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REPLY

We would like to thank the editor for inviting discussions on our paper focusing on the following issues: (1) Bayes or empirical Bayes approaches versus the classical decision theoretic approach; (2) the role of the loss function; (3) general adaptation of estimators and (4) practice. We would also like to thank the discussants for their constructive comments and suggestions for future research. Because of space limitations, we are able to respond only briefly to some of the issues raised by the discussants.

Like Professors Berger and Morris, we appreciate fully the advantage of using prior knowledge (if available) for the construction of Bayes and empirical Bayes estimators for the simultaneous estimation problem. In the Poisson means estimation problem under L_1 loss, there are Bayes and empirical Bayes estimators dominating the usual ones (see Clevenson and Zidek, 1975, Tsui, 1978b, Ghosh and Parsian, 1981, and Ghosh, 1983) and such estimators belong to the general class of estimators proposed in the present paper. One drawback of Bayes procedures is that they very often suffer from lack of robustness against the choice of priors. Moreover, if one evalutes the risk performance of Bayes procedures (rather than the Bayes risk performance), one finds that such estimators can have risk considerably larger than the usual ones if the true parameter vector differs significantly from the prior mean. Our dominating estimators, however, cannot perform worse than the usual ones against *any* prior and are thus robust against misspecified priors (see Berger, 1982). Professor Morris has derived an interesting estimator (equation (10)) which does not depend on the explicit form of the prior, as long as the prior is exchangeable. However, the assumption that it is appropriate to use a linear Bayes rule must be justified, because the resulting estimator is *non*linear.

Professor Morris suggested the possibility of using loss function L_c, giving in equation (2) of his discussion, with general positive c_i's. The loss function L_c was motivated by the problem of unequal sample sizes when estimating several Poisson means. Some improved estimators for these situations, where the c_i's are general positive numbers, are provided by Tsui (1979b) with $m_i = 0$ for all i, by Tsui and Press (1982), with $m_i = m^* > 0$ for all i, and by Hwang (1982) for the more general case of $m_i \geq 0$ for all i. Generalization of the results in the present paper to situations where L_c is appropriate follows directly from Theorems 3.1 and 4.1.

Both Professors Berger and Morris have noted the dependence of the dominating rules on the loss function. For estimating several Poisson means, Clevenson and Zidek (1975) obtained estimators dominating the usual ones under L_1 and, more generally, under L_m, as long as $m_i = m^* \geq 1$ for all i. At this point, it is not known whether there are estimators that uniformly improve upon δ^0 under both L_0 and L_1 when the underlying distributions are Poisson, or any other common distribution. The improved estimators' dependence on the loss function is found when distributions other than the Poisson are assumed (see, e.g., Berger, 1980). It should be noted that the empirical Bayes estimator (10) proposed by Professor Morris in general also depends on the choice of a loss function, even though it does not depend on the choice of c_i in his loss. Furthermore, the estimator (10) does not seem to us to be any simpler than our δ^2, which has intuitive appeal (see Section 2).

On the question of how to choose the weights $\theta_i^{-m_i}$ in the loss L_m, we merely point out that in many situations, inaccurate estimation for small θ_i's seems highly undesirable, and the choice of the m_i's reflects our belief as to the relative severity of inaccurate estimation for different components.

Dr. Hudson advocates finding shrinkage estimators which have great reduction in risk in specific situations. We agree that this is important. However, a general theory is also of value because it provides a broad perspective on research in the entire area of

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simultaneous estimation of the parameters of discrete random variables. The general theory can then be modified to suit specific situations.

To sum up, we are fortunate to have stimulating discussions on our paper by three eminent decision theorists. Their discussions clearly bring out the need for further research in simultaneous estimation and indicate several important directions in which to explore. Finally, it is our belief that a harmonious blend of *both* Bayesian and frequentist ideas is likely to produce worthwhile research in the simultaneous estimation problem.

REFERENCES

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