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ON THE ELIMINABILITY OF DE RE MODALITIES IN SOME SYSTEMS

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1* M. J. Cresswell has tried to show[1] in [1] that the so called *de re modalities* are not eliminable in the system S, where S = LPC + S5 + **Pr**. The axiom schema **Pr**, or

$$(a)(L\beta a \vee L \sim \beta a) \vee (a)(M\beta a \wedge M \sim \beta a),$$

is deemed by Cresswell to be a fair formal representative of von Wright's *principle of predication.*[2] In this form it is an extremely strong principle. Thus, it can be easily shown that (Lemma 2, section 3)

$$\text{LPC} + \text{T} + \mathbf{Pr} \vdash (x_1) \ldots (x_n) \{L(\alpha x_1 \ldots x_n \equiv \alpha y_1 \ldots y_n) \vee L(\alpha x_1 \ldots x_n \equiv \sim \alpha y_1 \ldots y_n)\},$$

where T is the "minimal" modal logic containing the axiom of necessity, the axiom of L-distribution over '$\supset$', and the rule of necessitation.

The above lemma shows, in semantic terms, that **Pr** is strong enough to trivialize modal logic to the extent that the behavior of any context with n free variables is completely determined in any given model by: (a) describing how it behaves "across" the model (i.e., in every "world" therein) for some arbitrary fixed n-tuple and (b) describing how it behaves for each other n-tuple at some world or another. Cresswell's work suggests that this trivialization may not be sufficient to render empty, semantically, the distinction between de re and de dicto modalities.†

Moreover Professor Cresswell suspects[3] that even the further addition of the schema:

$$L(\exists a)\beta \equiv (\exists a)L\beta \qquad \text{(ELC, henceforth)}$$

may not be equal to the job. We shall show by a simple proof, however,

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†(Added in proofs) But meanwhile we have proved in [6] that LPC + S5 + **Pr** $\vdash$ ELC and, therefore, by the present paper's results, that the distinction is rendered empty (even syntactically).

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that this is more than enough to bring about the eliminability of de re modalities. Indeed it will be shown that:

Theorem 3 (section **4**) *De re modalities of any type are eliminable in* LPC + T + **Pr** + ELC.

In addition we shall show (section **5**) that:

Theorem 4 LPC + S5 + ELC $\vdash$ **Pr**,

thus showing that adding ELC to quantified S5 suffices for the elimination of de re modalities. In this connection it is worth remarking that von Wright and Prior seemed to consider[4] the addition of ELC necessary to eliminate de re modalities—but this is not true.[5]

2 **Pr** *and* ELC *are intuitively unacceptable—so why bother?* We have already hinted why **Pr** is a little too extravagant (though this does not automatically discredit more modest formulations of von Wright's principle of predication). ELC seems to be even worse intuitively. Not only does it imply **Pr**, when added to quantified S5, but even subschemas of ELC which do not imply **Pr**—such as ELC restricted to β's belonging to LPC—do not seem to be acceptable. W.V.O. Quine remarked long ago[6] that if it is necessarily the case that there is someone who will win the game, it does not seem to follow that there is anyone who is a necessary winner. Such observations date back even to Thomas Aquinas.[7] Moreover even suggestions which have been made towards salvaging ELC by giving the quantifiers a scope dependent interpretation seem to lead into a blind alley. Thus it has been suggested that we read quantification into modal contexts as involving "intensional" objects. Yet a recent work by A. Bressan [3] shows, I believe, that if one follows this hunch to its logical conclusion the result is quite different than what is expected. By reading this pioneering and daring book it becomes clear that any move towards intensional interpretation of variables (or objects) should be accompanied by a similar move towards reinterpretation of singular properties, for instance, as intensional in the sense that in every "world" a predicate would have a quasi-extension consisting of a set of intensions! Once this is admitted ELC is no longer valid.

The question is then: why bother to prove the eliminability of de re modalities in intuitively unacceptable systems? It would seem to be much more instructive, philosophically, if we were to provide some intuitively unacceptable necessary condition for the eliminability of de re modalities. For then we would have at least shown by *reductio ad absurdum* that the de re/de dicto-dichotomy has an inalienable right to exist. The following remarks will point out a rationale for the effort invested in this article.

Firstly I believe that something akin to a *necessary condition*[8] for abolishing the dichotomy can be given, but this can be done most easily by establishing semantical analogues for the de re/de dicto-dichotomy in various systems—analogues which are achieved partly by the aid of such results as are dealt with in the present paper (see next paragraph). Thus, as far as I am capable of elaborating necessary conditions for abolishing the dichotomy, the present results appear to be quite relevant.

Much more important than the attempts to abolish the dichotomy is the fact that, for most of the familiar modal calculi (and for other intensional logics as well), the property of being eliminable by a *de dicto* formula (i.e., one which is not a de re modality) has a perfect *semantical analogue*. This, we believe, should facilitate discussion of problems related to quantification into modal contexts and should permit us to assess more accurately the price of abolishing such quantification. The results of this paper, and their like, prove to be extremely useful in establishing the existence and range of such analogues. Mathematically, at least, complete eliminability of de re modalities in intuitively strange systems provides an insight into the kinds of de re modalities which are eliminable in more "natural" systems.

However, since such results are beyond the scope of this paper,[9] we satisfy ourselves here with a few hints about the nature of the concepts involved and the particular role of results of the kind provided by this paper. The key concepts, in this direction, are special-type-generalizations of classical model theoretic relations, such as *elementary equivalence* and *isomorphism*. Given such a classical relation $\mathfrak{E}$, envisage the following relation, $\mathfrak{E}^*$, between ordered pairs of the type $\langle\mathfrak{M}, w\rangle$, where $\mathfrak{M}$ is a modal "model" and w is a "possible world" therein. Two such pairs, $\langle\mathfrak{M}, w\rangle$ and $\langle\mathfrak{M}', w'\rangle$, will stand in relation $\mathfrak{E}^*$ when there are maps, f, g, from the worlds of $\mathfrak{M}$ to those of $\mathfrak{M}'$ and vice versa, respectively, which satisfy (1) $f(w_0) = w_0'$ and $g(w_0') = w_0$, (2) images under f and g stand in relation $\mathfrak{E}$ to their sources (qua *classical* models), and (3) f and g preserve the *accessibility* structure of the models. This last clause means that w_2 is accessible (relatively possible) to w_1 in $\mathfrak{M}$ *iff* w_2' is accessible to w_1' in $\mathfrak{M}'$, *provided that* $w_1 = g(w_1')$ or $w_1' = f(w_1)$, *and that* $w_2 = g(w_2')$ or $w_2' = f(w_2)$. Now consider the following property of formulae: α will be called $\mathfrak{E}$-*invariant with respect to a family of modal structures*, $\mathfrak{F}$, iff for any two ordered pairs, $\langle\mathfrak{M}, w\rangle$, $\langle\mathfrak{M}', w'\rangle$, of the above kind, with $\mathfrak{M}$ and $\mathfrak{M}'$ in $\mathfrak{F}$, α has the same *truth value* in both $\langle\mathfrak{M}, w\rangle$ and $\langle\mathfrak{M}', w'\rangle$ whenever they stand in the relation $\mathfrak{E}^*$.

It turns out that in any modal calculus satisfying certain minimal conditions, closed de re modalities will be eliminable when and only when they are $\mathfrak{E}$-invariant with respect to a characteristic family. For example, for a calculus C, without identity, the following conditions will do: (1) validity of possibility of some tautology; (2) validity of distributing necessity over '$\supset$'; (3) admissibility of the rule of necessitation; and (4) some characteristic family for C is *closed* under the relation $\mathfrak{E}^*$. It is easy to verify that the quantified versions of T, S4, S5, for instance, satisfy these conditions, for appropriate choices of the classical $\mathfrak{E}$.

In establishing such results the harder part consists in showing the sufficiency of $\mathfrak{E}$-invariance for eliminability. It is here that "strange" extensions of a given calculus, C, prove to be instrumental. One looks for some such extension in which *every closed de re modality is eliminable*, a model of which is contained in *every* $\mathfrak{E}^*$-*equivalence class of* C-*models*.

An $\mathfrak{E}$-invariant α must be equivalent in such extensions to a *de dicto* wff

α_1. It then follows from the proved converse of what we seek to prove that both α and α_1 are **E**-invariant and, as can be easily seen from the definition, that $\ulcorner\alpha \equiv \alpha_1\urcorner$ is also **E**-invariant. Using now property (ii) it easily follows that $\ulcorner\alpha \equiv \alpha_1\urcorner$ holds in every model of C, and if our semantics are complete for C then $C \vdash \alpha \equiv \alpha_1$, which settles what we are after. Though this is admittedly only a mathematical justification for considering what happens to de re modalities in some strange modal systems, we believe that the elegance of the applications is correlated with deeper philosophical insights.[10]

3 *Results in* LPC + T + **Pr**[11]

Lemma 1 *In the system* LPC + T + **Pr** *we have*

$$\vdash (\exists x_1) \ldots (\exists x_n)(L\alpha \vee L \sim \alpha) \supset (x_1) \ldots (x_n)(L\alpha \vee L \sim \alpha)$$

for any α *and for any quantifier depth* n.

Proof (by induction): For $n = 1$ this is LPC-equivalent to the schema **Pr**. Suppose the above to be true for $n = r$ $(r \geqslant 1)$. Then by induction hypothesis

$$\vdash (\exists x_2) \ldots (\exists x_{r+1})(L\alpha \vee L \sim \alpha) \supset (x_2) \ldots (x_{r+1})(L\alpha \vee L \sim \alpha)$$

so that (by LPC)

(1) $\vdash (\exists x_1) \ldots (\exists x_{r+1})(L\alpha \vee L \sim \alpha) \supset (\exists x_1)(x_2) \ldots (x_{r+1})(L\alpha \vee L \sim \alpha)$.

Again by classical logic

(2) $\vdash (\exists x_1)(x_2) \ldots (x_{r+1})(L\alpha \vee L \sim \alpha) \supset (x_2) \ldots (x_{r+1})(\exists x_1)(L\alpha \vee L \sim \alpha)$

and by the case $n = 1$

$$\vdash (\exists x_1)(L\alpha \vee L \sim \alpha) \supset (x_1)(L\alpha \vee L \sim \alpha)$$

from which follows by classical logic

(3) $\vdash (x_2) \ldots (x_{r+1})(\exists x_1)(L\alpha \vee L \sim \alpha) \supset (x_2) \ldots (x_{r+1})(x_1)(L\alpha \vee L \sim \alpha)$

and from (1), (2), (3) we get the desired result by hypothetical syllogisms and permutation of universal quantifiers.

Lemma 2 *In the system* LPC + T + **Pr** *we have*

$$\vdash (x_1) \ldots (x_n) \{L(\alpha x_1 \ldots x_n \equiv \alpha y_1 \ldots y_n) \vee L(\alpha x_1 \ldots x_n \equiv \sim\alpha y_1 \ldots y_n)\}.$$

Proof: By using Lemma 1 we get

$$\vdash (\exists x_1) \ldots (\exists x_n)[L(\alpha x_1 \ldots x_n \equiv \alpha y_1 \ldots y_n) \vee L \sim (\alpha x_1 \ldots x_n \equiv \alpha y_1 \ldots y_n)] \supset (x_1) \ldots (x_n)[L(\alpha x_1 \ldots x_n \equiv \alpha y_1 \ldots y_n) \vee L \sim (\alpha x_1 \ldots x_n \equiv \alpha y_1 \ldots y_n)];$$

but since $\vdash \alpha y_1 \ldots y_n \equiv \alpha y_1 \ldots y_n$, we have by necessitation, addition, and existential generalization that the antecedent *holds*; therefore:

(1) $\vdash (x_1) \ldots (x_n)\{L(\alpha x_1 \ldots x_n \equiv \alpha y_1 \ldots y_n) \vee L \sim (\alpha x_1 \ldots x_n \equiv \alpha y_1 \ldots y_n)\}$,

but since $\vdash \sim(\alpha x_1 \ldots x_n \equiv \alpha y_1 \ldots y_n) \equiv (\alpha x_1 \ldots x_n \equiv \sim\alpha y_1 \ldots y_n)$ we get the desired result by the substitutivity of equivalents in any extension of LPC + T.

Lemma 3 *In the system* LPC + T + **Pr** *we have*

$$\vdash L\beta x_1 \ldots x_n .\equiv. \beta x_1 \ldots x_n \wedge (\exists x_1) \ldots (\exists x_n) L\beta x_1 \ldots x_n.$$

Proof: The left hand side obviously implies the right hand side by the axiom of necessity, existential generalization, and propositional logic. Suppose then that

(1) $\beta x_1 \ldots x_n \wedge (\exists x_1) \ldots (\exists x_n) L\beta x_1 \ldots x_n.$

Hence we have

(2)	$(\exists x_1) \ldots (\exists x_n)(L\beta \vee L \sim \beta)$	(from (1) by LPC)
(3)	$(x_1) \ldots (x_n)(L\beta \vee L \sim \beta)$	(by Lemma 2, from 2)
(4)	$L\beta x_1 \ldots x_n \vee L \sim \beta x_1 \ldots x_n$	(by instantiation from 3)

But

(5)	$\sim\sim\beta x_1 \ldots x_n$	(from (1))
(6)	$L \sim \beta x_1 \ldots x_n \supset \sim\beta x_1 \ldots x_n$	(axiom of necessity)
(7)	$\sim L \sim\beta x_1 \ldots x_n$	(5, 6 by modus tollens)
(8)	$L\beta x_1 \ldots x_n$	(4, 7 by disjunctive syl.)

4 *Results in* LPC + T + Pr + ELC

Lemma 4 *In the system* LPC + T + **Pr** + ELC:

$$\vdash L(\exists x_1) \ldots (\exists x_n)\alpha \equiv (\exists x_1) \ldots (\exists x_n)L\alpha.$$

Proof: Trivial by induction from ELC.

Theorem I *In the system* LPC + T + **Pr** + ELC *we have*

$$\vdash L\beta x_1 \ldots x_n .\equiv. \beta x_1 \ldots x_n \wedge L(\exists x_1) \ldots (\exists x_n)\beta x_1 \ldots x_n.$$

Proof: By Lemma 3 we have

$$\vdash L\beta x_1 \ldots x_n .\equiv. \beta x_1 \ldots x_n \wedge (\exists x_1) \ldots (\exists x_n)L\beta x_1 \ldots x_n$$

in any extension of LPC + T + **Pr**. Hence, using Lemma 4 and propositional logic we get the desired result. Q.E.D.

Theorem II *Every wff* α *is equivalent in* LPC + T + **Pr** + ELC *to some* α' *which is not a de re modality.*

We shall actually prove by induction:

Theorem III *Let* $\alpha\vec{x}$ *be a wff with the vector* $\vec{x} = \langle x_1, \ldots, x_r\rangle$ *representing all its free variables. Then there is a propositional function* Ψ_α *in* $n + m$ *places such that*

$$\text{LPC + T + } \mathbf{Pr} \text{ + ELC} \vdash \alpha \equiv \Psi_\alpha(\gamma_1, \ldots, \gamma_n, L\delta_1, \ldots, L\delta_m)$$

where $\gamma_1, \ldots, \gamma_n$ *are* LPC-*wffs which contain free only variables of the set* $\{\vec{x}\} = \{x_1, \ldots, x_r\}$ *and where* $\delta_1, \ldots, \delta_m$ *are closed de dicto (non-de re) wffs (n and m depend on* α, *of course).*

Proof (by strong-induction on the modal depth of α): If the model depth is 0 there are no modal operators and the theorem is obvious. Suppose the modal depth of α is $n+1$. Let $\alpha_1, \ldots, \alpha_s$ be the scopes of outermost occurrences of 'L'. (We assume that 'M' is to be rewritten as '$\sim L \sim$' everywhere in α.) Outermost occurrences of 'L' are those which are not in the scope of any modal operators. Obviously α is an LPC-construct (in terms of connectives and quantifiers) out of $L\alpha_1, \ldots, L\alpha_s$ and some LPC wffs $\beta_1, \ldots, \beta_k$. By using a prenex normal form for this construct we have

$$\vdash \alpha\vec{x} \equiv (\mathbf{Q}\vec{y})\,\Theta_\alpha(\beta_1\vec{x}\vec{y}, \ldots, \beta_k\vec{x}\vec{y}, L\alpha_1\vec{x}\vec{y}, \ldots, L\alpha_s\vec{x}\vec{y})$$

where $(\mathbf{Q}\vec{y})$ represents some quantifier-prefix using variables $y_1, \ldots, y_{k'}$ which do not belong to the set $\{\vec{x}\}$, and where Θ_α is an appropriate propositional function. All the α_i's have a modal depth $< n+1$. Hence by induction hypothesis

$$\vdash \alpha_i\vec{x}\vec{y} \equiv \Psi_{\alpha_i}(\gamma_1^i\vec{x}\vec{y}, \ldots, \gamma_{n_i}^i\vec{x}\vec{y}, L\delta_1^i, \ldots, L\delta_{m_i}^i)$$

where $\gamma_j^i \in \mathcal{L}(\text{LPC})$ and contains free only variables of $\{\vec{x}, \vec{y}\}$ and where $\delta_{k'}^i$ are *closed* de dicto wffs. By Theorem I:

$$\vdash L\alpha_i\vec{x}\vec{y} \equiv \Psi_{\alpha_i}(\langle\gamma_j^i\vec{x}\vec{y}\rangle, \langle L\delta_{k'}^i\rangle) \wedge L(\exists\vec{x})(\exists\vec{y})\,\Psi_{\alpha_i}(\langle\gamma_j^i\vec{x}\vec{y}\rangle_j, \langle L\delta_{k'}^i\rangle_{k'})$$

($\ulcorner(\exists\vec{x})\urcorner$ is short for '$(\exists x_1) \ldots (\exists x_n)$' and $\langle\gamma_j^i\rangle_j$, $\langle L\delta_{k'}^i\rangle_{k'}$ are obvious notations with the lower index as a running one).

The formulas:

$$\mu_i =_{Df} (\exists\vec{x})(\exists\vec{y})\,\Psi_{\alpha_i}(\langle\gamma_j^i\vec{x}\vec{y}\rangle_j, \langle L\delta_{k'}^i\rangle_{k'})$$

are obviously closed de dicto wffs. Thus we have

(1) $\vdash \alpha\vec{x} \equiv (\mathbf{Q}\vec{y})\,\Psi'_\alpha(\beta_1, \ldots, \beta_k, \langle\gamma_j^i\vec{x}\vec{y}\rangle_{i,j}, \langle L\delta_{k'}^i\rangle_{i,k'}, L\mu_1, \ldots, L\mu_s)$,

where Ψ'_α is an appropriate propositional function. Let $b_1^1, \ldots, b_{m_s}^s, c_1, \ldots, c_s$ be Boolean variables which take on values in the set $\{\mathbf{t}, \mathbf{f}\}$ where $\mathbf{t}$ may be represented as some LPC-tautology (closed) and where $\mathbf{f} = \sim\mathbf{t}$. Let

$$(\varphi)^b =_{Df} \begin{cases} \varphi \text{ if } Vb = \mathbf{t} \\ \sim\varphi \text{ if } Vb = \mathbf{f} \end{cases}, \text{ for any such variable } b.$$

Then

(2) $$\vdash \alpha\vec{x} \equiv \bigvee_{\langle Vb_1^1, \ldots, Vb_{m_s}^s, Vc_1, \ldots, Vc_s\rangle(\text{all possible valuations})} \ulcorner(L\delta_1^1)^{b_1^1} \wedge \ldots \wedge (L\delta_{m_s}^s)^{b_{n_s}^s}\urcorner$$
$$\wedge (L\mu_1)^{c_1} \wedge \ldots \wedge (L\mu_s)^{c_s} \wedge (\mathbf{Q}\vec{y})\Psi_{\alpha, b_1^1, \ldots, c_s},$$

where

$$\Psi_{\alpha, b_1^1, \ldots, c_s} =_{Df} \Psi'_\alpha(\beta_1, \ldots, \beta_s, \langle\gamma_j^i\vec{x}\vec{y}\rangle_{i,j}, \langle b_{k'}^i\rangle_{i,k'}, c_1, \ldots, c_s).$$

Proof of (2): Let $b_1^1, \ldots, b_{m_s}^s, c_1, \ldots, c_s$ take any of the possible values. We notice that $\vdash(\varphi)^b \equiv (\varphi \equiv b)$ for any formula φ and any such Boolean variable. Thus, by (1) and the above:

(3) $(L\delta_1^1)^{b_1^1} \wedge \ldots \wedge (L\delta_{m_s}^s)^{b_{m_s}^s} \wedge (L\mu_1)^{c_1} \wedge \ldots \wedge (L\mu_s)^{c_s} \wedge \alpha$

implies

$$(L\delta_1^1 \equiv b_1^1) \wedge \ldots \wedge (L\delta_{m_s}^s \equiv b_{m_s}^s) \wedge (L\mu_1 \equiv c_1) \wedge \ldots \wedge (L\mu_s \equiv c_s)$$
$$\wedge (\mathbf{Q}\vec{y})\, \Psi_\alpha'(\langle\beta_t\rangle_t, \langle\gamma_j^i\rangle_{i,j}, \langle L\delta_l^i\rangle_{i,l}, \langle L\mu_{i'}\rangle_{i'}).$$

We can transfer the quantifier to the prefix position (since the wffs preceding it are closed) and truth-functionally substitute, obtaining

$$(\mathbf{Q}\vec{y})\Psi_{\alpha, b_1^1, \ldots, c_s}$$

Therefore (3) implies also

$$(L\delta_1^1)^{b_1^1} \wedge \ldots \wedge (L\delta_{m_s}^s)^{b_{m_s}^s} \wedge (L\mu_1)^{c_1} \wedge \ldots \wedge (L\mu_s)^{c_s} \wedge (\mathbf{Q}\vec{y})\Psi_{\alpha, b_1^1, \ldots, c_s}.$$

Hence

$$\bigvee_{\langle Vb_1^1, \ldots, Vc_s\rangle} \alpha \wedge (L\delta_1^1)^{b_1^1} \wedge \ldots \wedge (L\mu_s)^{c_s} .\supset. \bigvee_{\langle Vb_1^1, \ldots, Vc_s\rangle} (L\delta_1^1)^{b_1^1} \wedge \ldots \wedge (L\mu_s)^{c_s}$$
$$\wedge (\mathbf{Q}\vec{y})\Psi_{\alpha, b_1^1, \ldots, c_s}$$

and the antecedent is truth-functionally equivalent to α. Conversely, suppose that we have $\phi_{\alpha, b_1^1, \ldots, c_s}$ where

$$\phi_{\alpha, b_1^1, \ldots, c_s} =_{Df} \bigwedge (L\delta_j^i)^{b_j^i} \wedge \bigwedge (L\mu_{i'})^{c_{i'}} \wedge (\mathbf{Q}\vec{y})\Psi_{\alpha, b_1^1, \ldots, c_s}$$

for some particular value of the Boolean variables. Since δ_j^i, $\mu_{i'}$ are closed, we have

$$\vdash \phi_{\alpha, b_1^1, \ldots, c_s} \equiv (\mathbf{Q}\vec{y}) \bigwedge (L\delta_j^i)^{b_j^i} \wedge \bigwedge (L\mu_{i'})^{c_{i'}} \wedge \Psi_{\alpha, b_1^1, \ldots, c_s}.$$

Therefore, since the matrix of the right hand side of this equivalence implies truth-functionally

$$(L\delta_1^1 \equiv b_1^1) \wedge \ldots \wedge (L\mu_s \equiv c_s) \wedge \Psi_{\alpha, b_1^1, \ldots, c_s}$$

which implies in turn (truth-functionally)

$$\Psi_\alpha'(\beta_1, \ldots, \beta_k, \gamma_1^1, \ldots, \gamma_{n_s}^s, L\delta_1^1, \ldots, L\delta_{m_s}^s, L\mu_1, \ldots, L\mu_s),$$

we obtain

$$\vdash \phi_{\alpha, b_1^1, \ldots, c_s} .\supset. (\mathbf{Q}\vec{y})\Psi_\alpha'(\langle\beta_t\rangle_t, \langle\gamma_j^i\rangle_{i,j}, \langle L\delta_l^i\rangle_{i,l}, \langle L\mu_{i'}\rangle_{i'})$$

whose consequent clearly implies α by (1). Hence

$$\bigvee_{\langle Vb_1^1, \ldots, Vc_s\rangle} \phi_{\alpha, b_1^1, \ldots, c_s} .\supset. \alpha.$$

This concludes the proof of (2) and thereby of the induction step (noticing that $\Psi_{\alpha, b_1^1, \ldots, c_s}$, for any value of the Boolean variables, belongs to LPC).

Q.E.D.

5 *System* LPC + S5 + ELC We now give a proof of **Pr** in the above system showing that the addition of ELC suffices for complete elimination of de re modalities. We note however with [1] that ELC is not a necessary requirement for such an elimination (see note 5).

Theorem 4 LPC + S5 + ELC $\vdash$ **Pr**.

Proof: (We prove in fact the validity of the schema $(\exists t)L\alpha \supset (t)(L\alpha \vee L \sim \alpha)$ which is equivalent to the schema **Pr**.) Let

$$\beta_\alpha =_{Df} \alpha \wedge M \sim \alpha$$

and let

$$\phi_\alpha =_{Df} (\alpha \wedge \sim (\exists t)\beta_\alpha) \vee \beta_\alpha .$$

Applying the schema ELC to the case ϕ_α, we get

(1) $L(\exists t)[(\alpha \wedge \sim (\exists t)\beta_\alpha) \vee \beta_\alpha] \supset (\exists t)L[(\alpha \wedge \sim (\exists t)\beta_\alpha) \vee \beta_\alpha]$.

Moreover:

(2) $\vdash (\exists t)(\alpha \wedge \sim (\exists t)\beta_\alpha \vee \beta_\alpha) \equiv (\exists t)\alpha \wedge \sim (\exists t)\beta_\alpha \vee (\exists t)\beta_\alpha$ (by LPC)
(3) $\vdash (\exists t)\alpha \wedge \sim (\exists t)\beta_\alpha \vee (\exists t)\beta_\alpha \equiv (\exists t)\alpha \vee (\exists t)\beta_\alpha$ (Prop.)
(4) $\vdash (\exists t)\alpha \vee (\exists t)\beta_\alpha \equiv (\exists t)(\alpha \vee \alpha \wedge M \sim \alpha)$ (LPC + Def. β_α)
(5) $\vdash (\exists t)(\alpha \vee \alpha \wedge M \sim \alpha) \equiv (\exists t)\alpha$ (by LPC)

Thus we get

(6) $\vdash (\exists t)\phi_\alpha \equiv (\exists t)\alpha$ (2, 3, 4, 5, Prop.)

Hence

(7) $\vdash L(\exists t)\phi_\alpha \equiv L(\exists t)\alpha$ (by necessitation and L-distribution over '$\equiv$')

On the other hand

(8) $\vdash L[\alpha \wedge \sim (\exists t)\beta_\alpha \vee \beta_\alpha] \equiv L[\alpha \wedge (\sim (\exists t)\beta_\alpha \vee M \sim \alpha)]$ (Prop. + T + Def. β_α)

so that

(9) $\vdash L\phi_\alpha \equiv L\alpha \wedge L(\sim (\exists t)\beta_\alpha \vee M \sim \alpha)$ (8, Prop. + T)
(10) $\vdash L(\sim (\exists t)\beta_\alpha \vee M \sim \alpha) \equiv L(L\alpha \supset (t) \sim \beta_\alpha)$ (by LPC + T)

Thus we have

(11) $\vdash L\phi_\alpha \equiv L\alpha \wedge L(L\alpha \supset (t) \sim \beta_\alpha)$ (9, 10)

Hence

(12) $\vdash L\phi_\alpha \supset L\alpha \wedge (LL\alpha \supset L(t) \sim \beta_\alpha)$ (11, LPC + T)
(13) $\vdash L\phi_\alpha \supset LL\alpha \wedge (LL\alpha \supset L(t) \sim \beta_\alpha)$
(from 12, in either LPC + S4 or LPC + S5)
(14) $L\phi_\alpha \supset L(t) \sim \beta_\alpha$ (12, 13, Prop.)
(15) $L(t) \sim \beta_\alpha \equiv L(t)(\alpha \supset L\alpha)$ (LPC + T + Def. β_α)
(16) $(\exists t)L\phi_\alpha \supset L(t)(\alpha \supset L\alpha)$ (14, 15, LPC)
(17) $L(\exists t)\alpha \supset L(t)(\alpha \supset L\alpha)$ (16, 7, 1, Prop.)

Using now the converse of the Barcan formula we get

(18) $L(\exists t)\alpha \supset (t)L(\alpha \supset L\alpha)$ (17, LPC + S4 or LPC + S5)
(19) $L(\exists t)\alpha \supset (t)L(\sim\alpha \vee L\alpha)$ (18, Prop.)

Since in LPC + T we have, as theorems, both $L(\alpha \vee \beta) \supset L\alpha \vee M\beta$ and $(\exists t)L\alpha \supset L(\exists t)\alpha$ (the "innocent" part of ELC), it follows from (19) that

(20) $(\exists t)L\alpha \supset (t)(L \sim \alpha \vee ML\alpha)$.

But in S5 $\vdash ML\alpha \equiv L\alpha$, so S5 + (20) imply

(21) $(\exists t)L\alpha \supset (t)(L\alpha \vee L \sim \alpha)$ Q.E.D.

6 *Remark on wffs in* LPC + S5 + ELC By employing the methods of section **4** one could actually show that every wff in LPC + S5 + ELC is provably equivalent to a de dicto (non-de re) wff of modal depth $\leqslant 1$.

NOTES

1. De re modalities will be taken here (as in Cresswell's paper) to be formulas in which a free variable appears in the scope of a modal operator. Cresswell's proof in [1] is defective, though a semantic proof of similar results has been given by P. Tichy for a modified system S* = LPC + S5 + **Pr*** where **Pr*** is the restriction of **Pr** to closed instances.
2. *Cf.* A. N. Prior [4], p. 211, and Hughes and Cresswell [2], pp. 184-188.
3. See [1], p. 330, footnote.
4. [4], pp. 211-214.
5. It is not generally true that we need the full schema ELC in any extension of quantified S5 in which de re modalities are eliminable. This point is made by Cresswell in [1], p. 330, and is suggested by him already in [2], pp. 186-187. One such construction is the following: Take as a basis quantified S5 with necessary identity and add the *schemata* (where β represents a *non-modal* formula)

$$(\exists x_1) \ldots (\exists x_n) \left[\bigwedge_{i<j\leqslant n} x_i \neq x_j \wedge L\beta x_1, \ldots, x_n\right] \supset (x_1) \ldots (x_n) \left[\bigwedge_{i<j\leqslant n} x_i \neq x_j \supset L\beta x_1, \ldots, x_n\right].$$

   It is not too difficult to show that in *such a system* every de re modality is eliminable, yet the *full* ELC is not valid.
6. This appears in [5], p. 149. The point is already made, very explicitly, in C. D. Broad's *Examination of McTaggart's Philosophy*, Vol. I (University Press, Cambridge, 1933), p. 363. Thanks are due to Professor R. Gale for pointing out to me this source.
7. By this I do not mean that our formal definition of de re modality captures what St. Thomas has in mind when he speaks about the distinctions between modalities de re and modalities de dicto on the level of singular statements. (*Cf. Summa Contra Gentiles*, i.67.) It seems to me more than plausible however that any successful formalization of St. Thomas' notions would entail the invalidity of ELC.

8. The necessary conditions we have in mind are of a mixed semantic-syntactic type. One such condition is the following: If C is an extension of quantified T, which permits the elimination of every closed de re modality, then either

   (i) every closed instance of $L(\exists\vec{x})\beta \supset (\exists\vec{x})L\beta$ is provable in C, when β is non-modal and identity free,

   or else,

   (ii) no semantically characteristic family of models of C is closed under $\mathfrak{E}$-invariance—where $\mathfrak{E}$ is the relation of *elementary equivalence.*

   For fuller details see [7] (chapter III in particular).

9. A fuller treatment of the subject is given in [7].

10. Odd as it may seem from an ordinary linguistic point of view, some of the "crazy" extensions considered in proving the above described results do seem to reflect some interesting philosophical predilections. Thus, the additional schemata considered in note 5 above would be exactly what one might expect to find in systems dealing with *bare particulars,* or such individuals as would lack any interesting essential properties. On the other hand, partial schemata of the type ELC (of the kind considered in note 8 above) may admit of an interpretation which makes them valid in the context of *intuitionistic metalogic.*

11. We shall be using essentially the same version of LPC as is used in [2]. 'Prop.' in the proof-annotations will refer to the propositional calculus.

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