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CORRECTION

A CENTRAL LIMIT THEOREM FOR STATIONARY PROCESSES AND THE PARAMETER ESTIMATION OF LINEAR PROCESSES

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Assumptions (i) and (ii) of Theorem 2.1 are insufficient to guarantee the central limit theorem. For the theorem to be valid, a Lindeberg-type or a related condition is needed. Hosoya (1992) used a Lindeberg-type condition and Findley and Wei (1992) proposed a Liapounov-type condition as an additional assumption to the theorem [Findley communicated to us about the error in our assumptions].

Specifically, a version of correction according to the approach of Hosoya (1992) is given as follows.

1. Condition (i) of Lemma A2.4 should be

$$\lim_{m\to\infty} \frac{1}{n(m)} \sum_{k=1}^{n(m)} E\Big[u_m(k)^2 I\{|u_m(k)| \geq \varepsilon n(m)^{1/2}\}\Big] = 0.$$

2. Theorem 2.1 (page 134) should have the following additional assumption:

(iii) *For any* $\varepsilon > 0$, *there exists* $B_\varepsilon > 0$ *such that uniformly in* N *and* r

$$E\Big[S(N,r)^2 I\{S(N,r) > B_\varepsilon\}\Big] < \varepsilon,$$

where $S(N,r) = [\sum_{\alpha=1}^{p}\{\sum_{t=1}^{N} x_\alpha(t+r)/N^{1/2}\}^2]^{1/2}$.

3. The line which contains (6.18) in the proof (pages 146 and 147) should be deleted and the following lines should be added after the last line of the proof:

It holds that

$$\begin{aligned} &E\Big\{\big(\eta_k - E(\eta_k|\mathscr{F}_{k-1}^*)\big)^2 I\big(|\eta_k - E(\eta_k|\mathscr{F}_{k-1}^*)| > 2B_\varepsilon\big)\Big\} \\ &\quad\leq 2E\Big\{\eta_k^2 I\big(\big||\eta_k| - |E(\eta_k|\mathscr{F}_{k-1}^*)|\big| > 2B_\varepsilon\big)\Big\} + 2E\Big\{E(\eta_k|\mathscr{F}_{k-1}^*)^2\Big\} \\ &\quad\leq 2E\Big\{\eta_k^2 I\big(|\eta_k| > B_\varepsilon\big)\Big\} + 2E\Big\{\eta_k^2 I\big(|E(\eta_k|\mathscr{F}_{k-1}^*)| > B_\varepsilon\big)\Big\} \\ &\qquad + 2E\Big\{E(\eta_k|\mathscr{F}_{k-1}^*)^2\Big\}, \end{aligned}$$

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where $2E\{\eta_k^2 I(|\eta_k| > B_\varepsilon)\} < 2\varepsilon$ in view of Assumption (iii). Since

$$\begin{aligned} E\{\eta_k^2 I(|E(\eta_k|\mathscr{F}_{k-1}^*)| > B_\varepsilon)\} &= E\big[E(\eta_k^2|\mathscr{F}_{k-1}^*)I(|E(\eta_k|\mathscr{F}_{k-1}^*)| > B_\varepsilon)\big] \\ &\le E\big|E(\eta_k^2|\mathscr{F}_{k-1}^*) - E(\eta_k^2)\big| \\ &\quad + E(\eta_k^2)Pr\{|E(\eta_k|\mathscr{F}_{k-1}^*)| > B_\varepsilon\}, \end{aligned}$$

the application of the Chebyshev inequality gives that

$$\begin{aligned} &\frac{1}{M}\sum_{k=1}^{M} E\{\eta_k^2 I(|E(\eta_k|\mathscr{F}_{k-1}^*)| > B_\varepsilon)\} \\ &\quad \le \frac{1}{M}\sum_{k=1}^{M} E\big|E(\eta_k^2|\mathscr{F}_{k-1}^*) - E(\eta_k^2)\big| + \frac{E(\eta_k^2)}{B_\varepsilon^2 M}\sum_{k=1}^{M} E\{E(\eta_k|\mathscr{F}_{k-1}^*)\}^2. \end{aligned}$$

As is seen in (6.17), the second member on the right-hand side tends to 0 as $N \to \infty$, and the first member tends to 0 in view of the inequality that comes after (6.19). Consequently, for sufficiently large N, it holds that

$$\frac{1}{M}\sum_{k=1}^{M} E\Big[\{\eta_k - E(\eta_k|\mathscr{F}_{k-1}^*)\}^2 I(|\eta_k - E(\eta_k|\mathscr{F}_{k-1}^*)| > B_\varepsilon)\Big] < \varepsilon,$$

from which the condition (i) of Lemma A2.4 follows by the line of argument given, for example, in Chow and Teicher [(1978), page 291].

4. Theorem 2.2 needs Assumption (v); namely:

(v) *For any* $\varepsilon > 0$ *and for any integer* $L \ge 0$, *there exists* $B_\varepsilon > 0$ *such that*

$$E\big[T(N,s)^2 E\{T(N,s) > B_\varepsilon\}\big] < \varepsilon$$

uniformly in N, s; *where*

$$T(N,s) = \left[\sum_{\alpha,\beta=1}^{p}\sum_{r=0}^{L}\left\{\sum_{t=1}^{N}\big(e_\alpha(t+s)e_\beta(t+s+r) - K_{\alpha\beta}\delta(0,r)\big)/N^{1/2}\right\}^2\right]^{1/2}.$$

5. The phrase "Assumptions (i) through (iv) of Theorem 2.2" in Theorem 3.1 and Proposition 4.1 is respectively changed to "Assumptions (i) through (v) of Theorem 2.2."

6. Hosoya (1989) also uses Theorem 2.2. Assumption (A) (page 403) of that paper should have the above Assumption (v) as additional Condition (iii).

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