsf{T}a_3$' does not directly semantically refer to anything, while '$a_2 = a_2 \,\&\, \mathsf{T}a_3$' directly semantically refers to a_3 but not a_2. Not only is it essential reference but it is essential *semantical* reference. A sentence containing only syntactical predicates (e.g., 'This sentence begins with a 't'.') may in a clear sense refer to themselves but they do not directly *semantically* refer to anything. My main motivation for these definitions lies in the problems posed by quantifiers. Where the quantifiers of $\mathcal{L}$ range over all the sentences of $\mathcal{L}$ we are in peril of having all quantified sentences being self-referential. But, on my definitions, '$(x)(x = a \supset \mathsf{T}x)$' directly semantically refers to a, rather than to everything and '$(x)(\phi x \supset \mathsf{T}x)$' where ϕ is a syntactical predicate directly semantically refers (in $\mathfrak{M}_i$) to just that class of things which is the extension (in $\mathfrak{M}_i$) of 'ϕ'.

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