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A NEGATIVE ANSWER TO A QUESTION OF TECK-CHEONG LIM ABOUT PSEUDO CONVERGENCE

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A bounded sequence (x_n) in a Banach space X is said to *pseudo-converge* to a point x_0, called a pseudo limit, if x_0 minimizes the function

$$f_s(x) = \limsup_m ||y_m - x||$$

for every subsequence $S = (y_n)$ of (x_n). In the recent paper [**2**] the author put the following question: *Is it true that in a general Banach space X, if (x_n) pseudo-converges to θ, then there exist a sequence (z_n) in X and a sequence (z_n^*) in X^* such that $z_n^* \in J(z_n)$ for all $n \in \mathbf{N}$, $z_n^* \xrightarrow{w^*} \theta$ and $\lim_n ||x_n - z_n|| = 0$?* (here J denotes a duality map; see [**2**]).

In this short note we want to show that when $X = l_\infty$ the answer to the above question is negative. Let us choose $x'_n = e_n$, the unit vector basis of c_0; it is well known that it converges weakly to θ. Furthermore, it is simple to see that it pseudo-converges to more than one point of l_∞ ([**2**]). Choose one of its nonzero pseudo-limits x'_0 and put $x_n = x'_n - x'_0$ for all $n \in \mathbf{N}$. Let us assume that there exist (z_n) and (z_n^*) as in the question above. It is clear that $z_n \xrightarrow{w} -x'_0$, too. Furthermore, $z_n^* \xrightarrow{w} \theta$ in $(l_\infty)^*$ (see [**1**, p. 103, Theorem 15]). Hence $(z_n^*, z_n) = (z_n^*, z_n + x'_0) + (z_n^*, -x'_0)$; using well-known results about $C(K)$ spaces (l_∞ is isomorphic to $C(\beta N)$!) (see [**1**, p. 113, Exercise 1]) we obtain that $(z_n^*, z_n) \to 0$ and so $z_n \xrightarrow{s} \theta$; this easily implies that $x'_0 = \theta$. This contradiction concludes the proof.

At the end we observe that each space X with the Dunford-Pettis property and the Grothendieck property, too, can be used to answer in the negative Lim's question as done above (for these definitions and

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useful reformulations, we refer to [**1**]), provided there exists in X a w-null sequence that pseudo-converges to a nonzero pseudo-limit.

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