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ON UNCONDITIONALLY CONVERGING AND WEAKLY PRECOMPACT OPERATORS

BY

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Introduction

In [1], the authors showed that if F is a Banach space such that F^* has the Radon Nikodym property and contains no subspace isomorphic to l_1, and if G is any Banach space and Ω a compact Hausdorff space, then an operator $T: C(\Omega, F) \longrightarrow G$ is unconditionally converging if and only if its adjoint T^* is weakly precompact and they asked whether or not the result is still true if one assumes only that F^* does not contain a subspace isomorphic to l_1. In this paper we give a positive answer to their question. We actually prove a more general result, namely we show that if E, F and G are Banach spaces such that E^* is isometric to an L_1-space, and F^* contains no subspace isomorphic to l_1, a bounded linear operator $T: E \hat{\otimes}_\varepsilon F \longrightarrow G$ is unconditionally converging if and only if its adjoint T^* is weakly precompact. The methods used to prove this result allow us to extend the result of [17], namely we will show that if E^* is isometric to an L_1-space and F is any Banach space, then l_1 is isomorphic to a complemented subspace of $E \hat{\otimes}_\varepsilon F$ if and only if l_1 is isomorphic to a complemented subspace of F.

Notations and definitions

Let X and Y be two Banach spaces. A bounded linear operator $T: X \longrightarrow Y$ is said to be **unconditionally converging** if T sends weakly unconditionally Cauchy series $\sum_{n=1}^{\infty} x_n$ in X into unconditionally convergent series, and T is said to be **weakly precompact** if every bounded sequence $(x_n)_{n \geq 1}$ has a subsequence $(x_{n_k})_{k \geq 1}$ such that $(T(x_{n_k}))_{k \geq 1}$ is weakly Cauchy. It follows from Rosenthal l_1 Theorem (see [16] or [9]) that T is weakly precompact if and only if the image by T of the unit ball of X does not contain a sequence equivalent to the l_1 basis. It follows from [8] see also

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[15] that an operator $T: X \longrightarrow Y$ is weakly precompact if and only if there exists a Banach space Z not containing l_1 and bounded linear operators $A: X \longrightarrow Z$ and $B: Z \longrightarrow Y$ such that $T = BA$. This shows that if $T: X \longrightarrow Y$ is weakly precompact then $T^*: Y^* \longrightarrow X^*$ is unconditionally converging. Of course if $T^*: Y^* \longrightarrow X^*$ is weakly precompact then T^{**} is unconditionally converging and hence T is unconditionally converging. In this paper we are interested in studying when unconditionally converging operators have weakly precompact adjoints. It is obvious that if F is a Banach space such that F^* does not contain a subspace isomorphic to l_1 then every bounded linear operator on F has a weakly precompact adjoint.

Here one should mention that the question we would like to address is closely related to the Pełczynski's property (V) of Banach spaces. For this recall that a Banach space X has **Pełczynski's property (V)** if every unconditionally converging operator T on X is weakly compact. The most known classical Banach spaces that have Pełczynski's property (V) are spaces $C(\Omega)$ of continuous functions on compact Hausdorff spaces [14], or more generally Banach spaces whose duals are isometric to L_1-spaces [12]. This last fact will be used later in this paper.

If Ω is a compact Hausdorff space and F is a Banach space, then $C(\Omega, F)$ will denote the Banach space of all continuous F-valued functions on Ω under the uniform norm. It is well known [10] that the dual of $C(\Omega, F)$ is isometrically isomorphic to the space $M(\Omega, F^*)$ of all regular F^*-valued measures on Ω that are of bounded variation. When F is the scalar field we will simply write $C(\Omega)$ and $M(\Omega)$. If $\mu \in M(\Omega, F^*)$ we will denote by $|\mu|$ the variation of μ which is an element of $M(\Omega)$ and for each $x \in F$ we will denote by $\langle x, \mu \rangle$ the element of $M(\Omega)$ such that for each Borel subset B of Ω we have

$$\langle x, \mu \rangle(B) = \mu(B)(x).$$

From this it follows that if $f \in C(\Omega)$ and $x \in F$ then

$$\langle x, \mu \rangle(f) = \mu(f \otimes x).$$

Where $f \otimes x$ is the element of $C(\Omega, F)$ defined by

$$f \otimes x(k) = f(k)x \text{ for all } k \in K.$$

If E is another Banach space, we denote by $E \otimes_\varepsilon F$ the algebraic tensor product of E and F endowed with the norm

$$\left\| \sum_{i=1}^{m} x_i \otimes y_i \right\| = \sup\left\{ \left| \sum_{i=1}^{m} x^*(x_i) y^*(y_i) \right| : \|x^*\|, \|y^*\| \le 1 \right\}.$$

The completion $E \hat{\otimes}_{\varepsilon} F$ of $E \otimes_{\varepsilon} F$ is called the injective tensor product of E and F. The space $C(\Omega, F)$ is isometrically isomorphic to $C(\Omega) \hat{\otimes}_{\varepsilon} F$.

If K is a compact convex subset of a locally convex topological Hausdorff space, then a measure $\mu \in M(K, F^*)$ is said to be a **boundary measure** or a **maximal measure** [19], if its variation $|\mu|$ is maximal in the sense of Choquet [7]. In what follows $M_m(K, F^*)$ will denote the space of all boundary measures. Throughout this paper we shall concentrate on the case where K is the unit ball of the dual of a complex Banach space equipped with its weak* topology. Let $\mathbf{T}$ be the unit circle and let λ denote the normalized Haar measure on $\mathbf{T}$. For each $t \in \mathbf{T}$, let $\sigma_t \colon K \longrightarrow K$ denote the affine weak*-homomorphism of K defined by $\sigma_t(p) = tp$ for all $p \in K$. Given any complex Banach space F, for each element $\mu \in M(K, F^*)$ we denote by $\sigma_t(\mu) = \mu \circ \sigma_t^{-1}$ the image of the measure μ by σ_t; it is immediate that $\sigma_t(\mu) \in M(K, F^*)$ for each $t \in \mathbf{T}$ and $\mu \in M(K, F^*)$. Following [11] we say that a measure $\mu \in M(K, F^*)$ is $\mathbf{T}$-homogeneous if $\sigma_t(\mu) = t\mu$ for all $t \in \mathbf{T}$. We also say that a function $\phi \in C(K, F)$ is $\mathbf{T}$-homogeneous if $\phi(tp) = t\phi(p)$ for all $t \in \mathbf{T}$ and $p \in K$. If $\phi \in C(K, F)$ we let $\hom_{\mathbf{T}}(\phi)$ denote the $\mathbf{T}$-homogeneous element of $C(K, F)$ such that for $p \in K$,

$$\hom_{\mathbf{T}}(\phi)(p) = \text{Bochner} - \int_{\mathbf{T}} t^{-1} \phi(tp)\, d\lambda(t).$$

It is clear that $\hom_{\mathbf{T}}$ defines a norm decreasing projection from $C(K, F)$ onto the subspace of all continuous $\mathbf{T}$-homogeneous functions. By duality, for $\mu \in M(K, F^*)$ we let $\hom_{\mathbf{T}}(\mu)$ denote the element of $M(K, F^*)$ such that

$$\hom_{\mathbf{T}}(\mu)(\phi) = \mu(\hom_{\mathbf{T}}(\phi))$$

for all $\phi \in C(K, F)$.

Finally, we shall denote by $M_{mh}(K, F^*)$ the subspace of $M_m(K, F^*)$ consisting of $\mathbf{T}$-homogeneous measures. If $F = \mathbb{C}$ we simply write $M_m(K)$ or $M_{mh}(K)$. All notations used here and not defined can be found in [9], [10] and [11].

Main result

In this section we suppose that all Banach spaces considered are over the complex field. The techniques we are using in the complex case [11] have their analog in the real case [13] and so all the results presented here are true in the real case also.

The next lemma is elementary and will be needed in the sequel.

LEMMA 1. *The mapping* $\mu \longrightarrow \hom_{\mathbf{T}}(\mu)$ *defines a norm decreasing projection from* $M(K, F^*)$ *onto the subspace of* **T**-*homogeneous measures. Moreover, if* $\mu \in M_m(K, F^*)$, *then* $\hom_{\mathbf{T}}(\mu) \in M_m(K, F^*)$.

Proof. The first assertion is easy and follows from the fact the map $\mu \longrightarrow \hom_{\mathbf{T}}(\mu)$ is adjoint of the operator $\hom_{\mathbf{T}}$ defined on $C(K, F)$. To prove the last assertion, let $\mu \in M_m(K, F^*)$. By [20] it is enough to show that for each $x \in F$, the measure

$$\langle x, \hom_{\mathbf{T}}(\mu) \rangle \in M_m(K).$$

For this note that for each $x \in F$, we have

$$\langle x, \hom_{\mathbf{T}}(\mu) \rangle = \hom_{\mathbf{T}} \langle x, \mu \rangle.$$

The result now follows from [11], where it is shown that

$$\text{if } \nu \in M_m(K) \text{ then } \hom_{\mathbf{T}}(\nu) \in M_m(K).$$

Let E be a Banach space, and denote by K its dual unit ball equipped with the weak* topology. Let F be another Banach space. We will view $E \hat{\otimes}_\varepsilon F$ as a subspace of $C(K, F)$ and we denote by $(E \hat{\otimes}_\varepsilon F)^\perp$ the annihilator of $E \hat{\otimes}_\varepsilon F$ in $M(K, F^*)$. With these notations we have the following theorem:

THEOREM 2. *The following statements are equivalent*:
(1) *The space* E^* *is isometric to an* L_1-*space*;
(2) *For any Banach space* F, *the intersection of* $(E \hat{\otimes}_\varepsilon F)^\perp$ *and* $M_{mh}(K, F^*)$ *is reduced to zero*;
(3) *For any Banach space* F, *the dual of* $E \hat{\otimes}_\varepsilon F$ *is isometrically isomorphic to* $M_{mh}(K, F^*)$.

Proof. To see that (1) $\Rightarrow$ (2), assume that E^* is isometric to an L_1-space. Let F be any arbitrary Banach space and let $\mu \in M_{mh}(K, F^*)$ such that $\mu = 0$ on $E \hat{\otimes}_\varepsilon F$. For each $x \in F$, the scalar measure $\langle x, \mu \rangle$ is then an element of $M_{mh}(K)$ and $\langle x, \mu \rangle = 0$ on E. It follows from [11] that $\langle x, \mu \rangle = 0$ for all $x \in F$, hence $\mu = 0$.

To show that (2) $\Rightarrow$ (3) consider an $L \in (E \hat{\otimes}_\varepsilon F)^*$. It follows from [20] that there exists an element $\mu_L \in M_m(K, F^*)$ such that

(i) $\mu_L = L$ on $E \hat{\otimes}_\varepsilon F$, and
(ii) $\|\mu_L\| = \|L\|$.

For each $L \in (E \hat{\otimes}_\varepsilon F)^*$, let $\nu_L = \hom_{\mathrm{T}} \mu_L$, where μ_L is an element of $M_m(K, F^*)$ satisfying conditions (i) and (ii) above. Since

$$\left(E \hat{\otimes}_\varepsilon F\right)^{\perp} \cap M_{mh}(K, F^*) = 0,$$

for each $L \in (E \hat{\otimes}_\varepsilon F)^*$, the element $\nu_L = \hom_{\mathrm{T}} \mu_L$ is the unique element of $M_{mh}(K, F^*)$ associated to L such that

$$\nu_L = L \text{ on } E \hat{\otimes}_\varepsilon F,$$

and

$$\|\nu_L\| = \|L\|.$$

It is clear then that the mapping $L \longrightarrow \nu_L = \hom_{\mathrm{T}} \mu_L$ defines a linear isometry from $(E \hat{\otimes}_\varepsilon F)^*$ onto $M_{mh}(K, F^*)$.

(3) $\Rightarrow$ (1) follows from [11], since assertion (3) implies that E^* is isometrically isomorphic to $M_{mh}(K)$ which is an L_1-space [11]. The following known proposition is useful in the proof of the next theorem:

PROPOSITION 2 [14]. *A Banach space X has the Pełczynski's property* (V) *if and only if the following is satisfied: A subset* $H \subset X^*$ *is relatively weakly compact whenever*

$$\lim_{n \to \infty} \sup_{x^* \in H} |x^*(x_n)| = 0$$

for any weakly unconditionally Cauchy series $\sum_{n=1}^{\infty} x_n$ *in X.*

THEOREM 3. *Let E, F and G be Banach spaces such that* E^* *is isometric to an* L_1 *space, and* F^* *contains no subspace isomorphic to* l_1. *Let* T: $E \hat{\otimes}_\varepsilon F \longrightarrow G$ *be a bounded linear operator. The following statements are equivalent*:

(i) *The operator T is unconditionally converging*;

(ii) *The adjoint* T^* *of T is weakly precompact.*

Proof. It is enough to show that (i) implies (ii). Suppose that T is an unconditionally converging operator. This implies that for any weakly unconditionally Cauchy series $\sum_{n=1}^{\infty} e_n$ in E we have

$$(*) \qquad \lim_{n \to \infty} \sup_{x \in F,\ \|x\| \le 1} \|T(e_n \otimes x)\| = 0$$

To see this, note that if this were not true, then there would exist a $\delta > 0$ and subsequence $(e'_n)_{n \ge 1}$ of $(e_n)_{n \ge 1}$ and a sequence $(x_n)_{n \ge 1}$ of elements of the unit ball of F such that

$$(**) \qquad \|T(e'_n \otimes x_n)\| > \delta \text{ for each } n \geq 1.$$

The series $\sum_{n=1}^{\infty} e'_n \otimes x_n$ is easily checked to be weakly unconditionally Cauchy in $E \hat{\otimes}_\varepsilon F$ since $\sum_{n=1}^{\infty} e'_n$ is weakly unconditionally Cauchy and $\|x_n\| \leq 1$ for all $n \geq 1$. Condition $(**)$ would then contradict the fact that T is unconditionally converging, thus we have condition $(*)$. For $x \in F$ and $y^* \in G^*$, consider the element $\langle x, T^*y^* \rangle \in E^*$ defined as follows:

$$\langle x, T^*y^* \rangle(e) = \langle T(e \otimes x), y^* \rangle \text{ for all } e \in E.$$

With this in mind, let

$$H = \{\langle x, T^*y^* \rangle; y^* \in G^*, x \in F \text{ with } \|y^*\| \leq 1 \text{ and } \|x\| \leq 1\}.$$

Hence $H \subset E^*$. Since E^* is isometric to an L_1-space, it follows from [12] that E has the Pełczynski's property (V). Notice now that if $e \in E$ and $\langle x, T^*y^* \rangle \in H$ then

$$|\langle x, T^*y^* \rangle(e)| \leq \|T(e \otimes x)\|.$$

Now apply condition $(*)$ and Proposition 2 to deduce that H is relatively weakly compact in E^*. Let

$$S: \left(E \hat{\otimes}_\varepsilon F\right)^* \longrightarrow M_{mh}(K, F^*)$$

denote the linear isometry which assigns to each element L in $(E \hat{\otimes}_\varepsilon F)^*$ the unique element $S(L)$ in $M_{mh}(K, F^*)$ such that $S(L) = L$ on $E \hat{\otimes}_\varepsilon F$. Similarly let

$$s: E^* \longrightarrow M_{mh}(K)$$

denote the isometry of E^* onto $M_{mh}(K)$. Then for each $x \in F$ and $y^* \in G^*$, we have

$$\langle x, S(T^*y^*) \rangle = s(\langle x, T^*y^* \rangle),$$

This follows from the fact that $s\langle x, T^*y^* \rangle$ and $\langle x, S(T^*y^*) \rangle$ are both elements of $M_{mh}(K)$ and they both agree on E, for if $e \in E$ one has

$$\langle x, S(T^*y^*) \rangle(e) = S(T^*y^*)(e \otimes x) = \langle T(e \otimes x), y^* \rangle,$$

and

$$s(\langle x, T^*y^* \rangle)(e) = \langle x, T^*y^* \rangle(e) = \langle T(e \otimes x), y^* \rangle.$$

Hence they are identical. The set

$$s(H) = \{\langle x, S(T^*y^*)\rangle; y^* \in G^*, x \in F \text{ with } \|y^*\| \leq 1 \text{ and } \|x\| \leq 1\}$$

is relatively weakly compact subset of $M_{mh}(K)$, and hence it is uniformly countably additive [9]. This in turn implies that the set

$$W = \{|S(T^*y^*)|; y^* \in G^* \text{ and } \|y^*\| \leq 1\}$$

is uniformly countably additive [10, page 8], here $|S(T^*y^*)|$ denotes the variation of the measure $S(T^*y^*)$. By a result of Grothendieck [9] the set W is relatively weakly compact subset of $M(K)$. If F^* contains no subspace isomorphic to l_1, it follows from [22] or from [4] and the methods used in [17], that the set

$$\{S(T^*y^*); y^* \in G^* \text{ and } \|y^*\| \leq 1\}$$

is weakly precompact. Since S is an isometry the set

$$\{T^*y^*; y^* \in G^* \text{ and } \|y^* \leq 1\}$$

is weakly precompact, and hence T^* is weakly precompact.

The following corollary solves positively the question asked in [1]. Before stating the corollary we need the following definition:

DEFINITION 4. We say that an operator T: $C(\Omega, F) \longrightarrow G$ is strongly bounded if its representing measure is continuous at the empty set (see [5] or [21]).

It is well known (see [5] or [21]) that if F contains no subspace isomorphic to c_0 then saying that T is strongly bounded is equivalent to saying that T is unconditionally converging. This fact together with Theorem 3 gives:

COROLLARY 5. *Let Ω be a compact Hausdorff space and let F and G be Banach spaces. If F^* does not contain a subspace isomorphic to l_1, then the following statements about a bounded linear operator T: $C(\Omega, F) \longrightarrow G$ are equivalent*:

(1) *The operator T is unconditionally converging*;
(2) *The operator T is strongly bounded*;
(3) *The adjoint operator T^* of T is weakly precompact.*

Proof. All one needs to notice is that if F^* does not contain a subspace isomorphic to l_1 then F^{**} does not contain a subspace isomorphic to c_0 and conclude using the remark preceding this corollary.

Discussions and remarks

In light of Theorem 3, one can ask the following question: Under what conditions on the Banach space X any operator $T\colon X \longrightarrow Y$ that is unconditionally converging has an adjoint T^* that is weakly precompact, where Y is any Banach space. To be able to answer this question let us agree to say that a Banach space X has the property **weak (V)** if it satisfies the following property: A subset H of its dual X^* is weakly precompact whenever it satisfies

$$(***) \qquad \lim_{n\to\infty} \sup_{x^*\in H} |\langle x_n, x^*\rangle| = 0$$

for every weakly unconditionally Cauchy series $\sum_{n=1}^{\infty} x_n$ in X.
It is clear that a Banach space X has the Pełczynski's property (V) if and only if X has **weak (V)** and X^* is weakly sequentially complete. One can quickly see that a Banach space X has weak (V) if and only if for any Banach space Y, any unconditionally converging operator $T\colon X \longrightarrow Y$ has a weakly precompact adjoint. To see that, let H be a subset of X^* that satisfy (***) above and let $(x_n^*)_{n\geq 1}$ be a sequence in H. Consider the following map $T\colon Y \longrightarrow c_0$ defined by $T(x) = (x_n^*(x))_{n\geq 1}$. This operator T is unconditionally converging since H satisfies (***), hence T^* is weakly precompact and therefore the sequence $({x^*}_n)_{n\geq 1}$ which is a subset of the image by T^* of the unit ball of l_1 is weakly precompact. Combining this observation with Theorem 3 we get that $E \hat{\otimes}_\varepsilon F$ has weak (V) whenever E and F are Banach spaces such that F^* is isometric to an L_1-space and F^* contains no subspace isomorphic to l_1. This in particular implies that if F^* contains no subspace isomorphic to l_1, then $C(\Omega, F)$ has the weak (V) property. Let us mention that it is still unknown whether $C(\Omega, F)$ has weak (V) whenever F has the same property. The best partial result in this direction was obtained in [6].

Consider now the following question: Under what conditions on the Banach space Y an operator $T\colon X \longrightarrow Y$ is weakly precompact as soon as T^* is unconditionally converging, where X is any Banach space. For this let us say that a Banach space has **weak** (V^*) [3] whenever it satisfies the following property: a subset H of X is weakly precompact whenever it satisfies

$$(****) \qquad \lim_{n\to\infty} \sup_{x\in H} |\langle x, x_n^*\rangle| = 0$$

for every weakly unconditionally Cauchy series $\sum_{n=1}^{\infty} x_n^*$ in X^*.

The Banach space X has (V^*) [14] if it has weak (V^*) and is weakly sequentially complete. It is clear that any Banach space Y that contains no subspace isomorphic to l_1 has weak (V^*), so c_0 has weak (V^*) but does not have (V^*). The results of [4] or [22] imply that if Y contains no subspace isomorphic to l_1, and if (Ω, Σ, ν) is a probability measure space, then $L_1(\nu, Y)$, the space of Bochner integrable Y-valued functions equipped with its usual norm [10], has the property weak (V^*). Here also it turns out that a Banach space Y has weak (V^*) if and only if for any Banach space X, any operator $T: X \longrightarrow Y$ is weakly precompact whenever its adjoint is unconditionally converging. To see that, let H be subset satisfying condition $(****)$ above and let $(x_n)_{n\geq 1}$ be a sequence in H. Consider the operator $T: l_1 \longrightarrow Y$ defined by $T((a_n)_{n\geq 1}) = \sum_{n=1}^{\infty} a_n x_n$. It is easy to check that T is well defined, linear and has a closed graph, so T is bounded. The adjoint T^* of T is unconditionally converging. For let $\sum_{n=1}^{\infty} y_n^*$ be a weakly unconditionally Cauchy series in Y^*. Since H satisfies $(****)$, one has

$$\lim_{k\to\infty} \sup_{n\geq 1} |y_k^*(x_n)| = 0.$$

A moment of reflection reveals that this implies that $\lim_{k\to\infty} \|T^*(y_k^*)\| = 0$ which in turn shows that T^* is unconditionally converging. Therefore T is weakly precompact and hence the sequence $(x_n)_{n\geq 1}$ is weakly precompact.

In [17] it was shown that if $C(\Omega, F)$ contains a complemented subspace isomorphic to l_1, then F contains a complemented subspace isomorphic to l_1. With the help of Theorem 2, we can extend this result as follows: Let E be a Banach space such that E^* is isometric to an L_1-space and let F be any other Banach space. If $E \hat{\otimes}_\varepsilon F$ contains a complemented subspace isomorphic to l_1, then F contains a complemented subspace isomorphic to l_1. First observe that if $E \hat{\otimes}_\varepsilon F$ contains a complemented subspace isomorphic to l_1, then $(E \hat{\otimes}_\varepsilon F)^*$ contains a subspace isomorphic to c_0. By Theorem 2, $(E \hat{\otimes}_\varepsilon F)^*$ is isomorphically isomorphic to $M_{mh}(K, F^*)$. This implies that $M(K, F^*)$ contain a subspace isomorphic to c_0. Apply now [17] to conclude that F^* contain a subspace isomorphic to c_0 and therefore F contains a complemented subspace isomorphic to l_1 [2].

Another application of Theorem 2 and [22] gives the following: Let E be a Banach space such that E^* is isometric to an L_1-space. Let F be a Banach space such that F^* is weakly sequentially complete then $(E \hat{\otimes}_\varepsilon F)^*$ is weakly sequentially complete. As before $(E \hat{\otimes}_\varepsilon F)^*$ is isomorphically isomorphic to $M_{mh}(K, F^*)$ which is a subspace of $M(K, F^*)$, but $M(K, F^*)$ is weakly sequentially complete by [22]. If one suppose that F is an addition a Banach lattice, then one can conclude using [18] that $(E \hat{\otimes}_\varepsilon F)^*$ has (V^*).

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