J. Math. Kyoto Univ. (JMKYAZ)
24-4 (1984) 705-712

Mixed problems for evolution equations II

by

Reiko SAKAMOTO

(Communicated by Prof. S. Mizohata December 14, 1983)

We discussed non-characteristic boundary value problems for single evolution equations in Part I([1]). In this paper, we consider character-fstic boundary value problems for single evolution equations. Under the assumption of mildness of characteristic, we can consider the problems in the similar framework to that in Part I. We shall be satisfied only to show the way to get the basic energy inequality, but it is easy to get the existence and uniqueness theorem in H^∞-sense by the similar way as in Part I.

§ 1. Assumptions.

Let $A(t, x, y; \tau, \xi, \eta)$ and $\{B_j(t, y; \tau, \xi, \eta)\}_{j=1,\dots,m_+}$, where we denote

$$B(t, y; \tau, \xi, \eta)=\begin{bmatrix} B_1(t, y; \tau, \xi, \eta) \\ \vdots \\ B_{m_+}(t, y; \tau, \xi, \eta) \end{bmatrix},$$

be polynomials with respect to (τ, ξ, η) with $\mathscr{B}^\infty$-coefficients in R^{n+1}, which are constant outside a ball in R^{n+1}. Let N be the Newton polygon of A in the sense of Part I, where we consider the following two cases in unified manner.

Case 1. The vertices of N are composed of the origin and

$$P_i=(\mu_{i+1}+\cdots+\mu_l, m_1+\cdots+m_i) \qquad (i=0,\dots, l),$$

where

$$m_i/\mu_i=p_i \text{ and } p_1>p_2>\dots>p_l=1 \qquad (\sum_{i=1}^l m_i=m, \sum_{i=1}^l \mu_i=\mu).$$

Case 2. The vertices of N are composed of the origin, $P_1, P_2, \dots, P_{l-1}$, and

$$P_l'=(1, m-1), \ P_l''=(0, m-1).$$

The principal part of N is

$$N_\circ=\begin{cases}\bigcup_{i=1}^{l}\overline{P_{i-1}P_i} & \text{in case } 1,\\ (\bigcup_{i=1}^{l-1}\overline{P_{i-1}P_i})\cup\overline{P_{l-1}P'_l} & \text{in case } 2.\end{cases}$$

Denotiong

$$A(t, x, y; \tau, \xi, \eta)=\sum_{(\sigma,\lambda+|\nu|)\in N} a_{\sigma\lambda\nu}(t, x, y)\tau^\sigma\xi^\lambda\eta^\nu,$$

$$A_\circ(t, x, y; \tau, \xi, \eta)=\sum_{(\sigma,\lambda+|\nu|)\in N_\circ} a_{\sigma\lambda\nu}(t, x, y)\tau^\sigma\xi^\lambda\eta^\nu,$$

$$A^{(i)}(t, x, y; \tau, \xi, \eta)=\sum_{(\sigma,\lambda+|\nu|)\in\overline{P_{i-1}P_i}} a_{\sigma\lambda\nu}(t, x, y)\tau^{\sigma-\mu_i}\xi^\lambda\eta^\nu$$

$$=\sum_{j=0}^{\mu_i}\overset{\circ}{a}_{m_1+\cdots+m_{i-1}+jp_i}(t, x, y; \xi, \eta)\tau^{\mu_i-j},$$

we assume

Assumption (A′).

i) $\overset{\circ}{a}_{m_1+\cdots+m_i}(t, x, y; \xi, \eta)\neq 0$ for $(t, x, y; \xi, \eta)\in R^{n+1}\times S^{n-1}$ $(i=1,\dots, l-1)$,

ii) p_i is even and $A^{(i)}(t, x, y; \tau, \xi, \eta)\neq 0$ for $(t, x, y; \xi, \eta)\in R^{n+1}\times S^{n-1}$ and $\operatorname{Im}\tau\leqq 0$ $(i=1,\dots, l-1)$,

iii) $p_l=1$, zeros of $A^{(l)}(t, x, y; \tau, \xi, \eta)$ with respect to τ are real and distinct for $(t, x, y; \xi, \eta)\in R^{n+1}\times S^{n-1}$,

iv) $\overset{\circ}{a}_m(t, x, y; 1, 0)=0$,

v) zeros of $A^{(l)}(t, x, y; \tau, \xi, \eta)$ with respect to ξ are bounded if (τ, η) is bounded.

Remark. It follows from (iv) that (0, 1, 0) is characteristic with respect to $A(t, x, y; \tau, \xi, \eta)$, and (v) means that it is mild.

Hereafter we only write $A(\tau, \xi, \eta)$ instead of $A(t, x, y; \tau, \xi, \eta)$ for simplicity.

Lemma 1.1. *Under the Assumption (A′), we have*

i) $\overset{\circ}{a}_{m-1}(1, 0)\neq 0$,

ii) $$A^{(l)}(\tau, \xi, \eta)=\theta\,\mathscr{A}^{(l)}(\theta, \xi, \eta)=\theta\sum_{j=0}^{m_l-1}\alpha_{m_1+\cdots+m_{l-1}+j}(\xi, \eta)\theta^{m_l-j-1},$$

where

$$\theta=\tau+(\overset{\circ}{a}_{m-1}(1, 0))^{-1}\sum_{j=1}^{n-1}\frac{\partial\overset{\circ}{a}_m}{\partial\eta_j}(1, 0)\eta_j$$

and $\alpha_j(\xi, \eta)$ *is a homogeneous polynomial of order* j *with respect to* (ξ, η),

iii) $\alpha_{m_1+\cdots+m_{l-1}}(\xi, \eta)\neq 0$ *and* $\alpha_{m_1+\cdots+m_l-1}(\xi, \eta)\neq 0$ *for* $(\xi, \eta)\in S^{n-1}$.

Proof. i) Since $a_m^\circ(1, 0)=0$ from $(A')-(\text{iv})$, we have

$$A^{(l)}(\tau, 1, 0)=a_{m-m_l}^\circ(1, 0)\tau^{m_l}+\cdots+a_{m-1}^\circ(1, 0)\tau,$$

whose zeros are distinct from $(A')-(\text{iii})$. Hence we have $a_{m-1}^\circ(1, 0)\neq 0$.

ii) Let us denote

$$A^{(l)}(\tau, \xi, \eta)=\sum_{j=1}^{m}\theta_j(\tau, \eta)\xi^{m-j},$$

where $\theta_j(\tau, \eta)$ is a homogeneous polynomial of order j with respect to (τ, η) $(j=1,\dots, m)$,

$$\begin{aligned}\theta_1(\tau, \eta)&=a_{m-1}^\circ(1, 0)\tau+\sum_{j=1}^{n-1}\frac{\partial a_m^\circ}{\partial\eta_j}(1, 0)\eta_j\\&=a_{m-1}^\circ(1, 0)\theta(\tau, \eta),\end{aligned}$$

and

$$\theta_j(\tau, \eta)=r_j(\eta)\qquad(j>m_l).$$

Deviding $\theta_j(\tau, \eta)$ by $\theta(\tau, \eta)$ for $j=1,\dots, m_l$, we have

$$\theta_j(\tau, \eta)=\theta(\tau, \eta)\tilde{\theta}_{j-1}(\tau, \eta)+r_j(\eta),$$

where $\tilde{\theta}_{j-1}(\tau, \eta)$ is a homogeneous polynomial of order $j-1$ with respect to (τ, η) and $r_j(\eta)$ is a homogeneous polynomial of order j with respect to η. Here we have

$$\begin{aligned}A^{(l)}(\tau, \xi, \eta)&=\theta(\tau, \eta)\{\tilde{\theta}_\circ\xi^{m-1}+\tilde{\theta}_1(\tau, \eta)\xi^{m-2}+\cdots\\&\quad\cdots+\tilde{\theta}_{m_l-1}(\tau, \eta)\xi^{m-m_l}\}+\{r_2(\eta)\xi^{m-2}+\cdots+r_m(\eta)\}\\&=\theta(\tau, \eta)\tilde{A}^{(l)}(\tau, \xi, \eta)+r(\xi, \eta),\end{aligned}$$

where $\tilde{\theta}_\circ=a_{m-1}^\circ(1, 0)\neq 0$.

Now we shall show that $r(\xi, \eta)\equiv 0$. In fact, if not, we have

$$r_2(\eta)\equiv\dots\equiv r_{h-1}(\eta)\equiv 0,\ \ r_h(\eta^\circ)\neq 0\quad(\eta^\circ\in S^{n-2}).$$

Since

$$\theta(\tau, \eta^\circ)=\tau-\tau(\eta^\circ),\ \ |\tau(\eta^\circ)|<M,$$

we have

$$\begin{aligned}A^{(l)}(\tau, \xi, \eta^\circ)&=\theta(\tau, \eta^\circ)\{\tilde{\theta}_\circ\xi^{m-1}+\tilde{\theta}_1(\tau, \eta^\circ)\xi^{m-2}\\&\quad+\cdots\cdots+\tilde{\theta}_{h-2}(\tau, \eta^\circ)\xi^{m-h+1}\\&\quad+(\tilde{\theta}_{h-1}(\tau, \eta^\circ)+r_h(\eta^\circ)/\theta(\tau, \eta^\circ))\xi^{m-h}+\cdots\}\end{aligned}$$

in $\{|\tau|<M \text{ and } \tau\neq\tau(\eta^\circ)\}$, where the coefficient of ξ^{m-h} is unbounded, which is the contradiction to $(A')-(\text{v})$.

By the change of variable τ to $\theta=\theta(\tau, \eta)$, we have

$$\tilde{A}^{(l)}(\tau,\ \xi,\ \eta)=\mathscr{A}^{(l)}(\theta,\ \xi,\ \eta)$$
$$=\alpha_{m_1+\cdots+m_{l-1}}(\xi,\ \eta)\theta^{m_l-1}+\cdots+\alpha_{m_1+\cdots+m_{l-1}}(\xi,\ \eta),$$

where $\alpha_j(\xi,\ \eta)$ is a homogeneous polynomial with respect to $(\xi,\ \eta)$ of order j.

iii) Since $\theta(\tau,\ \eta)=\tau-\tau(\eta)$, we have that the coefficient of τ^{m_l} in $A^{(l)}(\tau,\ \xi,\ \eta)$ is equal to the coefficient of θ^{m_l} in $\theta\mathscr{A}^{(l)}(\theta,\ \xi,\ \eta)$, that is,

$$\alpha_{m_1+\cdots+m_{l-1}}(\xi,\ \eta)=a_{m_1+\cdots+m_{l-1}}(\xi,\ \eta).$$

On the other hand, since $\tau=\tau(\eta)$ is the simple root of $A^{(l)}(\tau,\ \xi,\ \eta)$, we have

$$\alpha_{m_1+\cdots+m_{l-1}}(\xi,\ \eta)=\mathscr{A}^{(l)}(0,\ \xi,\ \eta)\neq 0. \quad ■$$

Now we consider of $A(\tau,\ \xi,\ \eta)$, whose Newton polygon is N, whose principal part is $N_\circ$. Let us denote

$$N'=\{N-(0,\ 1)\}\cap N.$$

We say that a polynomial is composed of lower order terms with respect to N, if its Newton polygon is contained in N', and we denote it l. o. t. (N').

Since

$$A(\tau,\ \xi,\ \eta)=\tau^\mu+\overset{\circ}{a}_{p_1}(\xi,\ \eta)\tau^{\mu-1}+\cdots+\overset{\circ}{a}_{m_1+\cdots+m_{l-1}-p_{l-1}}(\xi,\eta)\tau^{m_l+1}$$
$$+A^{(l)}(\tau,\ \xi\ \ \eta)+\text{l. o. t. }(N')$$
$$=\theta(\tau,\ \eta)^\mu+\overset{\circ}{a}_{p_1}(\xi,\ \eta)\theta(\tau,\ \eta)^{\mu-1}+\cdots+\overset{\circ}{a}_{m_1+\cdots+m_{l-1}-p_{l-1}}(\xi,\ \eta)\theta(\tau,\ \eta)^{m_l+1}$$
$$+\theta(\tau,\ \eta)\mathscr{A}^{(l)}(\theta(\tau,\ \eta),\ \xi,\ \eta)+\text{l. o. t. }(N'),$$

we have

$$A(\tau,\ \xi,\ \eta)=\theta(\tau,\ \eta)\mathscr{A}(\theta(\tau,\ \eta),\ \xi,\ \eta)+\text{l. o. t. }(N'),$$

where we denote

$$\alpha_j(\xi,\ \eta)=\overset{\circ}{a}_j(\xi,\ \eta) \qquad (j=1,\ldots,m_1+\ldots+m_{l-1}-1).$$

Since $\mathscr{A}^{(i)}(\theta,\xi,\eta)=A^{(i)}(\theta,\ \xi,\ \eta)$ $(i=1,\ldots,l-1)$, we have from Lemma 1.1

Proposition 1.2. *Under the assumption* (A') *for the polynomial* A, $\mathscr{A}$ *satisfies the following condition, which is the assumption* (A), *stated in Part I, for the polynomial* $\mathscr{A}$.

i) $\alpha_{m_1+\ldots+m_i}(\xi,\ \eta)\neq 0$ $(i=1,\ldots,l-1)$ *and* $\alpha_{m_1+\ldots+m_l-1}(\xi,\ \eta)\neq 0$ *for* $(\xi,\ \eta)\in S^{n-1}$,

ii) $\mathscr{A}^{(i)}(\theta, \xi, \eta) \neq 0 \quad (i=1, \dots, l-1)$ *for* $(\xi, \eta) \in S^{n-1}$ *and* $Im\, \theta \leqq 0$,

iii) *zeros of* $\mathscr{A}^{(l)}(\theta, \xi, \eta)$ *wtth respect to* θ *are real and distinct for* $(\xi, \eta) \in S^{n-1}$.

Now let

$\mathscr{A}(\theta, \xi, \eta) = c \prod_{j=1}^{m_+} (\xi - \xi_j^+(\theta, \eta)) \prod_{j=1}^{m_-} (\xi - \xi_j^-(\theta, \eta)) \quad (m_+ + m_- = m-1)$, where $\mathrm{Im}\, \xi_j^{\pm}(\theta, \eta) \gtrless 0$ if $\mathrm{Im}\, \theta < -K$ and $\eta \in R^{n-1}$. Let $B(\tau, \xi, \eta) = \{B_j(\tau, \xi, \eta)\}_{j=1,\dots,m_+}$ be boundary polynomials, and let us denote

$$B_j(\tau, \xi, \eta) = \mathscr{B}_j(\theta(\tau, \eta), \xi, \eta),$$
$$B(\tau, \xi, \eta) = \mathscr{B}(\theta(\tau, \eta), \xi, \eta).$$

Let $\mathscr{B}^{\sharp}(\theta, \xi, \eta)$ be the standardization of $\mathscr{B}(\theta, \xi, \eta)$ with respect to $\mathscr{A}(\theta, \xi, \eta)$. Let $R^{(i)}(\theta, \eta)$ be the i-th Lopatinski determinant with respect to $(\mathscr{A}, \mathscr{B})$, then we assume

Assumption (B′). $R^{(i)}(\theta, \eta) \neq 0 \quad (i=0, 1, \dots, l)$ for $\mathrm{Im}\, \theta \leqq 0$ and $\eta \in S^{n-1}$.

§2. Energy Inequality

2.1. In this section, we consider differential or seudo-differential operators in the shifted form:

$$A_\gamma = A(D_t - i\gamma, D_x, D_y),\ B_\gamma = B(D_t - i\gamma, D_x, D_y), \dots,$$

where γ is a large positive parameter. Denoting

$$\mathscr{N} = \{N - (1, 0)\} \cap N,\ \mathscr{N}' = \{\mathscr{N} - (0, 1)\} \cap \mathscr{N},$$

we define

$$|||u|||^2 = \sum_{(\sigma, s+\lambda) \in N'} ||\langle \theta_\gamma \rangle^\sigma \langle D_y \rangle^s D_x^\lambda u||^2,$$
$$|||u|||^2_{(-1)} = \sum_{(\sigma, s+\lambda) \in \mathscr{N}'} ||\langle \theta_\gamma \rangle^\sigma \langle D_y \rangle^s D_x^\lambda u||^2,$$

where

$$\langle \theta_\gamma \rangle = |\theta(D_t - i\gamma, D_y)|,\ \langle D_y \rangle = (|D_y|^2 + 1)^{\frac{1}{2}}.$$

It follows from the definition of above two norms that

$$|||u|||^2 = |||\theta_\gamma u|||^2_{(-1)} + \sum_{s+\lambda=m-1} ||\langle D_y \rangle^s D_x^\lambda u||^2.$$

Next, we shall introduce boundary norms:

$$\langle\!\langle u \rangle\!\rangle^2 = \sum_{(\sigma, s+\lambda) \in N'} \langle \langle \theta_\gamma \rangle^\sigma \langle D_y \rangle^s D_x^\lambda u \rangle^2,$$
$$\langle\!\langle u \rangle\!\rangle^2_{(-1)} = \sum_{(\sigma, s+\lambda) \in \mathscr{N}'} \langle \langle \theta_\gamma \rangle^\sigma \langle D_y \rangle^s D_x^\lambda u \rangle^2,$$

and

$$\langle\!\langle\!\langle u\rangle\!\rangle\!\rangle^2=\langle\!\langle\!\langle \theta_\gamma u\rangle\!\rangle\!\rangle^2_{(-1)}+\gamma^{q_1}\sum_{s+\lambda=m-1-\frac{1}{2}}\langle\langle D_y\rangle^s D_x^\lambda u\rangle^2.$$

2.2. We remark that

$$\begin{aligned}A_\gamma&=\mathscr{A}(\theta_\gamma,\ D_x,\ D_y)\theta_\gamma+A_1(\theta_\gamma,\ D_x,\ D_y)\\&=\theta_\gamma\mathscr{A}(\theta_\gamma,\ D_x,\ D_y)+A_2(\theta_\gamma,\ D_x,\ D_y),\\ \theta_\gamma\mathscr{B}^\sharp_\gamma&=\mathscr{B}^\sharp_\gamma\theta_\gamma+\mathscr{B}^\sharp_1(\theta_\gamma,\ D_x,\ D_y),\end{aligned}$$

where $A_1(\theta,\ \xi,\ \eta)$, $A_2(\theta,\ \xi,\ \eta)$ are l. o. t. (N') and $\mathscr{B}^\sharp_1(\theta,\ \xi,\ \eta)$ is l. o. t. (N''), where $N''=\{N-(0,\ 2)\}\cap N$.

Now let u satisfy

$$(P)\begin{cases}A_\gamma u=f & \text{for}\quad (t,\ x,\ y)\in R^1\times R^1_+\times R^{n-1},\\ \mathscr{B}^\sharp_\gamma u\,|_{x=0}=g & \text{for}\quad (t,\ y)\in R^1\times R^{n-1},\end{cases}$$

then we have

$$(P_1)\begin{cases}\mathscr{A}_\gamma v=f_1 & \text{for}\quad (t,\ x,\ y)\in R^1\times R^1_+\times R^{n-1},\\ \mathscr{B}^\sharp_\gamma v\,|_{x=0}=g_1 & \text{for}\quad (t,\ y)\in R^1\times R^{n-1},\end{cases}$$

where

$$v=\theta_\gamma u,\quad f_1=f-A_{1,\gamma}u,\quad g_1=\theta_\gamma g-\mathscr{B}^\sharp_{1,\gamma}u\,|_{x=0}.$$

Concerning to the problem (P_1), we have the following inequality from Part I.

Lemma 2.1. *There exist positive constants $\gamma_\circ$ and C such that*

$$\gamma^{\frac{1}{2}q_1}|||v|||_{(-1)}+\langle\!\langle v\rangle\!\rangle_{(-1)}\leqq C(\gamma^{-\frac{1}{2}q_1}||f_1||+\langle g_1\rangle)\qquad\text{for}\quad \gamma\geqq\gamma_\circ.$$

Since

$$\begin{aligned}&||f_1||\leqq||f||+||A_{1,\gamma}u||\leqq||f||+C|||u|||,\\ &\langle g_1\rangle\leqq\langle\theta_\gamma g\rangle+\langle\mathscr{B}^\sharp_{1,\gamma}u\rangle\leqq\langle\theta_\gamma g\rangle+C\gamma^{-q_1}\langle\!\langle\!\langle u\rangle\!\rangle\!\rangle,\end{aligned}$$

we have

Corollary.

$$\begin{aligned}&\gamma^{\frac{1}{2}q_1}|||\theta_\gamma u|||_{(-1)}+\langle\!\langle\theta_\gamma u\rangle\!\rangle_{(-1)}\\&\quad\leqq C(\gamma^{-\frac{1}{2}q_1}||f||+\langle\theta_\gamma g\rangle+\gamma^{-\frac{1}{2}q_1}|||u|||+\gamma^{-q_1}\langle\!\langle\!\langle u\rangle\!\rangle\!\rangle)\quad\text{for}\quad\gamma\geqq\gamma_\circ.\end{aligned}$$

On the other hand, since

$$\theta_\gamma\mathscr{A}_\gamma u=f-A_{2,\gamma}u,$$

we have

Lemma 2.2. $\gamma\|\mathscr{A}_\gamma u\|\leqq C(\|f\|+|||u|||)$.

2.3. Now, let us denote

$$\mathscr{A}_\gamma=(\theta_\gamma^{m-2}+\alpha_1\theta_\gamma^{m-3}+\cdots+\alpha_{m-2})\theta_\gamma+\alpha_{m-1}+\mathscr{A}_{1,\gamma},$$
$$\mathscr{B}_\gamma^\sharp=\tilde{\mathscr{B}}_\gamma^\sharp\theta_\gamma+\beta+\mathscr{B}_{2,\gamma}^\sharp,$$

where $\mathscr{A}_1$ is l. o. t. $(\mathscr{N}')$ and $\mathscr{B}_2^\sharp$ is l. o. t. $(\mathscr{N}'')$, where $\mathscr{N}''=\{\mathscr{N}-(0, 2)\}\cap\mathscr{N}$, then we have

$$(P_2)\left\{\begin{array}{l}\alpha_{m-1}u=F,\\ \beta u\,|_{x=0}=G,\end{array}\right.$$

where

$$F=\mathscr{A}_\gamma u-\mathscr{A}_{1,\gamma}u-(\theta_\gamma^{m-2}+\alpha_1\theta_\gamma^{m-3}+\cdots+\alpha_{m-2})v,$$
$$G=(\mathscr{B}_\gamma^\sharp u-\mathscr{B}_{2,\gamma}^\sharp u-\tilde{\mathscr{B}}_\gamma^\sharp v)\,|_{x=0}.$$

Since

$$\alpha_{m-1}(\xi, \eta)=\mathscr{A}(0, \xi, \eta),$$
$$\beta(\xi, \eta)=\mathscr{B}^\sharp(0, \xi, \eta),$$

(P_2) is an elliptic boundary value problem satisfying the Lopatinski condition. Here we have

Lemma 2.3.

$$\sum_{s+\lambda=m-1}\|\langle D_y\rangle^s D_x^\lambda u\|+\sum_{s+\lambda=m-1-\frac{1}{2}}\langle\langle D_y\rangle^s D_x^\lambda u\rangle$$
$$\leqq C\{\|F\|+\langle\langle D_y\rangle^{\frac{1}{2}}G\rangle+\|u\|\}.$$

Since

$$\|F\|\leqq\|\mathscr{A}_\gamma u\|+c\{\gamma^{-1}|||u|||+|||v|||_{(-1)}\}$$
$$\leqq C\{\gamma^{-q_1}\|f\|+\gamma^{-\frac{1}{2}q_1}\langle\theta_\gamma g\rangle+\gamma^{-q_1}|||u|||+\gamma^{-\frac{1}{2}q_1}\langle\!\langle\!\langle u\rangle\!\rangle\!\rangle\},$$
$$\langle\langle D_y\rangle^{\frac{1}{2}}G\rangle\leqq\langle\langle D_y\rangle^{\frac{1}{2}}g\rangle+C\{\gamma^{-\frac{1}{2}q_1}\langle\!\langle\!\langle u\rangle\!\rangle\!\rangle+\gamma^{-\frac{1}{2}q_1}\langle\!\langle\!\langle v\rangle\!\rangle\!\rangle_{(-1)}\}$$
$$\leqq C\{\gamma^{-q_1}\|f\|+\gamma^{-\frac{1}{2}q_1}\langle\theta_\gamma g\rangle+\langle\langle D_y\rangle^{\frac{1}{2}}g\rangle$$
$$+\gamma^{-q_1}|||u|||+\gamma^{-\frac{1}{2}q_1}\langle\!\langle\!\langle u\rangle\!\rangle\!\rangle\},$$

we have

Corollary.

$$\sum_{s+\lambda=m-1}\|\langle D_y\rangle^s D_x^\lambda u\|+\sum_{s+\lambda=m-1-\frac{1}{2}}\langle\langle D_y\rangle^s D_x^\lambda u\rangle$$
$$\leqq C\{\gamma^{-q_1}\|f\|+\gamma^{-\frac{1}{2}q_1}\langle\theta_\gamma g\rangle+\langle\langle D_y\rangle^{\frac{1}{2}}g\rangle$$
$$+\gamma^{-q_1}|||u|||+\gamma^{-\frac{1}{2}q_1}\langle\!\langle\!\langle u\rangle\!\rangle\!\rangle\}.$$

From the Corollary of Lemma 2.1, Lemma 2.2, and Corollary of Lemma 2.3, we have

$$\begin{aligned}
&\gamma^{\frac{1}{2}q_1}|||u|||+\langle\!\langle\!\langle u\rangle\!\rangle\!\rangle \\
&\quad\leqq C\{\gamma^{-\frac{1}{2}q_1}||f||+\langle\theta_\gamma g\rangle+\gamma^{\frac{1}{2}q_1}\langle\langle D_y\rangle^{\frac{1}{2}}g\rangle \\
&\qquad+\gamma^{-\frac{1}{2}q_1}|||u|||+\gamma^{-\frac{1}{2}q_1}\langle\!\langle\!\langle u\rangle\!\rangle\!\rangle\}.
\end{aligned}$$

Hence we have

Theorem. *There exist positive constants* γ_1 *and* C *such that*

$$\begin{aligned}
&\gamma^{\frac{1}{2}q_1}|||u|||+\langle\!\langle\!\langle u\rangle\!\rangle\!\rangle \\
&\quad\leqq C\{\gamma^{-\frac{1}{2}q_1}||A(D_t-i\gamma,\ D_x,\ D_y)u|| \\
&\qquad+\langle\theta(D_t-i\gamma,\ D_y)\mathscr{B}^\sharp(\theta(D_t-i\gamma,\ D_y),\ D_x,\ D_y)u\rangle \\
&\qquad+\gamma^{\frac{1}{2}q_1}\langle\langle D_y\rangle^{\frac{1}{2}}\mathscr{B}^\sharp(\theta(D_t-i\gamma,\ D_y),\ D_x,\ D_y)u\rangle\}
\end{aligned}$$

for $\gamma\geqq\gamma_1$.

DEPARTMENT OF MATHEMATICS,
NARA WOMEN'S UNIVERSITY.

References

[1] R. Sakamoto, Mixed problems for evolution equations I, J. Math. Kyoto Univ. 24 (1984).
[2] L. R. Volevich and S. G. Gindikin, The problem of Cauchy for differential operators with a dominant principal part, Functional Anal. Appl., 2 (1968).
[3] A. Majda and S. Osher, Initial-boundary value problems for hyperbolic equations with uniformly characteristic boundary, Comm. Pure Appl. Math., 28 (1975).