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AN ELEMENTARY PROOF OF THE IRREDUCIBILITY OF THE RANK g PICARD BUNDLE

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ABSTRACT. Over the Jacobi variety of a curve of genus g, we have the Picard vector bundles. In this paper, we show that over the complex numbers, when the Jacobian is a "general" Abelian variety, the rank g Picard bundle is irreducible. In the process, we also show that the irreducibility of a vector bundle over a "general" principally polarized Abelian variety can be guaranteed by a suitable top Chern class. We also show that this "general" condition is a generic property of Jacobians.

1. Introduction. Over the Jacobi variety of a curve of genus g, we have the Picard vector bundles χ^n of rank n for $n \geq g$ (see [**3, 4**] or [**7**]). Kempf [**5**] has shown that the rank g Picard vector bundles are stable. In this paper, we will show that these rank g Picard vector bundles are in fact irreducible, at least in most cases, where by an irreducible bundle we mean one that has no nontrivial subbundles. Umemura [**9**] has shown that in genus $g = 2$, the Picard bundles χ^n, $n \geq 2$, are stable; he also showed that with an additional generic assumption on the Jacobian that in genus $g > 2$ the Picard bundles χ^n, $n \geq g$, are also stable. The higher rank Picard bundles χ^n, $n > g$, are not irreducible because their rank exceeds the dimension of the Jacobi variety. In fact, each of the higher rank Picard bundles can be expressed as an extension (see [**3**]) $0 \to I \to \chi^n \to \chi^{n-1} \to 0$ for $n > g$, where I denotes the trivial line bundle. So the higher rank Picard bundles serve as examples of stable vector bundles which are not irreducible over a Jacobian. Our definition of irreducibility unfortunately does not imply stability, since for higher dimensional varieties, stability is a condition on subsheaves and not just subbundles. The Picard bundle of rank g ($g > 1$) does have nontrivial coherent subsheaves. For example, consider the unique section ϕ of the bundle χ^g as a map from the trivial

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line bundle $\phi : I \to \chi^g$ (see [**3**]). Thinking of this as a morphism of coherent sheaves, the image sheaf $\operatorname{Im}\phi$ will be a coherent subsheaf of χ^g. Since the section ϕ is zero at only one point (the canonical point on the Jacobian), the image sheaf $\operatorname{Im}\phi$ will be nontrivial.

The method will be to show that the total Chern class of the Picard bundle cannot be factored in the required manner if the bundle were not irreducible. This involves looking at the rational equivalence ring of algebraic cycles on the Jacobian and its image in the cohomology ring. In the case where the subring of cohomology classes carried by analytic cycles is generated by one element, we can apply a result of Schur on certain polynomials [**8**] to show that the Picard bundle is irreducible. More generally, if the above statement about analytic cycles is true for a principally polarized Abelian variety, which is "generally" the case by [**6**], then the irreducibility of a vector bundle whose rank does not exceed the dimension of the variety can be guaranteed by a suitable top Chern class. Finally, we show that this "general" condition is indeed a generic property of Jacobians.

1. Algebraic cycles. For any algebraic variety X over the complex numbers, denote its Chow ring of algebraic cycles under rational equivalence by $\mathcal{A}^*(X)$. So $\mathcal{A}^i(X)$ is the group generated by cycles of complex codimension i. The natural map

$$\mathcal{A}^i(X) \xrightarrow{[\bullet]} H^{2i}(X;\mathbf{Z})$$

takes algebraic cycles to their cohomology classes. Denote the image of this map by $\mathcal{B}^i(X)$. We are interested in this because the Chern classes of a vector bundle can be taken in either the Chow ring or the cohomology ring. Since usually it is easier to work in $H^*(X)$, we would like to know what $\mathcal{B}^*(X)$ is. Also, denote $\mathcal{B}^*(X;\mathbf{Q}) = \mathcal{B}^*(X)\otimes\mathbf{Q}$.

We now discuss principally polarized Abelian varieties. Let $V = \mathbf{C}^n/\mathcal{L}$ where $\mathcal{L} = \mathbf{Z}^n + \Omega\mathbf{Z}^n$ with $\Omega \in \mathcal{H}_g$, the Siegel upper half space. Mattuck has given a description of $\mathcal{B}^*(X;\mathbf{Q})$ for "general" Abelian varieties [**6**].

Definition. Let $A = \mathbf{C}^n/M\mathbf{Z}^{2n}$ be an Abelian variety with the normalized period matrix

$$M = \begin{pmatrix} e_1 & & 0 & \tau_{11} & \cdots & \tau_{1n} \\ & \ddots & & \vdots & & \vdots \\ 0 & & e_n & \tau_{n1} & \cdots & \tau_{nn} \end{pmatrix}$$

with $e_i \geq 1$, $e_i | e_{i+1}$, $\operatorname{Im}((\tau_{ij})) > 0$. Set $\tau_{ij} = a_{ij} + i b_{ij}$. A is *general* if a_{ij}, b_{ij}, $1 \leq i \leq j \leq n$, form $n(n+1)$ algebraically independent real numbers.

Theorem (Mattuck). *For a "general" Abelian variety A, we have $\mathcal{B}^*(A;\mathbf{Q}) = \mathbf{Q}[u]/(u^{n+1})$, where u is any nonzero element in $\mathcal{B}^1(A)$.*

The cohomology ring of V, with integer or rational coefficients, is an exterior algebra on $2n$ generators. Let

$$H^*(V;\mathbf{Z}) = \bigwedge[x_1, \ldots, x_n, y_1, \ldots, y_n] \ ,$$

where the generators are chosen so that the theta divisor $\Theta \in \mathcal{A}^1(V)$ has cohomology class

$$\theta = [\Theta] = \sum_{i=1}^{n} x_i \wedge y_i \in H^2(V;\mathbf{Z}) \ .$$

Note that $\theta^k = k! \sum_{j_1 < \cdots < j_k} x_{j_1} \wedge y_{j_1} \wedge \cdots \wedge x_{j_k} \wedge y_{j_k}$. So $\theta^k/k! \in H^k(V;\mathbf{Z})$. Note also that since $H^*(V;\mathbf{Z})$ is torsion free, we have $H^*(V;\mathbf{Z}) \subseteq H^*(V;\mathbf{Q})$. Also, note $\mathcal{B}^*(V;\mathbf{Q}) \subseteq H^*(V;\mathbf{Q})$.

Lemma 1. *Let V be a "general" principally polarized Abelian variety of dimension n, then for $0 \leq k \leq n$, $\mathcal{B}^k(V) = (\theta^k/k!)\mathbf{Z}$. Consequently, any vector bundle σ over V must have Chern classes of the form*

$$c_k(\sigma) = \frac{a_k}{k!}\theta^k$$

with $a_k \in \mathbf{Z}$.

Proof. By Mattuck's result, $\mathcal{B}^*(V;\mathbf{Q}) = \mathbf{Q}[\theta]/(\theta^{n+1})$. Take any element $z \in \mathcal{B}^k(V)$. Then $z \in \mathcal{B}^k(V;\mathbf{Q}) = \theta^k\mathbf{Q}$ implies $z = r\theta^k$ for some $r \in \mathbf{Q}$. Write

$$z = (rk!)\frac{\theta^k}{k!} = (rk!)\sum_{i_1<\cdots<i_k} x_{i_1}\wedge y_{i_1}\wedge\cdots\wedge x_{i_k}\wedge y_{i_k}\,.$$

Since $z \in H^k(V;\mathbf{Z}) \subseteq \bigwedge_{\mathbf{Z}}[x_1,\dots,y_n]$, each of the $x_{i_1}\wedge\cdots\wedge y_{i_k}$ must have an integer coefficient. Therefore $rk! \in \mathbf{Z}$, that is, $r \in (1/k!)\mathbf{Z}$, and we have $z \in (\theta^k/k!)\mathbf{Z}$. So $\mathcal{B}^k(V) \subseteq (\theta^k/k!)\mathbf{Z}$. Since the reverse inclusion is trivial, the proof is complete. □

Thus on a "general" principally polarized Abelian variety, the total Chern class of any vector bundle will be a polynomial in θ with rational coefficients. We will need a couple of facts about certain polynomials.

2. Polynomials and Schur's result.

Schur's result on the irreducibility of certain polynomials is the following [**8**]:

Theorem (Schur). *For any integers $a_1,\dots,a_{n-1} \in \mathbf{Z}$, the polynomial*

$$f(x) = 1 + \frac{a_1}{1!}x + \frac{a_2}{2!}x^2 + \cdots + \frac{a_{n-1}}{(n-1)!}x^{n-1} \pm \frac{1}{n!}x^n$$

is irreducible over $\mathbf{Q}$.

Of course, these polynomials resemble those of total Chern classes. Since the Chern classes are elements of a truncated polynomial ring, we will need the following simple observation.

Proposition 2. *Let $f,g,h \in \mathbf{Q}[x]$ be polynomials of degrees at most n. If*

$$f(x) = g(x)h(x) \quad \text{in } \mathbf{Q}[x]/(x^{n+1})$$

and

$$\deg g + \deg h \leq n$$

then we must have in fact $f(x) = g(x)h(x)$ in $\mathbf{Q}[x]$.

Proof. Because $\deg g + \deg h \le n$ and $\deg f \le n$ the product $g(x)h(x)$ and $f(x)$ will both generate terms involving $1, x, \dots, x^n$ at most. Then the equality $f(x) = g(x)h(x)$ modulo x^{n+1} will have to mean actual equality in $\mathbf{Q}[x]$. $\square$

3. Vector bundles. We now present the main theorem.

Theorem 3. *Let V be a "general" principally polarized Abelian variety of dimension n. Let σ be a vector bundle of rank $k \le n$. If $c_k(\sigma) = \pm(1/k!)\theta^k$, then σ is irreducible.*

Proof. By Lemma 1, the total Chern class of σ is

$$c(\sigma) = 1 + \frac{a_1}{1!}\theta + \frac{a_2}{2!}\theta^2 + \cdots + \frac{a_{k-1}}{(k-1)!}\theta^{k-1} \pm \frac{1}{k!}\theta^k$$

for some integers $a_1, \dots, a_{k-1} \in \mathbf{Z}$. Call this polynomial $f(\theta)$. Suppose by way of contradiction that σ is not irreducible. This means there is an exact sequence $0 \to \sigma' \to \sigma \to \sigma'' \to 0$, with σ', σ'' being nonzero vector bundles. We have $c(\sigma) = c(\sigma')c(\sigma'')$. Apply Lemma 1 to σ', σ'', their total Chern classes can be written as $c(\sigma') = g(\theta)$, $c(\sigma'') = h(\theta)$ where g, h are rational polynomials with $\deg g \le \operatorname{rank} c'$, $\deg h \le \operatorname{rank} c''$. Using the indeterminate x, $f(\theta) = g(\theta)h(\theta)$ means $f(x) = g(x)h(x)$ in $\mathbf{Q}[x]/(x^{n+1})$. Since

$$\deg g + \deg h \le \text{ rank } c' + \text{ rank } c'' = \text{ rank } c = \deg f = k \le n\ ,$$

we can apply Proposition 2 to obtain that $f(x) = g(x)h(x)$ in $\mathbf{Q}[x]$. Then $\deg g + \deg h = k$. The above inequality would then imply $\deg g = \operatorname{rank} c'$ and $\deg h = \operatorname{rank} c''$. Since c', c'' are nonzero, then g, h both have degree at least 1, that is, they are nonconstant polynomials. This implies that by $f(x) = g(x)h(x)$, $f(x)$ is not irreducible. But Schur's result says that $f(x)$ is an irreducible polynomial over $\mathbf{Q}$. This contradiction completes the proof. $\square$

For the specific case $k = n$, we have the following.

Corollary 4. *Let V be a "general" principally polarized Abelian variety of dimension n. If a rank n vector bundle σ has top Chern* *class*

$$c_n(\sigma) = \pm[\textit{one point}],$$

then it must be irreducible.

Proof. We only need to observe that the cohomology class associated to a point is $x_1 \wedge y_1 \wedge \cdots \wedge x_n \wedge y_n \in H^n(V; \mathbf{Z})$, which is of course $(\theta^n/n!)$. $\square$

We remark that the top Chern class of a rank n bundle may be sometimes calculated by looking at a meromorphic section of the bundle. If the meromorphic section has only one zero or only one pole, then the hypothesis of Corollary 4 is satisfied.

4. Picard bundles and Jacobians. Fix a genus $g \geq 1$. Let M be a marked compact Riemann surface of genus g. Let $J = J(M)$ be its Jacobi variety, which is a principally polarized Abelian variety of dimension g. Denote by χ^g the rank g Picard vector bundle. (See [**3, 4** or **7**].) We know that $c_k(\chi^g) = (1/k!)\theta^k$, $1 \leq k \leq g$. In particular, $c_g(\chi^g) = (1/g!)\theta^g = [\text{one point}]$. So as an application of Corollary 4, we have the following.

Theorem 5. *Let M be such that $J(M)$ is a "general" Abelian variety. Then the rank g Picard vector bundle over J is irreducible.*

For completeness, we now proceed to prove that being "general" is indeed a generic property of Jacobians. Let $\mathcal{H}_g$ be the Siegel upper half space of symmetric $g \times g$ matrices with positive definite imaginary part. Let $\mathcal{T}_g$ be the Teichmüller space of marked Riemann surfaces. Let $\phi : \mathcal{T}_g \to \mathcal{H}_g$ be the period map that sends a marked Riemann surface to its period matrix. Let $\mathcal{J}_g$ be the image of this map, the Jacobian locus. Note that an element $\Omega \in \mathcal{H}_g$ corresponds to the principally polarized Abelian variety $A_\Omega = \mathbf{C}^g/(\mathbf{Z}^g + \Omega\mathbf{Z}^g)$. For $\Omega = X + iY \in \mathcal{H}_g$ (X, Y real matrices), we have that A_Ω is not "general" if and only if there exists a nonzero polynomial p with integer coefficients in the $g(g+1)$ variables x_{jk}, y_{jk}, $1 \leq j \leq k \leq g$, vanishing at Ω. Thus if we let $\mathcal{P}$ be the set of all nonzero polynomials with integer coefficients in these $g(g+1)$ variables (note that $\mathcal{P}$ is a countable set) and let

$\mathcal{E}_g = \bigcup_{p\in\mathcal{P}} Z(p)$ where $Z(p)$ are the zeros of p in $\mathcal{H}_g$, we have that the set of "general" principally polarized Abelian varieties corresponds to the set $\mathcal{H}_g \setminus \mathcal{E}_g$. It is clear that each $Z(p)$ is a real codimension 1 subset of $\mathcal{H}_g$, and in particular, it is nowhere dense. So $\mathcal{E}_g$, being a countable union of nowhere dense subsets, is a meager Baire category 1 subset of $\mathcal{H}_g$. So we may say that being "general" is a generic property of principally polarized Abelian varieties. We will now proceed to prove that $\mathcal{E}_g \cap \mathcal{J}_g$ is a Baire category 1 subset of $\mathcal{J}_g$. We need a few lemmas.

Lemma 6. *Any real $g \times g$ matrix with determinant 1 can be expressed as a product of upper and lower triangular matrices with ones on the diagonal.*

Proof. Take any matrix Q with determinant 1. It is well known that any matrix can be expressed as the product of a lower triangular matrix, a diagonal matrix, and an upper triangular matrix, so let $Q = LDU$. Since L and U have no zeros on their diagonals (because $\det Q = 1$), we can let $L' = LD_1^{-1}$, $U = D_2^{-1}U$, $D = D_1 D D_2$, where D_1 and D_2 are the diagonal parts of L and U, respectively. Thus $Q = L'D'U'$, where L' is a lower triangular matrix and U' is an upper triangular matrix, both with ones on their diagonal, and D' is a diagonal matrix with determinant 1.

It now suffices to show that any diagonal matrix of determinant 1 can be written as the product of upper and lower triangular matrices with ones on the diagonal. Furthermore, since any diagonal matrix of determinant 1 can be written as a product of diagonal matrices,

$$\begin{bmatrix} a_2^{-1} & & & & \\ & a_2 & & & \\ & & 1 & & \\ & & & \ddots & \\ & & & & 1 \end{bmatrix} \begin{bmatrix} a_3^{-1} & & & & \\ & 1 & & & \\ & & a_3 & & \\ & & & \ddots & \\ & & & & 1 \end{bmatrix} \cdots \begin{bmatrix} a_g^{-1} & & & & \\ & 1 & & & \\ & & \ddots & & \\ & & & 1 & \\ & & & & a_g \end{bmatrix},$$

each of which has only ones on the diagonal except for two terms, then it suffices to show that any diagonal 2×2 matrix of determinant 1 can be written as a product of upper and lower diagonal matrices with ones on the diagonal. But just note that for $a \neq 0$, we have

$$\begin{bmatrix} a & 0 \\ 0 & 1/a \end{bmatrix} = \begin{bmatrix} 1 & a-1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1/a-1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix},$$

which can be easily verified. This completes the proof. □

Lemma 7. *If $p(X+iY)$ is a polynomial only in the entries y_{jk}, $j \leq k$, of Y, such that $p(\Omega) = 0$ for all $\Omega \in \mathcal{J}_g$, then in fact $p(\Omega) = 0$ for all $\Omega \in \mathcal{H}_g$, and hence p is the zero polynomial.*

Proof. Fix any $X_0 + iY_0 \in \mathcal{H}_g$, so Y_0 is real symmetric positive definite. We may diagonalize Y_0 by an orthogonal matrix Q, which in particular we can choose to be an orthogonal matrix of determinant 1. So $Y_0 = QDQ^{-1} = QD\,{}^tQ$, where Q has determinant 1, and D is some diagonal matrix with positive entries (because Y_0 is positive definite). By Lemma 6, $Q = \prod_{l=1}^{s} U_l L_l$, where U_l and L_l are respectively upper and lower triangular matrices with ones on the diagonals. Now, define a matrix function

$$V = \prod_{l=1}^{s} \begin{bmatrix} 1 & u_{11}^{(l)} & \cdots & u_{1g}^{(l)} \\ 0 & 1 & & u_{2g}^{(l)} \\ \vdots & & \ddots & \vdots \\ 0 & \ldots & & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & \cdots & 0 \\ t_{21}^{(l)} & 1 & & \\ \vdots & & \ddots & \vdots \\ t_{g1}^{(l)} & t_{g2}^{(l)} & & 1 \end{bmatrix},$$

which is a function of $sg(g-1)$ variables $u_{jk}^{(l)}$ and $t_{kj}^{(l)}$, $1 \leq j \leq k$. Note that, for any real values of these variables, we have $\det V = 1$. Now define the function,

$$h = p(iVD\,{}^tV),$$

on the variables $u_{jk}^{(l)}$ and $t_{kj}^{(l)}$. Note that h is a polynomial in the $u_{jk}^{(l)}$ and $t_{kj}^{(l)}$ variables, with the domain of these variables being any real numbers.

Recall that a $2g \times 2g$ integer matrix

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

is symplectic if $A\,{}^tD - B\,{}^tC = I$ and $A\,{}^tB$ and $C\,{}^tD$ are symmetric. And recall that any such symplectic matrix gives a map $\mathcal{H}_g \to \mathcal{H}_g$ via $\Omega \mapsto (A\Omega + B)(C\Omega + D)^{-1}$, which fixes the Jacobian locus $\mathcal{J}_g$. Now fix the arguments $u_{jk}^{(l)}$ and $t_{kj}^{(l)}$ to be integers. Since $\det V = 1$, V^{-1} will also be an integer matrix, and so

$$\begin{bmatrix} V & 0 \\ 0 & {}^tV^{-1} \end{bmatrix}$$

is a symplectic matrix. Note that this symplectic transformation takes $\Omega \mapsto V\Omega\,{}^tV$. Since D is a diagonal matrix with positive entries, then $iD \in \mathcal{H}_g$ is a diagonal matrix. Using the degeneration techniques in [**2**], one can find a family of hyperelliptic curves whose period matrices approach an arbitrary diagonal matrix in $\mathcal{H}_g$; thus, the diagonal matrices of $\mathcal{H}_g$ are in the closure of the Jacobian locus. Hence, $iD \in \overline{\mathcal{J}_g}$. Since the map $\Omega \mapsto V\Omega\,{}^tV$ is a homeomorphism of $\mathcal{H}_g$ that fixes $\mathcal{J}_g$, then it maps the closure of $\mathcal{J}_g$ to the closure of $\mathcal{J}_g$. Hence $iVD\,{}^tV \in \overline{\mathcal{J}_g}$. Since p is a continuous function that vanishes on $\mathcal{J}_g$, it must vanish on the closure as well. Thus, $p(iVD\,{}^tV) = 0$. Thus, we have shown that h vanishes when its arguments $u_{jk}^{(l)}$ and $t_{kj}^{(l)}$ are any integers. Since h is a polynomial, then h must be the zero polynomial.

Now, for some values of $u_{jk}^{(l)}$ and $t_{kj}^{(l)}$, $V = Q$. Thus $p(iQD\,{}^tQ) = 0$. That is, $p(iY_0) = 0$, whence $p(X_0 + iY_0) = 0$. Thus, we have shown that p vanishes on all of $\mathcal{H}_g$.

Since $\mathcal{H}_g$ is an open subset of the space of symmetric $g \times g$ matrices, then viewing p as a polynomial function on $\mathbf{R}^{g(g+1)/2}$, it vanishes on an open set. Hence, p must be the zero polynomial. □

Lemma 8. *If p is a polynomial in the $g(g+1)$ real and imaginary parts of the entries of $\Omega \in \mathcal{H}_g$ such that $p(\Omega) = 0$ for all $\Omega \in \mathcal{J}_g$, then in fact $p(\Omega) = 0$ for all $\Omega \in \mathcal{H}_g$, and hence p is the zero polynomial.*

Proof. Write p as a polynomial in x_{jk} and y_{jk}, $j \leq k$, where $\Omega = X + iY = [x_{jk}] + i[y_{jk}] \in \mathcal{H}_g$ (so X is any symmetric real matrix, and Y is real symmetric positive definite). Rewrite p by collecting all like powers of x_{jk}:

$$p(\Omega) = \sum_{\alpha} p_\alpha(iY)x_\alpha,$$

where x_α are products of certain powers of the x_{jk}, and each p_α is a polynomial only in the y_{jk}.

Fix any $\Omega_0 = X_0 + iY_0 \in \mathcal{J}_g$. Consider the function

$$h(X) = p(X + iY_0)$$

defined on real symmetric matrices X. So h is really a polynomial in $m = g(g+1)/2$ variables. In the case where X has integer entries, we have that

$$\begin{bmatrix} I & X - X_0 \\ 0 & I \end{bmatrix}$$

is a symplectic matrix, so that applying this symplectic transformation to $\Omega_0 \in \mathcal{J}_g$ yields that $\Omega_0 + X - X_0 \in \mathcal{J}_g$ as well. That is, $X + iY_0 \in \mathcal{J}_g$. Then by hypothesis, $p(X + iY_0) = 0$ whenever X has integer entries. So $h(X) = 0$ whenever X has integer entries. Since h is a polynomial that vanishes on all integer arguments, then h must be the zero polynomial. That is, the coefficients of h, which are $p_\alpha(iY_0)$, must be zero.

This proves that each $p_\alpha(iY_0) = 0$ for all $X_0 + iY_0 \in \mathcal{J}_g$. So by Lemma 7, each p_α must be the zero polynomial. Hence p is the zero polynomial. □

Proposition 9. *For any $p \in \mathcal{P}$, the set $Z(p) \cap \mathcal{J}_g$ is nowhere dense in $\mathcal{J}_g$.*

Proof. Suppose by contradiction that $Z(p) \cap \mathcal{J}_g$ contains an open subset of $\mathcal{J}_g$. Then the inverse image of this open subset under ϕ is an open subset of $\mathcal{T}_g$. Since p is a real analytic map and ϕ is a complex analytic map, then their composition $p \circ \phi : \mathcal{T}_g \to \mathbf{C}$ is a real analytic map. Since $p \circ \phi$ is zero on an open subset of $\mathcal{T}_g$ and $\mathcal{T}_g$ is a connected manifold (in fact, $\mathcal{T}_g$ is homeomorphic to a ball [**1**]), then $p \circ \phi$ must be identically zero. This means that $p(\Omega) = 0$ for all $\Omega \in \mathcal{J}_g$. Thus by Lemma 8, p must be the zero polynomial, a contradiction, thus proving the proposition. □

Since $\mathcal{J}_g \cap \mathcal{E}_g = \cup_{p \in \mathcal{P}} (Z(p) \cap \mathcal{J}_g)$ and each $Z(p) \cap \mathcal{J}_g$ is nowhere dense in $\mathcal{J}_g$, then $\mathcal{J}_g \cap \mathcal{E}_g$ is a meager Baire category 1 subset of $\mathcal{J}_g$. Since $\mathcal{J}_g \cap \mathcal{E}_g$ are exactly the Jacobians which are not "general," we may assert the following.

Theorem 10. *Being "general" is a generic property of Jacobians.*

REFERENCES

1. Lipman Bers, *Finite dimensional Teichmüller spaces and generalizations*, Bull. Amer. Math. Soc. **5** (1981), 131–172.

2. John Fay, *Theta functions on Riemann surfaces*, Springer-Verlag, New York, 1973.

3. R.C. Gunning, *Riemann surfaces and generalized theta functions*, Springer-Verlag, New York, 1976.

4. G. Kempf, *Inversions of abelian integrals*, Bull. Amer. Math. Soc. **6** (1982), 25–32.

5. ———, *Rank g Picard bundles are stable*, Amer. J. Math. **112** (1990), 397–401.

6. A. Mattuck, *Cycles on abelian varieties*, Proc. Amer. Math. Soc. **9** (1958), 88–97.

7. ———, *Symmetric products and Jacobians*, Amer. J. Math. **83** (1961), 189–206.

8. I. Schur, *Einige Sätze über Primzahlen mit Anwendungen auf Irreduzibilitätsfragen*, I. Sitzungsber. Berliner Akad. (1929), 125–136.

9. H. Umemura, *On a property of symmetric products of a curve of genus* 2, Proc. Int. Symp. on Algebraic Geometry, Kyoto (1977), 709–721.

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