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CORRECTION

A MODIFIED KOLMOGOROV–SMIRNOV TEST SENSITIVE TO TAIL ALTERNATIVES

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W. Stute and G. Dikta have kindly pointed out to us an error in the part of the proof of our Theorem 1 which is based on inequalities (8) and (9). This error is corrected by the following argument, replacing on page 935, line -11 to the sentence ending on line -5: First we require an elementary lemma, which we state without proof.

LEMMA. *Let* (X_n, Y_n), $n \geq 1$, *be a sequence of pairs of random variables such that*

(i) $$Y_n \to_p 1 \quad \text{as } n \to \infty,$$

and

(ii) $$M_n := \max(X_n, Y_n) \to_d M \quad \text{as } n \to \infty,$$

where M is a random variable such that

(iii) $$P\{M > 1\} = 1.$$

Then

(iv) $$P\{M_n = X_n\} \to 1 \quad \text{as } n \to \infty.$$

The part of the proof of Theorem 1 indicated above should now be replaced by the following two applications of this lemma.

APPLICATION 1. Set $X_n = L_{n,1}(k_n)$ and $Y_n = \sup\{u/G_n(u)\colon U_{k_n,n} < u < 1\}$. By Theorem 0 in Wellner (1978), $Y_n \to_p 1$ as $n \to \infty$. Now $M_n := L_{n,1} = \max(X_n, Y_n) \to_d M$, where M is a random variable with distribution $H(a,\infty)$ for $a > 1$. Since $H(1,\infty) = 0$ [see equation (2.10) of Rényi (1968)], $P\{M > 1) = 1$. Therefore by the lemma, $P\{L_{n,1} = L_{n,1}(k_n)\} \to 1$ as $n \to \infty$.

APPLICATION 2. Now set $X_n = L_{n,2}(k_n)$ and $Y_n := \sup\{G_n(u)/u\colon U_{k_n,n} < u < 1\}$. By (C1) and Theorem 0 in Wellner (1978), $Y_n \to_p 1$ as $n \to \infty$. Here, by Daniels (1945), $M_n := L_{n,2} = \max(X_n, Y_n) \to_d M$, where $P\{M > b\} = 1/b$ for $b \geq 1$. Again by the lemma, we see that $P\{L_{n,2} = L_{n,2}(k_n)\} \to 1$ as $n \to \infty$.

We are also very grateful to F. Calitz for making us aware of a mistake in our implementation of Noe's algorithm when we computed Table 1. A corrected Table 1 is presented here.

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TABLE 1
Weights and critical values for the L_n test.

Sample size	α 0.10 / w 0.2738	α 0.05 / w 0.2559	α 0.01 / w 0.2267
5	1.139	1.259	1.495
10	1.201	1.297	1.546
15	1.268	1.364	1.571
20	1.305	1.407	1.614
25	1.327	1.432	1.644
30	1.342	1.449	1.664
35	1.353	1.462	1.680
40	1.361	1.471	1.692
45	1.367	1.478	1.701
50	1.372	1.484	1.709
∞	1.425	1.544	1.788

We mention that Calitz (1987) has proposed a goodness-of-fit statistic, much in the same spirit as ours, that has shown some promise of being very sensitive to certain types of tail alternatives.

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