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BOUNDED REGRET OF A SEQUENTIAL PROCEDURE FOR ESTIMATION OF THE MEAN

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Let $X_1, X_2, \cdots$ be independent observations from a population with mean μ and variance σ^2, and suppose that given a sample of size n one wishes to estimate μ by $\bar{X}_n$, subject to the loss function $L_n = A\sigma^{2\beta-2}(\bar{X}_n - \mu)^2 + n$, $A > 0$, $\beta > 0$. If σ is known, then the optimal fixed sample size n_0 for minimizing the risk $R_n = EL_n$ can be computed, but if σ is unknown there is no fixed sample size procedure that will achieve the minimum risk. For the case when σ is unknown, a number of authors have investigated the performance of sequential estimation procedures designed to come close to attaining the minimum risk R_{n_0}. In this paper it is shown that for the class of sequential estimation procedures with stopping rules

$$T_A = \inf\{n \geq n_A : n^{-1} \sum_{i=1}^n (X_i - \bar{X}_n)^2 \leq A^{-1/\beta} n^{2/\beta}\}$$

the regret $R_{T_A} - R_{n_0}$ remains bounded as $A \to \infty$, under suitable assumptions on the moments of X_1 and the delay n_A, but (unlike previous results of bounded regret) without any assumption about the type of distribution of X_1.

1. Introduction and summary. Let $X_1, X_2, \cdots$ be independent observations from a population with mean μ and variance σ^2. Given a sample of size n, we wish to estimate μ by the sample mean $\bar{X}_n$, subject to the loss function

$$L_n = A\sigma^{2\beta-2}(\bar{X}_n - \mu)^2 + n, \qquad A > 0, \qquad \beta > 0. \tag{1.1}$$

For a fixed sample size n, the risk is

$$R_n = A\sigma^{2\beta-2}E(\bar{X}_n - \mu)^2 + n = A\sigma^{2\beta}n^{-1} + n, \tag{1.2}$$

which is minimized (when σ is known) by taking the sample size n_0, where

$$[A^{1/2}\sigma^\beta] \leq n_0 \leq [A^{1/2}\sigma^\beta] + 1, \tag{1.3}$$

with $[a]$ meaning integer part of a. The corresponding minimum risk is

$$R_{n_0} \cong 2A^{1/2}\sigma^\beta. \tag{1.4}$$

However, if σ is unknown there is no fixed sample size procedure that will attain the minimum risk R_{n_0}. For this case we use the stopping rule

$$\begin{aligned} T = T_A &= \inf\{n \geq n_A : n \geq A^{1/2}(n^{-1} \textstyle\sum_{i=1}^n (X_i - \bar{X}_n)^2)^{\beta/2}\} \\ &= \inf\{n \geq n_A : n^{-1} \textstyle\sum_{i=1}^n (X_i - \bar{X}_n)^2 \leq A^{-1/\beta} n^{2/\beta}\}, \end{aligned} \tag{1.5}$$

where n_A is a positive integer which may depend on A, and estimate μ by $\bar{X}_T$. This type of sequential estimation procedure was first considered by Robbins (1959), in the normal case.

The performance of the sequential procedure with stopping rule T is usually measured by the risk efficiency R_{n_0}/R_T, and by the regret $R_T - R_{n_0}$, where R_T is the risk using the

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sequential procedure. When the distribution of X_1 is normal, the asymptotic risk efficiency (i.e., $R_T/R_{n_0} \to 1$ as $A \to \infty$) has been established by Starr (1966) for $\beta = 1$ as well as for more general loss functions. Starr and Woodroofe (1969) have proved that the regret remains bounded as $A \to \infty$ in the normal case (again $\beta = 1$), and Woodroofe (1977) has given second order approximations for the expected sample size and the regret. In all three papers the delay n_A does not depend on A.

When X_1 has an exponential distribution, Starr and Woodroofe (1972) have obtained the bounded regret of a sequential estimation procedure whose stopping rule is different from (1.5), and Vardi (1979) has established a similar result in the Poisson case.

Recently Chow and Yu (1981) have proved the asymptotic risk efficiency of the sequential procedure with stopping rule T, without any assumption about the type of distribution of X_1, as long as $E|X_1|^{2p} < \infty$ for some $p > 1$ and the delay n_A obeys certain growth conditions as $A \to \infty$ (as shown in their paper, the delay n_A must depend on A in order to achieve asymptotic risk efficiency even in the Bernoulli case). Results of asymptotic risk efficiency have also been proved by Ghosh and Mukhopadhyay (1979), assuming $E|X_1|^8 < \infty$.

In this paper we obtain the bounded regret of the sequential procedure with stopping rule T, provided that $E|X_1|^{6p} < \infty$ for some $p > 1$ and that n_A grows at a certain rate as $A \to \infty$, but without any further assumptions about the nature of the distribution of X_1. The main results are summarized in the following two theorems, whose proofs are given in the next section.

THEOREM 1. *Let* $X_1, X_2, \cdots$ *be* i.i.d. *with* $EX_1 = \mu$, $\mathrm{Var}(X_1) = \sigma^2 > 0$, *and* $E|X_1|^{4p} < \infty$ *for some* $p > 1$. *For* $A > 0$ *and* $\beta > 0$, *define* T *by*

$$T = T_A = \inf\{n \ge n_A : n^{-1} \textstyle\sum_{i=1}^n (X_i - \bar{X}_n)^2 \le A^{-1/\beta} n^{2/\beta}\},$$

where $\delta A^{1/4} \le n_A = o(A^{1/2})$ *as* $A \to \infty$, *for some* $\delta > 0$. *Then*

$$ET - n_0 = O(1) \quad as \quad A \to \infty.$$

THEOREM 2. *Let* $X_1, X_2, \cdots$ *be* i.i.d *with* $EX_1 = \mu$, $\mathrm{Var}(X_1) = \sigma^2 > 0$, *and* $E|X_1|^{6p} < \infty$ *for some* $p > 1$. *For* $A > 0$ *and* $\beta > 0$, *define* T *by*

$$T = T_A = \inf\{n \ge n_A : n^{-1} \textstyle\sum_{i=1}^n (X_i - \bar{X}_n)^2 \le A^{-1/\beta} n^{2/\beta}\},$$

where $\delta A^{1/4} \le n_A = o(A^{1/2})$ *as* $A \to \infty$, *for some* $\delta > 0$. *Then*

$$R_T - R_{n_0} = 0(1) \quad as \quad A \to \infty.$$

The major difficulty in obtaining these results is that the difference $L_T - R_{n_0}$, unlike the ratio L_T/R_{n_0}, is not uniformly integrable in A: in fact,

$$E|L_T - R_{n_0}| \sim \lambda A^{1/2} \quad \text{as} \quad A \to \infty,$$

where λ is a positive constant. Thus uniform integrability results are not enough to prove boundedness of the regret, and some sort of cancellation is needed on taking the expectation of $L_T - R_{n_0}$. In the next section this cancellation is achieved using two main ideas. First, heavy use is made of the defining relation for T, as in equations (2.7) and (2.20) below, to write parts of the regret in terms of the stopped sum of squares. Second, Wald-type equations for moments of stopped martingales are applied to obtain the desired cancellation (see especially (2.8), (2.12), (2.17) and (2.22)).

2. Proofs of Theorems 1 and 2. Without loss of generality, assume $\mu = 0$ and $\sigma = 1$, and define $S_n = \sum_1^n X_i$, $\bar{X}_n = n^{-1}S_n$, and $V_n = \sum_1^n (X_i - \bar{X}_n)^2$. We shall make frequent use of the fact that $V_n \le V_{n+1}$ for all n. Also, for integrable f, define

$$E'(f) = E\{fI_{[T>n_A]}\} \quad \text{and} \quad E''(f) = E\{fI_{[T=n_A]}\} = E(f) - E'(f).$$

Equations (2.1), (2.2), (2.3), (2.5) and (2.6) below follow from Lemmas 2, 4 and 5 of Chow and Yu (1981), while (2.4) follows from Theorem IV-3 of Yu (1978).

$$E|X_1|^{2t} < \infty,\ t \ge 1,\ n_A = O(A^{1/2}) \quad \text{as} \quad A \to \infty \tag{2.1}$$
$$\Rightarrow \{(A^{-1/2}T)^t : A \ge 1\} \text{ is uniformly integrable;}$$

$$E|X_1|^2 < \infty,\ n_A > \delta A^{1/4} \quad \text{for some} \quad \delta > 0 \tag{2.2}$$
$$\Rightarrow \{(A^{-1/2}T)^{-q} : A \ge 1\} \quad \text{is uniformly integrable for all } q > 0;$$

$$E|X_1|^2 < \infty,\ n_A \ge \delta A^{1/4} \quad \text{for some} \quad \delta > 0 \Rightarrow P[T < (1-\theta)A^{1/2}] \tag{2.3}$$
$$= O(A^{-q}) \quad \text{as} \quad A \to \infty, \qquad \text{for all} \quad q > 0, \qquad \text{if } \theta \in (0, 1);$$

$$E|X_1|^{2t} < \infty,\ t \ge 2,\ \delta A^{1/4} \le n_A = o(A^{1/2}) \quad \text{as} \quad A \to \infty \tag{2.4}$$
$$\Rightarrow \{|A^{-1/4}(T - A^{1/2})|^t : A \ge 1\} \quad \text{is uniformly integrable;}$$

$$E|X_1|^{2t} < \infty,\ t \ge 1,\ n_A = O(A^{1/2}) \quad \text{as} \quad A \to \infty \tag{2.5}$$
$$\Rightarrow \{|A^{-1/4}S_T|^{2t} : t \ge 1\} \quad \text{is uniformly integrable;}$$

$$E|X_1|^{2t} < \infty,\ t \ge 2,\ n_A = O(A^{1/2}) \quad \text{as} \quad A \to \infty \tag{2.6}$$
$$\Rightarrow \{|A^{-1/4}(\textstyle\sum_1^T X_i^2 - T)|^t : A \ge 1\} \quad \text{is uniformly integrable.}$$

PROOF OF THEOREM 1. By the definition of T,

$$(T-1)^{1+2/\beta} \le A^{1/\beta}\, V_{T-1} \le A^{1/\beta} V_T \le A^{1/\beta} \textstyle\sum_1^T X_i^2 \tag{2.7}$$

on $\{T > n_A\}$, so by Wald's Lemma and (2.3), as $A \to \infty$,

$$\begin{aligned} E^{1+2/\beta}(T-1) &\le E[(T-1)^{1+2/\beta}] \le A^{1/\beta}E(T) + n_A^{1+2/\beta}P(T = n_A) \\ &\le A^{1/\beta}\{E(T-1) + O(1)\}. \end{aligned} \tag{2.8}$$

Therefore, by (2.1),

$$E^{2/\beta}(T-1) \le A^{1/\beta}\{1 + O(A^{-1/2})\}, \quad E(T-1) \le A^{1/2}\{1 + O(A^{-1/2})\}^{\beta/2} = A^{1/2} + O(1),$$

and hence

$$ET \le A^{1/2} + O(1). \tag{2.9}$$

To prove the reverse inequality, note that from the definition of T, a Taylor series expansion, and (2.3),

$$\begin{aligned} E(T - A^{1/2}) &= A^{1/2}E'(A^{-1/2}T - 1) + O(1) \ge A^{1/2}E'[(T^{-1}V_T)^{\beta/2} - 1] + O(1) \\ &= (\beta/2)A^{1/2}E'[T^{-1}(V_T - T)] \\ &\quad + (\beta/4)(\beta/2 - 1)A^{1/2}E'[\lambda^{\beta/2-2}(T^{-1}V_T - 1)^2] + O(1), \end{aligned} \tag{2.10}$$

where λ is a random variable lying between 1 and $T^{-1}V_T$. If $\beta/2 < 1$, by the defining property of T, Hölder's inequality, (2.2), (2.5) and (2.6),

$$\begin{aligned} E'[\lambda^{\beta/2-2}(T^{-1}V_T - 1)^2] &\le E'\{(T^{-1}V_T - 1)^2[1 + (T^{-1}V_T)^{\beta/2-2}]\} \\ &\le E'[(T^{-1}V_T - 1)^2\{1 + O(1)[(T-1)^{-1}V_{T-1}]^{\beta/2-2}\}] \\ &\le E'[(T^{-1}V_T - 1)^2\{1 + O(1)[A^{-1/\beta}(T-1)^{2/\beta}]^{\beta/2-2}\}] \\ &\le E'[(T^{-1}V_T - 1)^2\{1 + O(1)A^{-1/2+2/\beta}T^{1-4/\beta}\}] = O(A^{-1/2}). \end{aligned} \tag{2.11}$$

By (2.3), Wald's Lemma, Hölder's inequality, (2.2), (2.4) and (2.6), since $E|X_1|^{4p} < \infty$,

$$
\begin{aligned}
A^{1/2}E'[T^{-1}(V_T - T)] &= A^{1/2}E[T^{-1}(V_T - T)] + O(1) \\
&= A^{1/2}E[T^{-1}(\textstyle\sum_1^T X_i^2 - T)] + O(1) \qquad (2.12)\\
&= E[T^{-1}(A^{1/2} - T)(\textstyle\sum_1^T X_i^2 - T)] + O(1) = O(1).
\end{aligned}
$$

From (2.10), (2.12), (and (2.11) if $\beta/2 < 1$), as $A \to \infty$,

$$E(T - A^{1/2}) \geq O(1), \qquad (2.13)$$

yielding Theorem 1 by (2.9).

REMARK. The results of Lai and Siegmund (1979) are designed to give second order approximations to expected stopping times in a wide variety of situations. In the present case, applying their results would require checking a number of rather complicated conditions (as in their Theorem 3), and undoubtedly would involve a much higher moment assumption than the one needed here. Their paper also gives second order approximations to the variance of the stopping time in the special case of ordinary renewal theory; since the variance of T is an important quantity in the proof of Theorem 1 above, it would be of interest to obtain such second order approximations in this case as well.

The following lemma is needed for the proof of Theorem 2.

LEMMA. *Let* $X_1, X_2, \cdots$ *be* i.i.d. *with* $EX_1 = 0$, $EX_1^2 = 1$, *and* $E|X_1|^4 < \infty$. *For* $A > 0$ *and* $\beta > 0$, *define* T *by*

$$T = T_A = \inf\{n \geq n_A : n^{-1} \textstyle\sum_{i=1}^n (X_i - \bar{X}_n)^2 \leq A^{-1/\beta} n^{2/\beta}\},$$

where $n_A = O(A^{1/2})$. *Then for every* $A \geq 1$,

$$E(S_T^2 - T)^2 = 4E \textstyle\sum_{j=1}^T S_{j-1}^2 + E(X_1^2 - 1)^2 ET + 4E(X_1^3)E(TS_T) < \infty, \qquad (2.14)$$

$$E(S_T^2 - \textstyle\sum_{i=1}^T X_i^2)^2 = 4E \sum_{j=1}^T S_{j-1}^2 < \infty, \; E(\sum_1^T X_i^2 - T)^2 = E(X_1^2 - 1)^2 ET < \infty, \qquad (2.15)$$

$$E\{(S_T^2 - T)(\textstyle\sum_1^T X_i^2 - T)\} = O(A^{1/2}), \quad \text{as } A \to \infty. \qquad (2.16)$$

PROOF. $\{(S_n^2 - n), \mathscr{F}_n\}$ is a martingale with martingale differences $(X_n^2 - 1) + 2X_nS_{n-1}$, where $\mathscr{F}_n = \sigma(X_1, \cdots, X_n)$. Also, from (2.1), $ET^2 < \infty$, and Chow, Robbins and Teicher (1965) (see Chow and Teicher, 1978, Theorem 7, page 241), for fixed A, as $n \to \infty$

$$\int_{[T>n]} |S_n^2 - n| \, dP \leq \int_{[T>n]} S_n^2 \, dP + \int_{[T>n]} T \, dP = E(S_{T\wedge n}^2) - \int_{[T\leq n]} S_T^2 \, dP + o(1) = o(1).$$

Therefore, from Theorem 1 and Lemmas 6 and 8 of Chow, Robbins and Teicher (1965),

$$
\begin{aligned}
E(S_T^2 - T)^2 &= E \textstyle\sum_{j=1}^T E\{(X_j^2 - 1 + 2X_jS_{j-1})^2 \mid \mathscr{F}_{j-1}\} \\
&= E \textstyle\sum_{j=1}^T \{E(X_1^2 - 1)^2 + 4S_{j-1}^2 + 4E(X_1^3)S_{j-1}\} \\
&= E(X_1^2 - 1)^2 ET + 4E \textstyle\sum_{j=1}^T S_{j-1}^2 + 4E(X_1^3)E(TS_T) < \infty,
\end{aligned}
$$

proving (2.14). Similarly for (2.15). Finally, by (2.14), (2.15), Wald's lemma and (2.1),

$$
\begin{aligned}
2E\{(S_T^2 - T)(\textstyle\sum_1^T X_i^2 - T)\} &= -\{E(S_T^2 - \textstyle\sum_1^T X_i^2)^2 - E(S_T^2 - T)^2 \\
&\quad - E(\textstyle\sum_1^T X_i^2 - T)^2\} \\
&= -\{4E \textstyle\sum_1^T S_{j-1}^2 - E(X_1^2 - 1)^2 ET \qquad (2.17)\\
&\quad - 4E(X_1^3)E(TS_T) - 4E \textstyle\sum_1^T S_{j-1}^2 - E(X_1^2 - 1)^2 ET\} \\
&= 4E(X_1^3)E[(T - A^{1/2})S_T] + 2E(X_1^2 - 1)^2 ET
\end{aligned}
$$

$$= 4E(X_1^3)E[(T - A^{1/2})S_T] + O(A^{1/2}), \quad A \to \infty.$$

But from (2.4), (2.5) and Hölder's inequality,

$$E|(T - A^{1/2})S_T| \le E^{1/2}|T - A^{1/2}|^2 E^{1/2}|S_T|^2 = O(A^{1/2}) \quad \text{as} \quad A \to \infty, \tag{2.18}$$

and (2.16) follows from (2.17) and (2.18).

PROOF OF THEOREM 2. From Theorem 2 of Chow, Robbins and Teicher (1965),

$$\begin{aligned} R_T - R_{n_0} &= E[S_T^2(AT^{-2})] + ET - 2A^{1/2} = E[S_T^2(AT^{-2} - 1)] + 2ET - 2A^{1/2} \\ &= E[S_T^2(AT^{-2} - 1)] + O(1) \quad \text{as} \quad A \to \infty, \end{aligned}$$

by Theorem 1 above. It therefore suffices to prove that

$$E[S_T^2(AT^{-2} - 1)] = O(1) \quad \text{as} \quad A \to \infty.$$

By Taylor's Theorem,

$$E[S_T^2(AT^{-2} - 1)] = -2E[S_T^2(A^{-1/2}T - 1)] + 3E[S_T^2\lambda^{-4}(A^{-1/2}T - 1)^2], \tag{2.19}$$

where λ is a random variable lying between 1 and $A^{-1/2}T$. From (2.2), (2.4), (2.5) and Schwartz's inequality, the second term on the right side of (2.19) is bounded in A. The main point is therefore to establish that

$$E[S_T^2(A^{-1/2}T - 1)] = O(1) \quad \text{as} \quad A \to \infty.$$

Using the definition of T, (2.5), (2.3) and (2.1), for some λ between 1 and 2,

$$\begin{aligned} E\{S_T^2(T^{-1}V_T)^{\beta/2}\} &\le E\{S_T^2(A^{-1/2}T)\} \le E\{S_T^2[A^{-1/2}(T-1)]\} + O(1) \\ &\le E'\{S_T^2[(T-1)^{-1}V_{T-1}]^{\beta/2}\} + O(1) \\ &\le E\{S_T^2[(T-1)^{-1}V_T]^{\beta/2}\} + O(1) \\ &= E\{S_T^2(T^{-1}V_T)^{\beta/2}\} + E\{S_T^2(T^{-1}V_T)^{\beta/2} \\ &\quad \cdot([T/(T-1)]^{\beta/2} - 1)\} + O(1) \\ &\le E\{S_T^2(T^{-1}V_T)^{\beta/2}\} + E\{S_T^2(A^{-1/2}T)(\beta/2)[(T-1)^{-1} \\ &\quad + (\tfrac{1}{2})(\beta/2 - 1)(T-1)^{-2}\lambda^{\beta/2-2}]\} + O(1) \\ &= E\{S_T^2(T^{-1}V_T)^{\beta/2}\} + O(1). \end{aligned} \tag{2.20}$$

Hence

$$\begin{aligned} E\{S_T^2(A^{-1/2}T - 1)\} &= E\{S_T^2[(T^{-1}V_T)^{\beta/2} - 1]\} + O(1) \\ &= (\beta/2)E'[S_T^2T^{-1}(V_T - T)] \\ &\quad + (\beta/4)(\beta/2 - 1)E'[S_T^2\lambda^{\beta/2-2}(T^{-1}V_T - 1)^2] + O(1), \end{aligned} \tag{2.21}$$

where λ is a random variable lying between 1 and $T^{-1}V_T$. As in (2.10), the second term on the right side of (2.21) is bounded in A. Therefore, from (2.21), (2.3), (2.5) and (2.2), Wald's Lemma, and (2.4), (2.5), (2.6) together with Hölder's inequality,

$$\begin{aligned} E[S_T^2(A^{-1/2}T - 1)] &= (\beta/2)E'[S_T^2T^{-1}(V_T - T)] + O(1) \\ &= (\beta/2)E[S_T^2T^{-1}(V_T - T)] + O(1) \\ &= (\beta/2)E[S_T^2T^{-1}(\textstyle\sum_1^T X_i^2 - T)] + O(1) \\ &= (\beta/2)E[(S_T^2 - T)T^{-1}(\textstyle\sum_1^T X_i^2 - T)] + O(1) \\ &= (\beta/2)A^{-1/2}E[(S_T^2 - T)(\textstyle\sum_1^T X_i^2 - T)] + O(1). \end{aligned} \tag{2.22}$$

It follows from (2.22) and the Lemma that

$$E[S_T^2(A^{-1/2}T - 1)] = O(1) \quad \text{as} \quad A \to \infty, \tag{2.23}$$

which proves Theorem 2.

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