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A NOTE ON METHOD OF CONSTRUCTION OF AFFINE RESOLVABLE BALANCED INCOMPLETE BLOCK DESIGNS

BY H. L. AGRAWAL AND B. S. BOOB

University of Rajasthan

A new method of construction of affine resolvable balanced incomplete block design from a given affine resolvable balanced incomplete block design is given.

Introduction and summary. The balanced incomplete block design (BIBD) with parameters v, b, r, k and λ when affine resolvable can be expressed as a function of two parameters m and u where m is the number of blocks in a replication and u is the number of treatments common between any two blocks belonging to different replications. More explicitly, the affine resolvable BIBD denoted by $A(u, m)$ have the parameters (Bose (1942))

$$v = m^2u\,, \qquad b = m\frac{m^2u-1}{m-1}\,, \qquad r = \frac{m^2u-1}{m-1}\,,$$

$$k = mu \qquad \text{and} \qquad \lambda = \frac{mu-1}{m-1}\,.$$

Griffiths and Mavron (1972) have shown that the existence of $A(u, 3)$ implies the existence of $A(3u, 3)$ and Ratnalikar (1975) proved the existence of $A(4u, 4)$ from $A(u, 4)$.

In this note it has been established that if m is prime power, the existence of $A(u, m)$ implies the existence of $A(mu, m)$.

Construction. Let $\alpha_i^{\,t}$ be the column of the incidence matrix of $A(u, m)$ corresponding to the ith block in the tth replication $i = 0, 1, \cdots, (m-1)$; $t = 1, 2, \cdots, r$. Since m is a prime power, we can construct $(m-1)$ mutually orthogonal latin squares of side m in symbols $0, 1, \cdots, (m-1)$ with the initial rows in the natural order, i.e. the initial rows are $(0, 1, \cdots, m-1)$. Let these be $L_1, L_2, \cdots, L_{m-1}$ and construct the matrix L as

$$L = [L_1|L_2|\cdots|L_{m-1}|L_m]\,,$$

where

$$L_m = \begin{bmatrix} 0 & 1 \cdots m-1 \\ 0 & 1 \cdots m-1 \\ \vdots & \vdots \qquad \vdots \\ 0 & 1 \cdots m-1 \end{bmatrix} m \times m\,.$$

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Now in L, replacing the integer i by α_i^t for fixed t, we get the matrix N_t. Define

$$N = [N_1 \quad N_2 \cdots N_r \mid N_{r+1}] \\ = [M_1 \mid N_{r+1}] ,$$

where

$$N_{r+1} = \begin{bmatrix} E_{v1} & 0 & \cdots & 0 \\ 0 & E_{v1} & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & E_{v1} \end{bmatrix} mv \times m .$$

E_{v1} is a $v \times 1$ column matrix with all elements unity.

We now prove that N is the incidence matrix of $A(mu, m)$. Consider two treatments i and j.

Case (i).

$$sv + 1 \leqq i \neq j \leqq (s+1)v \qquad s = 0, 1, \cdots, m-1 .$$

As in the rows of L every symbol occurs m times, the first v rows of M_1 are the m replications of $A(u, m)$. Thus the treatments i and j occur together in $m\lambda$ blocks and once in N_{r+1}. Thus i and j occur together in $m\lambda + 1 = r$ blocks.

Case (ii).

$$sv + 1 \leqq i \leqq (s+1)v ; \\ s'v + 1 \leqq j \leqq (s'+1)v \qquad s \neq s' = 0, 1, \cdots, m-1 .$$

It is obvious from L that i and j occur together only once in N_t for $t = 1, \cdots, r$ and do not occur in N_{r+1}. Hence they occur together in r blocks.

Thus N is the incidence matrix of BIBD with parameters

$$v^* = m^3u , \qquad b^* = m\frac{m^3u - 1}{m-1} , \qquad r^* = \frac{m^3u - 1}{m-1} ,$$

$$k^* = m^2u \qquad \text{and} \qquad \lambda^* = \frac{m^2u - 1}{m-1} .$$

It is obvious that N is resolvable and as $b^* = v^* + r^* - 1$ (Bose 1942), N is affine resolvable.

Thus N is the incidence matrix of $A(mu, m)$.

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Department of Statistics
University of Rajasthan
Jaipur-302004 (India)