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LIMITS OF DIRICHLET FINITE FUNCTIONS ALONG CURVES

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Let R be a noncompact connected orientable real analytic Riemannian n-manifold. In formulating the Dirichlet principle in the absence of a border for R various types of behavior at the ideal boundary have been used. Brelot [1] considered limits of functions along a family of curves tending to the ideal boundary, the collection of Green's lines. Royden [11] introduced a compactification R^* of R to which Dirichlet finite functions extend continuously and considered values of functions on Δ, the harmonic part of $R^*\backslash R$, as boundary values. Nakai [6], [7] showed that for Dirichlet finite functions these two modes of behavior are in a sense the same. Subsequently Ohtsuka [8] used limits along arbitrary curves tending to the ideal boundary and extremal length to specify boundary behavior. Since Δ is a tractable analytic device and extremal length is related to the geometry of R, it is important to determine the connection between the latter sort of boundary behavior and the former ones.

Let $\tilde{M}(R)$ denote the space of Tonelli functions on R with finite Dirichlet integrals, $D_R(f) = \int_R df \wedge * df < +\infty$. We shall show that an $f \in \tilde{M}(R)$ has limit 0 along almost every curve (in the sense of extremal length) joining a fixed parametric ball to the ideal boundary if and only if the values of f on Δ are 0. In particular, given a function $g \in \tilde{M}(R)$ the solution to the Dirichlet problem having the same boundary values as g does not depend on which of the above meanings is assigned to the term boundary values. As an application we give a criterion for R to carry nonconstant Dirichlet finite harmonic functions.

1. We begin by organizing some terminology for later use. We say that an open set $\mathcal{O} \subset R$ is an *end of R* if the relative boundary $\partial\mathcal{O}$ is piecewise smooth and compact in R whereas $\bar{\mathcal{O}}$ is not compact. A region $\Omega \subset R$ will be called *regular* if $\bar{\Omega}$ is compact and $\partial\Omega$ is piecewise smooth. The *relative harmonic measure* of an end $\mathcal{O}$ of R, $\omega = \omega(\cdot\,; \mathcal{O}, R)$ is defined as follows. Let $\{R_m | m = 1, 2, \ldots\}$ be an exhaustion of R by regular regions with $\partial\mathcal{O} \subset R_1$ and let $\omega_m = \omega_m(\cdot\,; \mathcal{O}, R)$ be in $\tilde{M}(R)$ such that $\omega_m | R\backslash\mathcal{O} = 0$, $\omega_m | \mathcal{O}\backslash R_m = 1$ and $\omega_m | \mathcal{O} \cap R_m$ is harmonic. By the maximum principle $0 \leqq \omega_{m+1} \leqq \omega_m \leqq 1$. Hence by the Harnack principle we may define

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$\omega(\cdot\,; \mathcal{O}, R) = \lim_{m\to+\infty} \omega_m(\cdot\,; \mathcal{O}, R)$. The function ω is continuous on R with $\omega|R\backslash\mathcal{O} = 0$, $\omega|\mathcal{O}$ harmonic. For any $k > m$, $D_R(\omega_k - \omega_m, \omega_k) = 0$ by the harmonicity of ω_k on $\mathcal{O} \cap R_k$ and consequently $0 \leqq D_R(\omega_m - \omega_k) = D_R(\omega_m) - D_R(\omega_k)$. From this we see that $\{\omega_m\}$ is Cauchy with respect to the Dirichlet seminorm and consequently by Fatou's lemma we conclude that

$$\omega(\cdot\,; \mathcal{O}, R) = D\text{-}\lim_{m\to+\infty} \omega_m(\cdot\,; \mathcal{O}, R). \tag{1}$$

Let $\mathcal{O}$ be an end of R such that $R\backslash\mathcal{O}$ is compact. Then R is said to be *parabolic* if $\omega = 0$ and *hyperbolic* otherwise. This is equivalent to the other definitions and in particular is independent of $\mathcal{O}$ (cf. **[4]**, **[12]**).

2. We shall be concerned with the critical points E of a nonconstant harmonic function on a subregion U of R. Since such functions are real analytic, we see by Lemma 12 of [2] that E is a union of countably many submanifolds of dimensions at most $n - 2$ and in particular E is polar. We shall need the following stronger statement.

LEMMA. *Let R be hyperbolic and U be a subregion of R with ∂U piecewise smooth. Let E be the set of critical points of a nonconstant harmonic function on U and assume that $E \cap \partial U = \varnothing$. Then there is a function $\varphi \in \tilde{M}(R\backslash E)$ such that $\varphi|R\backslash U = 0$ and for every $p_0 \in E$*

$$\lim_{R\backslash E\ni p\to p_0} \varphi(p) = +\infty. \tag{2}$$

Take an exhaustion $\{G_k|k = 1, 2, \ldots\}$ of R by regular regions and set $U_k = U \cap G_k$, $E_k = E \cap \bar{G}_k$, a compact set, $k = 1, 2, \ldots$. Fix k for the time being and consider $U_{k+1}\backslash E_k$ as an end of $R\backslash E_k$. Set $\omega^{(k)} = \omega(\cdot\,; U_{k+1}\backslash E_k, R\backslash E_k)$, the relative harmonic measure of $U_{k+1}\backslash E_k$ in $R\backslash E_k$. Since E is polar and R is hyperbolic, there is a positive superharmonic function s on R with $s|E = +\infty$. For an arbitrary $\varepsilon > 0$ the function $\omega^{(k)} - \varepsilon s$ is subharmonic on $U_{k+1}\backslash E_k$, bounded above and has nonpositive limit superior at each point of the relative boundary of $U_{k+1}\backslash E_k$ in R. Thus $\omega^{(k)} \leqq \varepsilon s$ which implies that $\omega^{(k)} = 0$. In view of (1) we may choose a matrix of positive integers $\{m_{kj}\}$ such that the defining sequence $\omega_m^{(k)} = \omega_m(\cdot\,; U_{k+1}\backslash E_k, R\backslash E_k)$ for $\omega^{(k)}$ has the property that

$$D_{R\backslash E_k}(\omega_{m_{kj}}^{(k)}) < 4^{-j}.$$

Also define $\omega_m^{(k)}|E_k = 1$ and note that this makes $\omega_m^{(k)}$ continuous on R.

Let $\{C_i\}$ be an exhaustion of $U\backslash E$ by compact sets. For each positive integer i we may choose k_i such that $C_i \subset U_{k_i+1}\backslash E_{k_i}$ and then we may pick $j_i \geqq i$ such that

$$w_i = \omega_{m_{k_i j_i}}^{(k_i)}$$

is harmonic on C_i. Of course $D_{R\setminus E}(w_i) < 4^{-i}$. Defining $\varphi_i = \sum_1^i w_i$ gives a D-Cauchy sequence. In fact for $i' > i$

(3) $$D^{1/2}_{R\setminus E}(\varphi_{i'} - \varphi_i) \leqq \sum_i^{i'} D^{1/2}_{R\setminus E}(w_j) < 2^{1-i}.$$

For any compact $C \subset U\setminus E$ all but finitely many w_i are harmonic on C and thus the sequence $\{\varphi_i\}$ converges uniformly on compact subsets of $U\setminus E$ to a function φ or to $+\infty$.

Each point of ∂U has a compact neighborhood N with $N \subset R\setminus E$. Given any compact set A contained in the interior of N there is a constant a such that $u(p) \leqq aD_N(u)$ for every $u \in \tilde{M}(N)$ with $u|N\setminus U = 0$, u harmonic on the interior of $N \cap U$ and every $p \in A$ (cf. **[4]**). Since $\varphi_i|R\setminus U = 0$, we can deduce from this and (3) that $\{\varphi_i\}$ actually converges uniformly on compact subsets of $R\setminus E$ to a function φ. By Kawamura's lemma (cf. **[4]**, **[12]**) we conclude that $\varphi \in \tilde{M}(R)$. Finally, for any $p_0 \in E$ there are infinitely many w_j with $w_j(p_0) = 1$. Since $\lim \inf_{R\setminus E \ni p \to p_0} \varphi(p) \geqq \varphi_i(p_0)$ for any i, we conclude that (2) holds.

3. Let $\mathscr{H}$ be a family of locally rectifiable curves in R. A nonnegative Borel measurable function ρ is called *admissible* with respect to $\mathscr{H}$ if $\int_\gamma \rho ds \geqq 1$ for each curve $\gamma \in \mathscr{H}$. The *modulus* of $\mathscr{H}$ (which is the reciprocal of the extremal length of $\mathscr{H}$) is defined as $\operatorname{mod} \mathscr{H} = \inf \int_R \rho^2 dV$, where the infimum is taken over all admissible ρ with respect to $\mathscr{H}$. A property is said to hold for *almost every* curve in $\mathscr{H}$ if it holds for all curves in $\mathscr{H}\setminus\mathscr{H}_0$, where $\mathscr{H}_0$ is a subfamily with $\operatorname{mod} \mathscr{H}_0 = 0$. The well known elementary properties of modulus and extremal length continue to hold for $n > 2$ with the above definition and we shall use them freely.

The following is an analogue of a result of Brelot-Choquet **[2]** and will play a fundamental role here.

LEMMA. *Let R, U, E be as in Lemma 2 and $\mathscr{H}$ a family of locally rectifiable curves in U. Denote by $\mathscr{H}_E$ those curves in $\mathscr{H}$ which do not pass through or terminate at points of E. Then* $\operatorname{mod} \mathscr{H}_E = \operatorname{mod} \mathscr{H}$.

Clearly $\operatorname{mod} \mathscr{H}_E \leqq \operatorname{mod} \mathscr{H}$. To establish the reverse inequality let $\eta > 0$ be arbitrary. Then there is a ρ admissible with respect to $\mathscr{H}_E$ with $\operatorname{mod} \mathscr{H}_E + \eta > \int_R \rho^2\, dV$. Let φ be the function of Lemma 2. For any curve $\gamma \in \mathscr{H}\setminus\mathscr{H}_E$ and an arbitrary $\varepsilon > 0$, the function $\varepsilon\,|\nabla\varphi|$ has the property $\int_\gamma \varepsilon|\nabla\varphi|\, ds \geqq \int_\gamma \varepsilon|d\varphi| = +\infty$. Thus the function $\rho_\varepsilon = \max(\rho, \varepsilon|\nabla\varphi|)$ is admissible with respect to $\mathscr{H}$ and we obtain

$$\operatorname{mod} \mathscr{H} \leqq \int_R \rho_\varepsilon^2\, dV \leqq \int_R \rho^2\, dV + \varepsilon^2 D_R(\varphi) < \operatorname{mod} \mathscr{H}_E + \eta + \varepsilon^2 D_R(\varphi),$$

which establishes $\operatorname{mod} \mathscr{H} \leqq \operatorname{mod} \mathscr{H}_E$.

4. Fix a parametric ball B in R and denote by $\mathscr{G}$ the family of all curves in $R \backslash B$ joining ∂B to the ideal boundary of R; i.e. $\gamma \in \mathscr{G}$ if γ: $[a, b) \to R\backslash B$, (b may be $+\infty$) is locally rectifiable, $\gamma(a) \in \partial B$, $\gamma(t) \in R \backslash \bar{B}$ if $t \neq a$ and for any compact set $C \subset R$ there is a $t_0 \in [a, b)$ such that $\gamma(t) \notin C$ for every $t > t_0$. The notion of parabolicity is related to modulus as follows.

PROPOSITION. *R is parabolic if and only if* mod $\mathscr{G} = 0$.

For $n > 2$ this is due to *Ow* **[9]**. We shall present a new proof to which we shall refer later. Consider the functions $\omega_m = \omega_m(\cdot\,; R\backslash\bar{B}, R)$ defining ω. For any $\gamma \in \mathscr{G}$. $\int_\gamma |\nabla \omega_m|\, ds \geqq \int_\gamma d\omega_m = 1$ which shows that $|\nabla \omega_m|$ is admissible with respect to $\mathscr{G}$. Therefore, $0 \leq$ mod $\mathscr{G} \leq \lim_m D_R(\omega_m)$. If R is parabolic, then (1) implies that mod $\mathscr{G} = 0$.

Conversely assume that R is hyperbolic. The function $\omega = \omega(\cdot\,; R\backslash\bar{B}, R)$ can be assumed to be harmonic on $R\backslash B$ by suitably redefining it in a neighborhood of ∂B. Let E be the critical points of ω in $R\backslash B$. We may further assume that $E \cap \partial B = \emptyset$ for if this were not the case we would shrink B slightly and this would only decrease mod $\mathscr{G}$. Denote by $\mathscr{H}$ the family of maximal integral curves of $\nabla\omega$ starting at points of B. Every point of B is the starting point of a curve in $\mathscr{H}$. Consider $\mathscr{H}_E$ the subfamily of curves in $\mathscr{H}$ which do not terminate at points of E. Clearly $\mathscr{H}_E \subset \mathscr{G}$. Let ρ be an admissible density with respect to $\mathscr{H}$. Then for any $\gamma \in \mathscr{H}$ $1 \leqq (\int_\gamma \rho ds)^2 = (\int_\gamma \rho/|\nabla\omega|\, d\omega)^2 \leqq \int_\gamma d\omega \int_\gamma \rho^2/|\nabla\omega|^2\, d\omega \leqq \int_\gamma \rho^2/|\nabla\omega|^2\, d\omega$. Here we have used the fact that along an integral curve of $\nabla\omega$, $|\nabla\omega| ds = d\omega$. We choose the orientation for ∂B that gives $*d\omega > 0$ on ∂B, multiply the above inequality by $*d\omega$ and integrate to obtain $\int_{\partial B} *d\omega \leqq \int_{\partial B}(\int_\gamma \rho^2/|\nabla\omega|^2 d\omega)*d\omega = \int_H \rho^2 dV \leqq \int_R \rho^2 dV$, where H is the subset of R foliated by the curves in $\mathscr{H}$. Thus by Lemma 3 we conclude that $0 < \int_{\partial B} *d\omega \leqq$ mod $\mathscr{H} =$ mod $\mathscr{H}_E \leqq$ mod $\mathscr{G}$.

5. It is easily seen that for any $f \in \tilde{M}(R)$ the limit $\lim_{t\to b} f(\gamma(t))$ exists as a finite real number for almost every $\gamma \in \mathscr{G}$. We denote this limit simply by $f(\gamma)$. In addition we shall use the notation $e(\gamma)$ for the end part of $\gamma \in \mathscr{G}$ in the Royden compactification R^*; i.e., $e(\gamma) = \bar{\gamma} \cap \Gamma$, where $\bar{\gamma}$ denotes the closure in R^* of the image set under γ. It is easy to verify that for $f \in \tilde{M}(R)$, $f(\gamma)$ exists and is equal to a if and only if $f|e(\gamma)$ is the constant a.

LEMMA. *Let $\mathscr{G}_0$ be a subfamily of $\mathscr{G}$ such that $F = cl(\bigcup_{\gamma\in\mathscr{G}_0} e(\gamma))$ is disjoint from Δ, the harmonic boundary. Then* mod $\mathscr{G}_0 = 0$.

The parabolicity of R is equivalent to $\Delta = \emptyset$ (cf. **[4]**, **[12]**). In case R is parabolic the assertion follows from Proposition 4. If R is hyperbolic, then there is a nonnegative superharmonic function $v \in \tilde{M}(R)$ such that $v|F = +\infty$, $v|\Delta = 0$ (cf. **[4]**, **[12]**). For any $\gamma \in \mathscr{G}_0$ we have $e(\gamma) \subset F$ and therefore $v(\gamma) = +\infty$. For an arbitrary $\varepsilon > 0$ and $\gamma \in \mathscr{G}_0$ we see that $\int_\gamma \varepsilon|\nabla v|\, ds \geqq \int_\gamma \varepsilon dv = +\infty$; i.e., $\varepsilon|\nabla v|$ is admissible with respect to $\mathscr{G}_0$. We conclude that mod $\mathscr{G}_0 \leqq \varepsilon^2 D_R(v)$ and the assertion follows.

6. The converse of the above lemma is of course not true. Simply consider a single line segment in the open unit disk with one end point on the unit circle. The following is a partial converse.

LEMMA. *Let $\mathscr{G}_0$ be a subfamily of $\mathscr{G}$ with* mod $\mathscr{G}_0 = 0$. *Then the set* $K = \mathrm{cl}(\bigcup_{\gamma \in \mathscr{G} \setminus \mathscr{G}_0} e(\gamma))$ *contains* Δ.

Of course we only need to consider $\Delta \neq \emptyset$. In this case we see by Proposition 4 that $\mathrm{mod}(\mathscr{G}\setminus\mathscr{G}_0) > 0$ and then by Lemma 5 that $K \cap \Delta \neq \emptyset$. Assume that, contrary to the assertion, there is a point $p^* \in \Delta \setminus K$. Choose a function $h \in HD(R)$ such that $h(p^*) = 1$, $h|K \cap \Delta = 0$ and $0 < h < 1$ on R. Furthermore we may pick $\alpha \in (0, 1)$ such that α is not a critical value of h and the level surface $\{h = \alpha\}$ intersects B. Let U be a component of the set $\{h > \alpha\}$ that intersects B and set $S = \partial U \cap B$. By the choice of α each point of S is the initial point of an integral curve of ∇h in U and we denote by $\mathscr{H}$ the family of all such integral curves which are maximal. By the same argument as in the proof of Lemma 3 we see that $0 < \int_S {}^*dh \leqq \mathrm{mod}\,\mathscr{H}$ with the appropriate orientation on S. Let E be the critical points of h in U and $\mathscr{H}_E$ the subfamily of curves in $\mathscr{H}$ not terminating at points of E. By Lemma 3 we have mod $\mathscr{H}_E > 0$. The curves of $\mathscr{H}_E$ tend to the ideal boundary of R and we form a new family $\mathscr{H}_1$ consisting of the portions of the curves in $\mathscr{H}_E$ joining ∂B to the ideal boundary of R. Then mod $\mathscr{H}_1 \geqq$ mod $\mathscr{H}_E > 0$ and $\mathscr{H}_1$ is a subfamily of $\mathscr{G}$.

Now define $K_1 = \mathrm{cl}(\bigcup_{\gamma \in \mathscr{H}_1 \setminus \mathscr{G}_0} e(\gamma))$. Since mod $(\mathscr{H}_1 \setminus \mathscr{G}_0) > 0$, we see by Lemma 5 that $K_1 \cap \Delta \neq \emptyset$. On the one hand, $K_1 \cap \Delta \subset K \cap \Delta$ and consequently $h \mid K_1 \cap \Delta = 0$. But on the other hand, for every $\gamma \in \mathscr{H}_1$ we have $h(\gamma) > \alpha$ which implies $h \mid K_1 \geqq \alpha$. The contradiction completes the proof.

7. We are ready to establish our main result.

THEOREM. *Let $f \in \tilde{M}(R)$. Then $f(\gamma) = 0$ for almost every $\gamma \in \mathscr{G}$ if and only if $f|\Delta = 0$.*

Again no proof is necessary if R is parabolic and we turn to the hyperbolic case. The necessity is a simple consequence of Lemma 6. For the proof of the sufficiency assume that $f|\Delta = 0$. By considering the positive and negative parts of f separately we may restrict our attention to the case $f \geqq 0$. For each positive integer k consider the family $\mathscr{G}_k = \{\gamma \in \mathscr{G} | f(\gamma)$ exists and $f(\gamma) \geqq k^{-1}\}$. Also define $K_k = \mathrm{cl}(\bigcup_{\gamma \in \mathscr{G}_k} e(\gamma))$. In view of $f \mid \Delta = 0$ we see that K_k is disjoint from Δ and hence by Lemma

5 mod $\mathscr{G}_k = 0$. Let $\mathscr{G}_0$ be a family of curves in $\mathscr{G}$ such that mod $\mathscr{G}_0 = 0$ and $f(\gamma)$ exists for each $\gamma \in \mathscr{G}\backslash\mathscr{G}_0$. Then $\mathscr{G}_\infty = \bigcup_0^\infty \mathscr{G}_k$ has mod $\mathscr{G}_\infty = 0$ and $f(\gamma) = 0$ for every $\gamma \in \mathscr{G}\backslash\mathscr{G}_\infty$.

8. There are a number of extremal length criteria for the nonexistence of nonconstant Dirichlet finite harmonic functions on R (cf. **[10]**, **[5]**, **[14]**). We have the following result.

COROLLARY. dim $HD(R) = 1$ *if and only if for each* $f \in \tilde{M}(R)$ *there is a constant* c_f *such that* $f(\gamma) = c_f$ *for almost every* $\gamma \in \mathscr{G}$.

If R is parabolic, then dim $HD(R) = 1$ and mod $\mathscr{G} = 0$ which means that the corollary holds. Assume that R is hyperbolic. If dim $HD(R) = 1$, then Δ consists of one point p^* and for any $f \in \tilde{M}(R)$ the required constant c_f is $f(p^*)$. Indeed, since $f - c_f | \Delta = 0$, the theorem implies that $(f - c_f)(\gamma) = 0$ for almost every $\gamma \in \mathscr{G}$. If dim $HD(R) > 1$, then there is a nonconstant bounded function $h \in HD(R)$ which may be normalized to satisfy $h(R) = (0, 1)$ (cf., e.g., **[4]**). By the technique in the proof of Lemma 6 we can produce an $\alpha \in (0, 1)$ and $\mathscr{H}_1 \subset \mathscr{G}$ with mod $\mathscr{H}_1 > 0$ and $h(\gamma) > \alpha$ for every $\gamma \in \mathscr{H}_1$, as well as a subfamily $\mathscr{H}_2 \subset \mathscr{G}$ with mod $\mathscr{H}_2 > 0$ and $h(\gamma) < \alpha$ for every $\gamma \in \mathscr{H}_2$. Thus for h there is no constant c_h satisfying the condition of the corollary.

As an illustration consider $\mathbf{R}^n$, $n \geqq 3$. Uspenskiĭ **[13]** showed that a smooth function f on $\mathbf{R}^n$ with $|\nabla f| \in L^p(\mathbf{R}^n)$, $1 < p < n$, has the same limit along all rays except for a collection of rays piercing the unit sphere in a set of $(n - 1)$-dimensional measure zero. It is easily seen that dim $HD(\mathbf{R}^n) = 1$ (cf., e.g., **[1]**). Thus the corollary contains the case $p = 2$ of Uspenskiĭ's result. Indeed, by the corollary there is a subset $\mathscr{G}_f$ of $\mathscr{G}$ with modulus zero and a constant c_f such that $f(\gamma) = c_f$ for every $\gamma \in \mathscr{G}\backslash\mathscr{G}_f$. The argument of No. 4 shows that a collection of radial lines piercing the unit sphere in a set of positive $(n - 1)$-dimensional outer measure has positive modulus and thus is not contained in the exceptional set $\mathscr{G}_f$.

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