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Absence of the affine lines on the homology planes of general type

By

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Introduction

Let X be a nonsingular algebraic surface defined over the complex field $\mathbf{C}$. We call X a *homology plane* (resp. $\mathbf{Q}$-*homology plane*) if the homology groups $H_i(X; \mathbf{Z})$ (resp. $H_i(X; \mathbf{Q})$) vanish for all $i > 0$. A purpose of the present article is to show the following result.

Main Theorem. *Let X be a $\mathbf{Q}$-homology plane of Kodaira dimension* 2. *Then there lies no curve C on X which is topologically isomorphic to the affine line* $\mathbf{A}^1$.

The core of a proof is to show that X and $X - C$ are respectively embedded as Zariski open sets into almost minimal pairs (cf. [9]; see below) and that the inequality of Miyaoka-Yau type (cf. [5], [10]), after a relevant modification, can be applied to derive a contradiction if one assumes the existence of a curve topologically isomorphic to $\mathbf{A}^1$.

M. Zaidenberg [11] informed us of the following theorem which overlaps our main theorem and whose proof is to be published in Math. USSR, Izvestija.

Theorem of Zaidenberg. *Let X be a homology plane which is not isomorphic to* $\mathbf{A}^2$. *Then the following conditions are equivalent to each other*:

(1) *There exists a curve Γ_0 in X which is isomorphic to* $\mathbf{A}^1$;

(2) *There exists a simply connected curve Γ_0 in X which is* a posteriori *isomorphic to* $\mathbf{A}^1$;

(3) *There exists an isotrivial family of curves $X \to C$, which is not a singular* $\mathbf{C}^{**}$-*family*;

(4) *There exists a regular map $X \to \mathbf{P}^1$ with $\mathbf{C}^*$ as a general fiber*;

(5) *X has Kodaira dimension* 1.

1. Almost minimal surfaces and inequalities of Miyaoka-Yau type

Let (V, D) be a pair consisting of a nonsingular projective surface V and a reduced effective divisor D with simple normal crossings. Denote by K_V the

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canonical divisor of V. By the theory of peeling [9], we can decompose the divisor D uniquely into a sum of effective $\mathbf{Q}$-divisors $D = D^* + Bk(D)$ so that

(i) $Bk(D)$ has the negative definite intersection form;

(ii) $(D^* + K_V \cdot Z) = 0$ for every irreducible component Z of all maximal twigs, rods and forks which are admissible and rational;

(iii) $(D^* + K_V \cdot Y) \geq 0$ for every irreducible component Y of D except the irrelevant components of twigs, rods and forks which are all non-admissible and rational.

The divisors D^* and $Bk(D)$ are called respectively the *stripped form* and the *bark* of the divisor D.

We call the pair (V, D) *almost minimal* if, for every irreducible curve C on V, either $(D^* + K_V \cdot C) \geq 0$ or $(D^* + K_V \cdot C) < 0$ and the intersection matrix of $C + Bk(D)$ is not negative definite.

We recall the following two results.

Lemma 1.1 [9, Th. 1.11]. *Let (V, D) be as above. Then there exists a birational morphism $\mu\colon V \to \tilde{V}$ onto a nonsingular projective surface $\tilde{V}$ such that, with $\tilde{D} = \mu_*(D)$, the following conditions are satisfied*:

(1) $\dim H^0(V, n(D + K_V)) = \dim H^0(\tilde{V}, n(\tilde{D} + K_{\tilde{V}}))$ *for every integer* $n \geq 0$;

(2) $\mu_* Bk(D) \leq Bk(\tilde{D})$ *and* $\mu_*(D^* + K_V) \geq \tilde{D}^* + K_{\tilde{V}}$;

(3) *the pair* $(\tilde{V}, \tilde{D})$ *is almost minimal.*

The birational morphism μ is obtained as a composite of the following operations:

(1) Find an exceptional curve E of the first kind, i.e., a (-1) curve, which is an irreducible component of D and can be contracted so that the image of D under the contraction is still a divisor with simple normal crossings. E is called a *superfluous component* of D. If there is such a component E, contract E.

(2) If there is no superfluous component in D then consider $D^* + K_V$ and $Bk(D)$.

(3) Find a (-1) curve E such that $E \not\subset \mathrm{Supp}\,(D)$, $(D^* + K_V \cdot E) < 0$ and the intersection matrix of $E + Bk(D)$ is negative definite. If there is none, then we are done. If there is one, E meets D (Supp $(Bk(D))$, indeed) transversally in at most two smooth points. Contract E and all components of D which become subsequently (-1) curves. Repeat this operation as long as there exist (-1) curves like E above.

(4) Repeat these operations (1), (2) and (3) all over again.

The pair $(\tilde{V}, \tilde{D})$ is called *an almost minimal model of* (V, D). For the next result, we refer to [9, Th. 1.12 and Remark at p. 227].

Lemma 1.2. *Let (V, D) be as above. Then $\kappa(V - D) \geq 0$ if and only if $D^* + K_V$ is nef, i.e., $(D^* + K_V \cdot C) \geq 0$ for every irreducible curve C on V. Moreover, $D^* + K_V$ is big and nef if and only if $\kappa(V - D) = 2$.*

We shall next consider a slight modification of the inequality of Miyaoka-Yau type (cf. [5], [10]). The authors were informed after the completion of this article that Kobayashi [6, Th. 1 in Sect. 3] generalized the inequality to the case of a surface with log-canonical singularities; indeed, if one sets $b_i = \infty$ for every i in the situation treated by Kobayashi, we obtain the inequality of Miyaoka-Yau type that we need in this article.

Let V be a nonsingular projective surface and let D be a reduced effective divisor with simple normal crossings. Let Γ be a set of nonsingular curves and let $\Gamma_1, \dots, \Gamma_r$ be the connected components of Γ. We assume that the following conditions are satisfied:

(1) $D^* + K_V$ is a nef and big $\mathbf{Q}$-divisor;

(2) $(D^* + K_V \cdot C) = 0$ for every irreducible component of Γ;

(3) Every irreducible component C of Γ has self-intersection $(C^2) \leq -2$;

(4) $\mathrm{Supp}\,(Bk(D)) \subset \Gamma$;

(5) There is no (-1) curve E such that $E \not\subset \mathrm{Supp}\,(\Gamma \cup D)$ and E meets $\mathrm{Supp}\,(D)$ transversally in one smooth point.

By the condition (1), $(D^* + K_V)^2 > 0$. Hence we know that, by the pluri-quasicanonical morphism $\Phi_{|N(D^*+K_V)|}$ (cf. [3], [7]) every connected component Γ_i is contracted algebraically to a singular point. So, let $f: V \to W$ be the contraction of Γ and let $P_i := f(\Gamma_i)$. Let $\Delta := f_*(D)$ as a divisor. Then we have the following:

Lemma 1.3. *With the above notations and assumptions, the connected components Γ_i are classified into the following types*:

(1) *If $P_i \notin \mathrm{Supp}\,(\Delta)$ and $\mathrm{Supp}\,(\Gamma_i) \not\subset \mathrm{Supp}\,(D)$, then P_i is a rational double point.*

(2) *If $P_i \notin \mathrm{Supp}\,(\Delta)$ and $\mathrm{Supp}\,(\Gamma_i) \subset \mathrm{Supp}\,(D)$, one of the following cases takes place*:

(2-1) *Γ_i consists of a nonsingular elliptic curve and P_i is an elliptic singular point*;

(2-2) *P_i is a quasi-elliptic singular point, i.e., it is a quotient of an elliptic singular point under a finite group action fixing the elliptic singular point; the resolution graph is given in* [7, Chap. III, Lemma 2.4];

(2-3) *Γ_i consists of a cycle of nonsingular rational curves, one of which has self-intersection ≤ -3, and P_i is a cuspidal singular point*;

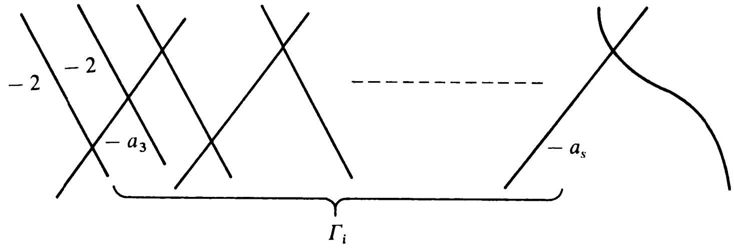
(2-4) *P_i is a quasi-cuspidal singular point, i.e., it is a quotient of a cuspidal singular point under a finite group action fixing the cuspidal singular point.*

(2-5) *P_i is a quotient singular point and Γ_i is either an admissible rational rod or an admissible rational fork.*

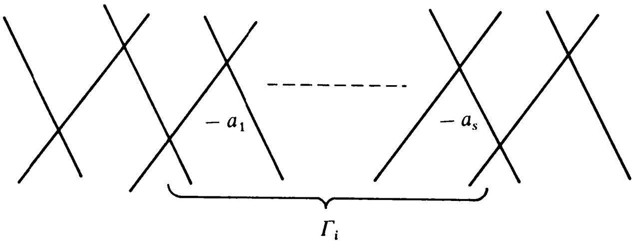
(3) *If $P_i \in \mathrm{Supp}\,(\Delta)$ and $\mathrm{Supp}\,(\Gamma_i) \subset \mathrm{Supp}\,(D)$, then one of the following cases takes place*:

(3-1) *P_i has a cyclic quotient singularity and Γ_i is an admissible rational maximal twig*;

(3-2) *P_i is a quotient singular point and Γ_i has the configuration as given* *in Figure* 1:



(3-3) *P_i has a cyclic quotient singularity and Γ_i has the configuration given in Figure* 2:



Proof. (1) See [7, Chap. III, Lemma 2.1].

(2) Suppose $\mathrm{Supp}(\Gamma_i) \subset \mathrm{Supp}(Bk(D))$. Then, since $P_i \notin \mathrm{Supp}(\Delta)$, Γ_i is a connected component of D, and Γ_i is either an admissible rational rod or an admissible rational fork. So, suppose $\mathrm{Supp}(\Gamma_i) \not\subset \mathrm{Supp}(Bk(D))$. If Γ_i contains an irrational component, we fall into the case (2-1) (cf. [7, Chap. III, Lemma 2.1]. Assume that all irreducible components are nonsingular rational curves. If Γ_i contains a cycle, then we get to the case (2-3) (cf. [7, Chap. III, Lemma 2.3]). The remaining case is reduced to the cases (2-3) and (2-4) (cf. [7, Chap. III, Lemma 2.4]). More precisely, among 13 cases classified there, all the cases except for the case (i) are quasi-elliptic and the case (i) is quasi-cuspidal.

(3) If Γ_i contains an irrational component, we get to the case (2-1). So, every irreducible component of Γ_i is rational. If $\mathrm{Supp}(\Gamma_i) \subset \mathrm{Supp}(Bk(D))$, then Γ_i is an admissible rational maximal twig. If $\mathrm{Supp}(\Gamma_i) \not\subset \mathrm{Supp}(Bk(D))$ and $\mathrm{Supp}(\Gamma_i) \cap \mathrm{Supp}(Bk(D)) \neq \emptyset$, then we get to the case (3-2). If $\mathrm{Supp}(\Gamma_i) \cap \mathrm{Supp}(Bk(D)) = \emptyset$, Γ_i must be a rational linear chain as in the case (3-3).

Now, we shall revise (weaken, in a sense) the inequality of Miyaoka-Yau type proved by Kobayashi [5] to the effect that it can be applied to a proof of our main theorem. Conforming to the classification in Lemma 1.3 of the connected components of Γ, we denote by $\Gamma(1)$ ($\Gamma(2)$ or $\Gamma(3)$, resp.) the union of all connected components of type (1) ((2) or (3), resp.).

Theorem 1.4 [5]. *Let V be a nonsingular projective surface and let D be a reduced effective divisor with simple normal crossings. Let Γ be a set of nonsingular* *curves. We assume that the five conditions listed before Lemma* 1.3 *are satisfied and that* $\Gamma(1) = \Gamma(2) = \varnothing$. *Then the following assertions hold true*:

(1) *There exists a complete Ricci-negative Einstein-Kähler metric on* $V - D$ *with finite volume, which is unique up to multiplication by positive numbers.*

(2) $(D^* + K_V)^2 \leq 3(e(V) - e(D))$, *where* $e(V)$ *and* $e(D)$ *are the Euler numbers of* V *and* D, *respectively.*

Employing the notations of Lemma 1.3, we let $\varphi: V \to \bar{V}$ be the contraction of all connected components of Γ of type (3-1) and (-2) curves of all connected components of type (3-2). Namely, φ is the contraction of all connected components of Supp $(Bk(D))$. Let $\bar{D} = \varphi_*(D)$. Then $\bar{V}$ acquires only cyclic quotient singularities lying on the component of $\bar{D}$.

Lemma 1.5. *Let* P *be a cyclic quotient singular point lying on an irreducible component* Z *of* $\bar{D}$. *Then there exists a neighbourhood of* P *satisfying the following conditions*:

(i) $(U, P) \simeq (\hat{U}/G, G \cdot 0)$, *where* $\hat{U}$ *is a neighbourhood of the origin of* $\mathbf{C}^2$ *isomorphic to* $\Delta \times \Delta$ *with the unit disk* Δ *and* G *is a finite cyclic subgroup of* $GL(2, \mathbf{C})$ *acting diagonally on* $\mathbf{C}^2$;

(ii) $\pi^{-1}(Z \cap U)$ *is irreducible and* $\hat{U} - \pi^{-1}(Z \cap U) \simeq \Delta^* \times \Delta$, *where* $\pi: \hat{U} \to U$ *is the quotient morphism and* Δ^* *is the punctured unit disk*;

(iii) (U, P) *admits a V-metric in the sense of* [5].

Proof. Let $D_1, \ldots, D_r$ be irreducible components of $\varphi^{-1}(P)$, which constitute a maximal twig of D and let C be the proper transform of Z. Let $(D_i^2) = -a_i$ $(1 \leq i \leq r)$ and let n be the determinant of the $(r \times r)$-matrix $(-(D_i \cdot D_j))$. In the present proof, let D^* denote the $\mathbf{Q}$-divisor $D - Bk(T)$, where $Bk(T) = \sum_{i=1}^r \alpha_i D_i$ is the bark of a twig $T := D_1 + \cdots + D_r$. By [9], we know that n is the smallest positive integer such that nD^* is an integral divisor. Note that $(D^* + K_V \cdot D_i) = 0$ and $0 < \alpha_i < 1$ for $1 \leq i \leq r$, and that Supp $(Bk(T))$ = Supp (T). Hence $n(D^* + K_V)$ is linearly equivalent to a Cartier divisor disjoint from T. In view of a relation

$$n(D + K_V) = n(D^* + \mathrm{K}_V) + nBk(T)$$

which is, locally near $T + C$, equivalent to $nBk(T)$. So, we can consider an n-ple cyclic covering $\rho: \tilde{V} \to V$ ramifying totally over Supp $(nBk(T))$. As a normal surface, $\tilde{V}$ may still have cyclic quotient singularities. So, let $\sigma: \hat{V} \to \tilde{V}$ be the minimal resolution of singularities of $\tilde{V}$, and let $q = \rho \cdot \sigma$. In the sequel, we argue locally near $q^{-1}(T + C)$ or its images. We know that $q^{-1}(T)$ is a linear chain of nonsingular rational curves. Let B be a reduced, effective divisor supported by Supp $(q^{-1}(T))$. Note that $q^{-1}(C)$ is irreducible. Indeed, if $(C \cdot D_r) = 1$, then $n = (q^*(C) \cdot q^*(D_r)) = n(q^*(C) \cdot \hat{D}_r)$, where $q(\hat{D}_r) = D_r$ and $q^*(D_r) = n\hat{D}_r +$ other components. Since $(q^*(C) \cdot \hat{D}_r) = 1$, $q^*(C)$ consists of a single irreducible component $\hat{C}$ which is—smooth at the unique point $q^{-1}(C \cap D_r)$. By the logarithmic ramification formula written locally near $\hat{C} + B$

$$(\hat{C} + B + K_{\hat{V}}) = q^*(D + K_V) + R_q$$
$$\equiv q^*(D^* + K_V) + q^*(Bk(T)) + R_q\,,$$

where R_q is an effective divisor and the last equality is considered up to numerical equivalence of **Q**-divisors. It is known that $q^*(Bk(T))$ is an integral divisor (cf. Hirzebruch [2]). Hence $q^*(D^* + K_V)$ is numerically equivalent to an integral divisor. On the other hand, since the intersection matrix of $q^*(Bk(T)) + R_q$ is negative definite and $(q^*(D^* + K_V) \cdot A) = 0$ for every component A of B, we know that

$$\hat{C} + B^* + K_{\hat{V}} = q^*(D^* + K_V)$$

near $\hat{C} + B$. This implies that B is contractible to a smooth point.

Let $\tau\colon \hat{V} \to V_1$ be the contraction of B, let $Q = \tau(B)$ and let $Z_1 = \tau(\hat{C})$. Then $\varphi \cdot q\colon \hat{V} \to \bar{V}$ decomposes to $\psi \cdot \tau\colon \hat{V} \to V_1 \xrightarrow{\psi} \bar{V}$, where ψ is a finite morphism such that $\psi^{-1}(P) = Q$. The Galois group G ($\simeq \mathbf{Z}/n\mathbf{Z}$) action on $\hat{V}$ descends down to a G-action on V_1 in such a way that ψ ramifies only over the point P and the curve Z_1 is G-stable. Since the point Q is smooth, we can choose local coordinates (z_1, z_2) at the point Q so that the curve Z_1 is given by $z_1 = 0$ and the group G acts on (z_1, z_2) by $(z_1, z_2) \mapsto (\zeta Z_1, \zeta^a z_2)$, where ζ is a primitive n-th root of the unity. Now, the function $F(z_1, z_2) = (\log|z_1|^2)^{-1}(1 - |z_2|^2)^{-1}$ should give a desired V-metric on $\bar{V}$ near P.

Proof of Theorem 1.4. One can follow *verbatim* the proof of Theorems 1 and 2 in [5] only by showing additionally that the local contribution of the point P (with the above notations) in the computation of $\int_{V-D} \tilde{c}_2$ is zero. With the same notations g and g_0 as in [5] and with the notations of Lemma 1.5, it suffices to note that

$$0 = \frac{e(\hat{U}_-)}{|G|} = \int_{U_-} e(g) = \int_{V-C\cup T} e(g_0) = \int_{V-C} e(g_0)$$

where $\hat{U}_- = \hat{U} - Z_1$ and $U_- = U - Z$.

2. Proof of Main Theorem

Let X be a **Q**-homology plane of Kodaira dimension 2. Let (V, D) be a pair of a nonsingular projective surface and a reduced effective divisor with simple normal crossings such that X is isomorphic to a Zariski open set $V - D$. We may assume without loss of generality that the image of D is not a divisor with simple normal crossings under the contraction of any (-1) curve component of D. We refer to [1] and [8] for the following result.

Lemma 2.1. (1) *X is an affine surface.*

(2) *Every component of D is a nonsingular rational curve and the dual graph of D is a tree.*

(3) $p_g(V) = q(V) = 0$, *where $q(V)$ is the irregularity of V.*

(4) $|D + K_V| = \emptyset$.

(5) *If X is a homology plane then V is a rational surface.*

Proof. For the assertion (4), we refer to [7, Lemma 2.1.1].

If the pair (V, D) is not almost minimal, there is a (-1) curve E such that $E \not\subset \mathrm{Supp}(D)$ and E meets D transversally in at most two smooth points of D. If E meets D in two points, the dual graph of $E + D$ contains a loop. Hence, by the construction of an almost minimal model $(\tilde{V}, \tilde{D})$ of (V, D), the dual graph of $\tilde{D}$ would contain a loop. Thus, $|\tilde{D} + K_{\tilde{V}}| \neq \emptyset$ by [7, Lemma 2.1.1]. This is a contradiction to Lemma 1.1. Therefore, if a (-1) curve E as above exists at all, E meets D transversally only in one smooth point.

Lemma 2.2. *Let X and (V, D) be the same as above. Let $(\tilde{V}, \tilde{D})$ be an almost minimal model of (V, D) and let $\tilde{X} = \tilde{V} - \tilde{D}$. Then the following assertions hold true*:

(1) *$\tilde{X}$ is a Zariski open set of X, and $X - \tilde{X}$ is a disjoint union of curves isomorphic to the affine line.*

(2) *$\tilde{X}$ has Kodaira dimension 2, and the Euler number ≤ 0 provided $\tilde{X} \subsetneqq X$.*

Proof. The assertion (1) follows from the construction of $(\tilde{V}, \tilde{D})$. Since $\tilde{X}$ is a Zariski open set, the Kodaira dimension $\kappa(\tilde{X})$ is not less than that of X. Note that $e(X) = 1$ and $e(\tilde{X}) = e(X) - N$, where N is the number of irreducible components of $\tilde{X} - X$. Thence follows the second assertion.

Let $(\tilde{V}, \tilde{D})$ be as above. If there is a (-1) curve E on $\tilde{V}$ such that $E \not\subset \mathrm{Supp}(\tilde{D})$ and E meets $\tilde{D}$ transversally in one smooth point, then consider a pair $(\tilde{V}, \tilde{D} + E)$ instead of $(\tilde{V}, \tilde{D})$ and pass to an almost minimal model of $(\tilde{V}, \tilde{D} + E)$. We will be thus endowed with an almost minimal pair $(\tilde{V}, \tilde{D})$ such that

(i) $\tilde{X} := \tilde{V} - \tilde{D}$ has Kodaira dimension 2;

(ii) $e(\tilde{X}) \leq 1$, and $e(\tilde{X}) = 1$ if and only if $\tilde{X}$ coincides with the given **Q**-homology plane X;

(iii) There is no (-1) curve E on $\tilde{V}$ such that $E \not\subset \mathrm{Supp}(\tilde{D})$ and E meets $\tilde{D}$ transversally in one smooth point.

Then $(\tilde{V}, \tilde{D})$ satisfies all conditions $(1) \sim (5)$ for (V, D) listed before Lemma 1.3 as well as the condition $\Gamma(1) = \Gamma(2) = \emptyset$, where Γ consists of all admissible rational maximal twigs of $\tilde{D}$. Then Theorem 1.4 asserts that

$$(\tilde{D}^* + K_{\tilde{V}})^2 \leq 3(e(\tilde{V}) - e(\tilde{D})) = 3e(\tilde{X}).$$

Since $\tilde{X}$ has Kodaira dimension 2, we have $(\tilde{D}^* + K_{\tilde{V}})^2 > 0$, whence $e(\tilde{X}) > 0$. This implies that $\tilde{X}$ coincides with the given **Q**-homology plane X. Summarizing the above arguments, we have the following:

Theorem 2.3. *Let X be a* $\mathbf{Q}$*-homology plane. Then there exists an almost minimal pair (V, D) such that $X = V - D$ and there is no (-1) curve $E \not\subset \mathrm{Supp}(D)$ meeting D transversally in one smooth point.*

Now, suppose there exists a curve C on X which is topologically isomorphic to the affine line. The divisor $D + \bar{C}$ on V may not be a divisor with simple normal crossings. Then there exists a birational morphism $\mu: V_1 \to V$ from a nonsingular projective surface V_1 onto V such that $D_1 := \mu^*(D + \bar{C})_{red}$ is an effective reduced divisor with simple normal crossings and $V_1 - D_1 \simeq X - C$. Clearly, $X_1 := X - C$ has Kodaira dimension 2 and $e(X_1) = 0$. We apply to the pair (V_1, D_1) the same arguments as made use of to prove Theorem 2.3, and we can conclude that there exists an almost minimal pair (V_2, D_2) such that $X_1 = V_2 - D_2$ and there is no (-1) curve $E \not\subset \mathrm{Supp}(D_2)$ meeting D_2 transversally in one smooth point. Theorem 1.4 then implies that

$$0 < (D_2^* + K_{V_2})^2 \leq 3e(X_1) = 0.$$

which is a contradiction. This completes a proof of Main Theorem.

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