The Annals of Statistics
1996, Vol. 24, No. 6, 2560–2568

SEARCHING PROBABILITIES FOR NONZERO EFFECTS IN SEARCH DESIGNS FOR THE NOISY CASE

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Consider search designs for identifying at most k nonzero effects under a search linear model. In the noisy case, the procedure for identifying these effects, which is based on the sum of squares due to error, is considered and its stochastic properties are studied. The errors of observations are assumed to be distributed independently with the normal distribution $N(0, \sigma^2)$ and, in particular, the case of $k = 1$ is considered. Some results on a searching probability under the procedure are presented. Probabilities with respect to search designs which are main effect plus one plans constructed in earlier works, are presented. A comparison between two search designs is given by using a criterion based on probability.

1. Introduction.

Consider the search linear model

$$\mathbf{y} = A_1\boldsymbol{\xi}_1 + A_2\boldsymbol{\xi}_2 + \mathbf{e}, \qquad V(\mathbf{e}) = \sigma^2 I, \tag{1.1}$$

where $\mathbf{y}(N \times 1)$ is a vector of observations, $A_i(N \times \nu_i)$ are known design matrices and $\boldsymbol{\xi}_i(\nu_i \times 1)$ are vectors of effects for $i = 1, 2$, $\mathbf{e}(N \times 1)$ is an error vector, σ^2 is the error variance and I is the identity matrix of appropriate order. It is known that at most k effects of $\boldsymbol{\xi}_2$ are nonzero, where k is assumed to be a quite small positive integer compared with ν_2, and the remaining effects of $\boldsymbol{\xi}_2$ are zero. However, it is not known which are the nonzero effects. It is assumed that the effects of $\boldsymbol{\xi}_1$ are completely unknown. The problem is to search (or identify) the k nonzero effects of $\boldsymbol{\xi}_2$ and estimate them and also estimate the effects in $\boldsymbol{\xi}_1$. This concept was introduced by Srivastava (1975), who gave the following basic mathematical formulation of the problem.

THEOREM 1.1. *Consider the model* (1.1) *and let* $\sigma^2 = 0$. *A necessary and sufficient condition that the search and inference problem can be completely solved is that for every submatrix* $A_{22}(N \times 2k)$ *of* A_2,

$$\operatorname{rank}(A_1 \colon A_{22}) = \nu_1 + 2k \tag{1.2}$$

holds.

Received January 1994; revised January 1996.
AMS 1991 *subject classification*. Primary 62K15.
Key words and phrases. Search designs, noisy case, sum of squares due to error, normal distribution, searching probability.

The case when $\sigma^2 = 0$ is called the noiseless case. However, in practical experiments, we have the noisy case $\sigma^2 > 0$, in which the condition (1.2) is still necessary. Henceforth, we consider a design (called a search design) such that condition (1.2) holds. For the noisy case, Srivastava (1975) also proposed a procedure for solving the problem, which corresponds to the minimization of the sum of squares due to error (SSE). The purpose of this paper is to study stochastic properties of SSE for a given search design. Under the procedure, it is interesting to learn how large the probability is that a search design could search the nonzero effects. Ghosh (1987) studied equivalence properties for the minimization of SSE and also gave an estimator of the number k of nonzero effects in ξ_2.

In Section 2, some results on the minimization of SSE are given and a search probability is discussed. In Sections 3 and 4, the components of the error vector $\mathbf{e}$ are assumed to be distributed independently with the normal distribution $N(0, \sigma^2)$ and, in particular, the case of $k = 1$ is considered. In Section 3, the searching probability is more deeply investigated and a criterion based on it is proposed for selecting a search design. Section 4 presents computational results of the searching probabilities with respect to search designs which are main effect plus one (MEP.1) plans for 2^m factorials given by Ghosh (1981), Ohnishi and Shirakura (1985), Shirakura (1991) and Shirakura and Tazawa (1991). A comparison between the two plans is given by using the criterion in Section 3.

2. Procedure for searching nonzero effects. Consider a search design T with N treatments. Suppose $\boldsymbol{\zeta}(k \times 1)$ is a vector of nonzero effects of $\boldsymbol{\xi}_2$. Then model (1.1) reduces to

$$\mathbf{y} = A_1\boldsymbol{\xi}_1 + A_{21}(\boldsymbol{\zeta})\boldsymbol{\zeta} + \mathbf{e}, \tag{2.1}$$

where $A_{21}(\boldsymbol{\zeta})$ is the $N \times k$ submatrix of A_2 corresponding to $\boldsymbol{\zeta}$ ($\subset \boldsymbol{\xi}_2$). The BLUE of $\boldsymbol{\theta} = (\boldsymbol{\xi}_1', \boldsymbol{\zeta}')'$ and the estimate of $\mathbf{y}$ are given by

$$\hat{\boldsymbol{\theta}} = \widehat{\begin{pmatrix} \boldsymbol{\xi}_1 \\ \boldsymbol{\zeta} \end{pmatrix}} = M(\boldsymbol{\zeta})^{-1}A(\boldsymbol{\zeta})'\mathbf{y} \quad \text{and} \quad \hat{\mathbf{y}} = A(\boldsymbol{\zeta})\hat{\boldsymbol{\theta}}, \tag{2.2}$$

respectively, where

$$A(\boldsymbol{\zeta}) = [A_1 : A_{21}(\boldsymbol{\zeta})], \qquad M(\boldsymbol{\zeta}) = A(\boldsymbol{\zeta})'A(\boldsymbol{\zeta}). \tag{2.3}$$

Then the SSE $s(\boldsymbol{\zeta})^2$ can be written as

$$s(\boldsymbol{\zeta})^2 = \|\mathbf{y} - \hat{\mathbf{y}}\|^2 = \mathbf{y}'(I - Q(\boldsymbol{\zeta}))\mathbf{y}, \tag{2.4}$$

where $\|\cdot\|$ is the length of a vector and

$$Q(\boldsymbol{\zeta}) = A(\boldsymbol{\zeta})M(\boldsymbol{\zeta})^{-1}A(\boldsymbol{\zeta})'.$$

Note that $Q(\boldsymbol{\zeta})$ is an idempotent matrix of order N satisfying

$$Q(\boldsymbol{\zeta})' = Q(\boldsymbol{\zeta}), \qquad Q(\boldsymbol{\zeta})^2 = Q(\boldsymbol{\zeta}) \quad \text{and} \quad A(\boldsymbol{\zeta}) = Q(\boldsymbol{\zeta})A(\boldsymbol{\zeta}).$$

Srivastava (1975) proposed the folowing procedure for searching out the nonzero effects of $\boldsymbol{\zeta}_2$: for an observation vector $\mathbf{y}$, calculate $s(\boldsymbol{\zeta})^2$ for all possible $k \times 1$ vectors $\boldsymbol{\zeta}$ of $\boldsymbol{\xi}_2$. Find a vector $\boldsymbol{\zeta}^*$ $(k \times 1)$ for which $s(\boldsymbol{\zeta}^*)^2$ turns out to be a minimum. Then take $\boldsymbol{\zeta}^*$ as the possibly nonzero vector of ξ_2. Also, the BLUE of $(\boldsymbol{\xi}_1, \boldsymbol{\zeta}^*)$ is given by replacing $\boldsymbol{\zeta}$ with $\boldsymbol{\zeta}^*$ in $\boldsymbol{\theta}$ of (2.2).

Now assume the true model of (1.1) to be

$$\mathbf{y} = A_1\boldsymbol{\xi}_1 + A_{21}(\boldsymbol{\zeta}_0)\boldsymbol{\zeta}_0 + \mathbf{e}, \tag{2.5}$$

where $\boldsymbol{\zeta}_0$ is the $k \times 1$ vector of possibly nonzero effects of $\boldsymbol{\xi}_2$. Then it follows that the expectation of $s(\boldsymbol{\zeta})^2$ of (2.4) is given as

$$E\big(s(\boldsymbol{\zeta})^2\big) = \boldsymbol{\theta}_0' B(\boldsymbol{\zeta}; \boldsymbol{\zeta}_0)\boldsymbol{\theta}_0 + (N - \nu_1 - k)\sigma^2,$$

where $\boldsymbol{\theta}_0 = (\boldsymbol{\xi}_1', \boldsymbol{\zeta}_0')'$ and

$$B(\boldsymbol{\zeta} : \boldsymbol{\zeta}_0) = A(\boldsymbol{\zeta}_0)'(I - Q(\boldsymbol{\zeta}))A(\boldsymbol{\zeta}_0).$$

Therefore, the following theorem can easily be established:

THEOREM 2.1. *Under* (2.5), $E(s(\boldsymbol{\zeta})^2) \geq (N - \nu_1 - k)\sigma^2$. *Furthermore,* $E(s(\boldsymbol{\zeta})^2) = (N - \nu_1 - k)\sigma^2$ *holds for any values of elements in* $\boldsymbol{\theta}_0$ *if and only if* $\boldsymbol{\zeta} = \boldsymbol{\zeta}_0$.

We study more deeply the stochastic properties of $s(\boldsymbol{\zeta})^2$. Let

$$M = A_1'A_1 \quad \text{and} \quad Q = A_1 M^{-1} A_1'$$

and define

$$h(\boldsymbol{\zeta}, \mathbf{y}) = \mathbf{y}'(Q(\boldsymbol{\zeta}) - Q)\mathbf{y}. \tag{2.6}$$

Then $s(\boldsymbol{\zeta})^2$ can be expressed as

$$s(\boldsymbol{\zeta})^2 = \mathbf{y}'(I - Q)\mathbf{y} - h(\boldsymbol{\zeta}, \mathbf{y}).$$

We thus establish the following theorem.

THEOREM 2.2. *For an observation vector* $\mathbf{y}$, *the minimization of* $s(\boldsymbol{\zeta})^2$ *on* $\boldsymbol{\zeta}$ $(\subset \boldsymbol{\xi}_2)$ *is equivalent to the maximization of* $h(\boldsymbol{\zeta}, \mathbf{y})$ *on* $\boldsymbol{\zeta}$ $(\subset \boldsymbol{\xi}_2)$.

In view of the procedure considered here for searching $\boldsymbol{\zeta}_0$ in (2.5), it is most desirable that if $\boldsymbol{\zeta}^*$ $(\subset \boldsymbol{\xi}_2)$ maximizes $h(\boldsymbol{\zeta}, \mathbf{y})$, then $\boldsymbol{\zeta}^* = \boldsymbol{\zeta}_0$ is obtained exactly. However, this is not ensured for the noisy case. So, for $\mathbf{y}$ composed of random variables, consider the minimum probability

$$P = \min_{\boldsymbol{\zeta}_0 \subset \boldsymbol{\xi}_2} \min_{\boldsymbol{\zeta} \in \mathscr{A}(\boldsymbol{\xi}_2; \boldsymbol{\zeta}_0)} P\big(h(\boldsymbol{\zeta}_0, \mathbf{y}) > h(\boldsymbol{\zeta}, \mathbf{y})\big), \tag{2.7}$$

where $\mathscr{A}(\boldsymbol{\xi}_2; \boldsymbol{\zeta}_0)$ denotes the set of all possible $\boldsymbol{\zeta}$ of $\boldsymbol{\xi}_2$, of which at least one effect is not of $\boldsymbol{\zeta}_0$. This means that the larger the value of P, the higher the confidence with which the true vector could be searched by the procedure. In this sense, P is called a searching probability.

Since the matrix $M(\boldsymbol{\zeta})$ in (2.3) is positive definite, the following two lemmas can easily be obtained.

LEMMA 2.3. *The inverse of* $M(\boldsymbol{\zeta})$ *is expressed as*

$$M(\boldsymbol{\zeta})^{-1} = \left[\begin{array}{c|c} M^{-1} + M^{-1}A_1' A_{21}(\boldsymbol{\zeta}) C(\boldsymbol{\zeta}) A_{21}(\boldsymbol{\zeta})' A_1 M^{-1} & -M^{-1}A_1' A_{21}(\boldsymbol{\zeta}) C(\boldsymbol{\zeta}) \\ \hline -C(\boldsymbol{\zeta}) A_{21}(\boldsymbol{\zeta})' A_1 M^{-1} & C(\boldsymbol{\zeta}) \end{array}\right],$$

where

$$C(\boldsymbol{\zeta}) = \left(A_{21}(\boldsymbol{\zeta})'(I-Q)A_{21}(\boldsymbol{\zeta})\right)^{-1}.$$

LEMMA 2.4. $h(\boldsymbol{\zeta}, \mathbf{y})$ *of* (2.6) *can be expressed as*

$$h(\boldsymbol{\zeta}, \mathbf{y}) = \mathbf{y}'(I-Q)A_{21}(\boldsymbol{\zeta})C(\boldsymbol{\zeta})A_{21}(\boldsymbol{\zeta})'(I-Q)\mathbf{y}. \tag{2.8}$$

These two lemmas may be useful in evaluating the searching probability P.

3. Searching probabilities for one nonzero effect. Consider the case of $k = 1$, where there exists at most one nonzero effect in $\boldsymbol{\xi}_2$. Then model (2.1) becomes

$$\mathbf{y} = A_1\boldsymbol{\xi}_1 + \mathbf{a}(\zeta)\zeta + \mathbf{e},$$

where ζ is an effect in $\boldsymbol{\xi}_2$ and $\mathbf{a}(\zeta)$ $[= A_{21}(\zeta)]$ is the $N \times 1$ column of A_2 corresponding to ζ. Suppose now the components of the error vector $\mathbf{e}$ are distributed independently with a normal distribution $N(0, \sigma^2)$. The $h(\zeta, \mathbf{y})$ of (2.8) reduces to

$$h(\zeta, \mathbf{y}) = \{\mathbf{a}(\zeta)'(I-Q)\mathbf{y}\}^2 / r(\zeta), \tag{3.1}$$

where $r(\zeta) = \mathbf{a}(\zeta)'(I-Q)\mathbf{a}(\zeta)$ is a positive value for any ζ. By (3.1), it is clear that the true model

$$\mathbf{y} = A_1\boldsymbol{\xi}_1 + \mathbf{a}(\zeta_0)\zeta_0 + \mathbf{e}$$

yields

$$\begin{aligned} h(\zeta, \mathbf{y}) &= \left(\frac{\mathbf{a}(\zeta)'(I-Q)\mathbf{a}(\zeta_0)}{\sqrt{r(\zeta)}}\zeta_0 + \frac{\mathbf{a}(\zeta)'(I-Q)\mathbf{e}}{\sqrt{r(\zeta)}} \right)^2 \\ &= \left\{ \mathbf{b}(\zeta)'\mathbf{b}(\zeta_0)\sqrt{r(\zeta_0)}\,\zeta_0 + \mathbf{b}(\zeta)'\mathbf{e} \right\}^2, \end{aligned}$$

where $\mathbf{b}(\zeta) = (I-Q)\mathbf{a}(\zeta)/\sqrt{r(\zeta)}$, which is the unit vector, that is, $\|\mathbf{b}(\zeta)\| = 1$. Denote

$$Z = \mathbf{b}(\zeta)'\mathbf{e} + \mathbf{b}(\zeta)'\mathbf{b}(\zeta_0)\delta_0 \quad \text{or} \quad Z_0 = \mathbf{b}(\zeta_0)'\mathbf{e} + \delta_0,$$

according as $\zeta \neq \zeta_0$ or $\zeta = \zeta_0$, where $\delta_0 = \sqrt{r(\zeta_0)}\,\zeta_0$. Then it is easy to see that for $\zeta \neq \zeta_0$,

$$\begin{aligned} P(h(\zeta_0, \mathbf{y}) > h(\zeta, \mathbf{y})) &= P(Z_0^2 > Z^2) \\ &= P((Z_0 + Z)(Z_0 - Z) > 0) \\ &= P(X_1 > 0, X_2 > 0) + P(X_1 < 0, X_2 < 0), \end{aligned} \tag{3.2}$$

where

$$
\begin{aligned}
X_1 &= Z_0 - Z = (\mathbf{b}(\zeta_0) - \mathbf{b}(\zeta))'\mathbf{e} + (1 - \mathbf{b}(\zeta)'\mathbf{b}(\zeta_0))\delta_0, \\
X_2 &= Z_0 + Z = (\mathbf{b}(\zeta_0) + \mathbf{b}(\zeta))'\mathbf{e} + (1 + \mathbf{b}(\zeta)'\mathbf{b}(\zeta_0))\delta_0.
\end{aligned} \tag{3.3}
$$

It follows from condition (1.2) that $\mathbf{b}(\zeta_0) \neq \pm\mathbf{b}(\zeta)$ for any ζ_0 and $\zeta(\neq \zeta_0) \in \boldsymbol{\xi}_2$. Furthermore, $(\mathbf{b}(\zeta_0) - \mathbf{b}(\zeta))'(\mathbf{b}(\zeta_0) + \mathbf{b}(\zeta)) = 0$. Therefore, the following result is true.

LEMMA 3.1. *Random variables X_1 and X_2 defined in* (3.3) *are stochastically independent and have the normal distributions $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$, respectively, where*

$$
\begin{aligned}
\mu_1 &= (1 - \mathbf{b}(\zeta)'\mathbf{b}(\zeta_0))\delta_0, & \mu_2 &= (1 + \mathbf{b}(\zeta)'\mathbf{b}(\zeta_0))\delta_0, \\
\sigma_1^2 &= 2(1 - \mathbf{b}(\zeta)'\mathbf{b}(\zeta_0))\sigma^2, & \sigma_2^2 &= 2(1 + \mathbf{b}(\zeta)'\mathbf{b}(\zeta_0))\sigma^2.
\end{aligned}
$$

By this lemma, the following theorem is now established.

PROPOSITION 3.2. *The probability of* (3.2) *can be written as*

$$
\begin{aligned}
G(x, d) = 1 &- \Phi(d\sqrt{1-x}) - \Phi(d\sqrt{1+x}) \\
&+ 2\Phi(d\sqrt{1-x})\Phi(d\sqrt{1+x}),
\end{aligned} \tag{3.4}
$$

where

$$
\begin{aligned}
x &= \mathbf{b}(\zeta)'\mathbf{b}(\zeta_0), \qquad -1 \le x \le 1, \\
d &= \frac{\delta_0}{\sqrt{2}\,\sigma} = \sqrt{\frac{r(\zeta_0)}{2}}\,\frac{\zeta_0}{\sigma}
\end{aligned} \tag{3.5}
$$

and $\Phi(x)$ is the distribution function of the standard normal distribution $N(0, 1)$.

THEOREM 3.3. *The function $G(x, d)$ of* (3.4) *has the following properties*:

(i) *$G(-x, d) = G(x, d)$ and $G(x, -d) = G(x, d)$ for $-1 \le x \le 1$.*
(ii) *$G(1, d) = 0.5$ and $G(x, 0) = 0.5$ for $-1 \le x \le 1$.*
(iii) *For a fixed d $(\neq 0)$, $G(x, d)$ is strictly decreasing on $0 < x < 1$.*
(iv) *For a fixed x $(-1 < x < 1)$, $G(x, d)$ is strictly increasing on $d > 0$.*

PROOF. The proofs for (i) and (ii) are trivial. For (iii), the differential of $G(x, d)$ on x is

$$
\begin{aligned}
\frac{\partial}{\partial x} G(x, d) = \frac{d}{\sqrt{2\pi}} \Bigg\{ &\exp\left(-\frac{d^2(1-x)}{2}\right) \frac{0.5 - \Phi(d\sqrt{1+x})}{\sqrt{1-x}} \\
&- \exp\left(-\frac{d^2(1+x)}{2}\right) \frac{0.5 - \Phi(d\sqrt{1-x})}{\sqrt{1+x}} \Bigg\}.
\end{aligned}
$$

For any $d \neq 0$, it can easily be checked that this function is negative on $0 < x < 1$. Similarly, we can prove (iv). $\square$

From Theorems 3.2 and 3.3(iii), note that, for an effect ζ_0 and any $\zeta(\neq \zeta_0) \in \boldsymbol{\xi}_2$, the probability of (3.2) is larger (or smaller) when the value of $|x| = |\mathbf{b}(\xi)'\mathbf{b}(\zeta_0)|$ in (3.5) is smaller (or larger). For a fixed $\zeta_0 \in \boldsymbol{\xi}_2$, therefore, denote

$$x(\zeta_{l_0}) = \left|\mathbf{b}(\zeta_{l_0})'\mathbf{b}(\zeta_0)\right| = \max_{\zeta \in \mathscr{A}(\boldsymbol{\xi}_2;\, \zeta_0)} |\mathbf{b}(\zeta)'\mathbf{b}(\zeta_0)|. \tag{3.6}$$

Then the searching probability of (2.7) can be written as

$$P = G(x_0, q_0\rho) = \min_{\zeta_0 \in \boldsymbol{\xi}_2} G\left(x(\zeta_{l_0}), \sqrt{\frac{r(\zeta_{l_0})}{2}}\,\rho\right) \tag{3.7}$$

[$= P_T(\rho)$, say; clearly, P is dependent on a searching design T], where ρ is the actual quantity of the effect ζ_0/σ. Since $P_T(\rho) = P_T(-\rho)$ from Theorem 3.3(i), $P_T(\rho)$ can be considered only for $\rho > 0$. From Theorem 3.3(ii) and (iv), also note that $P_T(\rho) > 0.5$ and is strictly increasing for $\rho > 0$. Besides taking an interest in the amount of $P_T(\rho)$ for a given T and for each ρ, the searching probability may be used for competing search designs.

DEFINITION 3.1. Let T_1 and T_2 be two search designs with N treatments. Then T_1 is said to be better (for searching the true effect) than T_2 if $P_{T_1}(\rho) \geq P_{T_2}(\rho)$ holds for each $\rho > 0$.

THEOREM 3.4. *Suppose $r(\zeta)$ is constant for $\zeta \in \boldsymbol{\xi}_2$. Then the searching probability $P_T(\rho)$ of* (3.7) *can be expressed as*

$$P_T(\rho) = G(x_0^*, q\rho), \tag{3.8}$$

where $q = \sqrt{r(\zeta)/2}$ and for $x(\zeta_{l_0})$ in (3.6),

$$x_0^* = \max_{\zeta_0 \in \boldsymbol{\xi}_2} x(\zeta_{l_0}).$$

The proof follows easily from Theorem 3.3(iii).

The utility of this theorem will be presented in the next section.

4. Searching probabilities with respect to MEP.1 plans. Consider a factorial experiment with m factors each at two levels. A treatment is represented by $(t_1, \ldots, t_m)$, where t_i, the level of ith factor, equals 0 or 1. Note that a design can be written as a $(0, 1)$ matrix whose rows are treatments. In model (1.1), consider $\boldsymbol{\xi}_1$ to be a vector composed of the general mean and the main effects ($\nu_1 = m + 1$) and $\boldsymbol{\xi}_2$ to be of certain ν_2 interactions. This implies that the remaining interactions are assumed negligible in advance. In this setting, a search design [satisfying (1.2)] is called a main effect plus k (MEP.k) plan. In particular, consider the case of $k = 1$. Let T be a MEP.1 plan with N treatments. In the following discussion, consider two types of $\boldsymbol{\xi}_2$ according to the situations for the effects in which a nonzero effect should be searched and estimated.

TYPE 1. $\boldsymbol{\xi}_2$ is the vector composed only of two-factor interactions where $\nu_2 = \binom{m}{2}$.

Denote by $\Omega(m, j)$, the $\binom{m}{j} \times m$ matrix composed of all distinct treatments in which the number of ones is j ($j = 0, \dots, m$). Then, as shown by Shirakura and Tazawa (1991), the deisgns

$$(4.1) \qquad T = \begin{cases} [\Omega(4,0)' : \Omega(4,1)' : \Omega(4.3)']', & \text{for } m = 4, \\ [\Omega(m,1)' : \Omega(m, m-1)']', & \text{for } m > 5, \end{cases}$$

are MEP.1 plans with $N = 9$ and $N = 2m$ treatments, respectively. These MEP.1 plans are considered here. After some claculations, it is easy to observe that $r(\zeta)$ is constant for all $\zeta \in \boldsymbol{\xi}_2$; that is,

$$\mathbf{r}(\zeta) = \begin{cases} 112/13, & \text{for } m = 4, \\ 16(m-2)/m, & \text{for } m \geq 5, \end{cases}$$

and that $|x| = |\mathbf{b}(\zeta)'\mathbf{b}(\zeta_0)|$ in (3.5) have two values for each m as follows:

$$|x| = \begin{cases} \dfrac{1}{14} \text{ and } \dfrac{6}{7}, & \text{for } m = 4, \\ \dfrac{(m-4)}{2(m-2)} \text{ and } \dfrac{2}{m-2}, & \text{for } m \geq 5. \end{cases}$$

Therefore, by Theorem 3.4, $P_T(\rho)$ in (3.8) can be expressed as

$$P_T(\rho) = \begin{cases} G\left(\dfrac{6}{7}, \sqrt{\dfrac{56}{13}}\,\rho\right), & \text{for } m = 4, \\ G\left(\dfrac{2}{m-2}, \sqrt{\dfrac{8(m-2)}{m}}\,\rho\right), & \text{for } 5 \leq m \leq 8, \\ G\left(\dfrac{m-4}{2(m-2)}, \sqrt{\dfrac{8(m-2)}{m}}\,\rho\right), & \text{for } m \geq 9, \end{cases}$$

In Table 1, the values of $P_T(\rho)$ with respect to T in (4.1) are given for $\rho = 1.0$, 1.5, 2.0 and 2.5 for each m ($4 \leq m \leq 9$). It turns out that the searching probabilities are larger than 0.94 except for the cases of ($m = 4$; $\rho = 1.0, 1.5$) and ($m = 5$; $\rho = 1.0$). In particular, for $m \geq 6$ and $\rho \geq 1.5$, the search designs can almost exactly identify the nonzero effect in $\boldsymbol{\xi}_2$ under the procedure.

TYPE 2. $\boldsymbol{\xi}_2$ is the vector composed of the two-factor and three-factor interactions, where $\nu_2 = \binom{m}{2} + \binom{m}{3}$.

Consider MEP.1 plans ($4 \leq m \leq 8$) listed in Table 2 of Ohnishi and Shirakura (1985), which are the corrected results of Ghosh (1981). In Table 2, the computational results for $P_T(\rho)$ in (3.7) for each MEP.1 plan T are given

TABLE 1
*The searching probabilities of $P_T(\rho)$**

	m						
ρ	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1.0	0.78228	0.89519	0.94666	0.96577	0.97501	0.97550	0.97581
1.5	0.88033	0.97109	0.99283	0.99274	0.99863	0.99864	0.99864
2.0	0.94166	0.99429	0.99945	0.99989	0.99996	0.99996	0.99996
2.5	0.97506	0.99921	0.99997	0.99999	0.99999	0.99999	0.99999

* *The figures are omitted below the sixth of decimals.*

in the same format as Table 1.

Next let

$$T_1 = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}'.$$

Then, as shown by Shirakura (1991), the design composed of the first 15 rows of T_1 is a MEP.1 plan for $m = 7$ and $N = 15$. Therefore, this T_1 is also a MEP.1 plan for $N = 17$. Note that the plan of the first eight rows of T_1 is the same as in the example in Ghosh (1980). For the plan T_1, the searching probability can also be given by

$$P_{T_1}(\rho) = 0.91365,\ 0.98134,\ 0.99726 \text{ and } 0.99974$$
$$\text{according to } \rho = 1.0,\ 1.5,\ 2.0 \text{ and } 2.5.$$

As a competitive search design, let T_2 be a MEP.1 plan for $m = 7$ and $N = 17$ [of Ohnishi and Shirakura (1985)] considered above. Figure 1 gives the graphs of $P_{T_1}(\rho)$ and $P_{T_2}(\rho)$ for $\rho > 0$. Clearly, T_1 is better than T_2 for searching the true effect in $\boldsymbol{\xi}_1$.

TABLE 2
The searching probabilities of $P_T(\rho)$

	m				
ρ	**4**	**5**	**6**	**7**	**8**
1.0	0.85898	0.79747	0.83819	0.72546	0.75268
1.5	0.95469	0.89942	0.93136	0.81713	0.84769
2.0	0.98824	0.95595	0.97622	0.88610	0.91446
2.5	0.99764	0.98349	0.99337	0.93416	0.95645

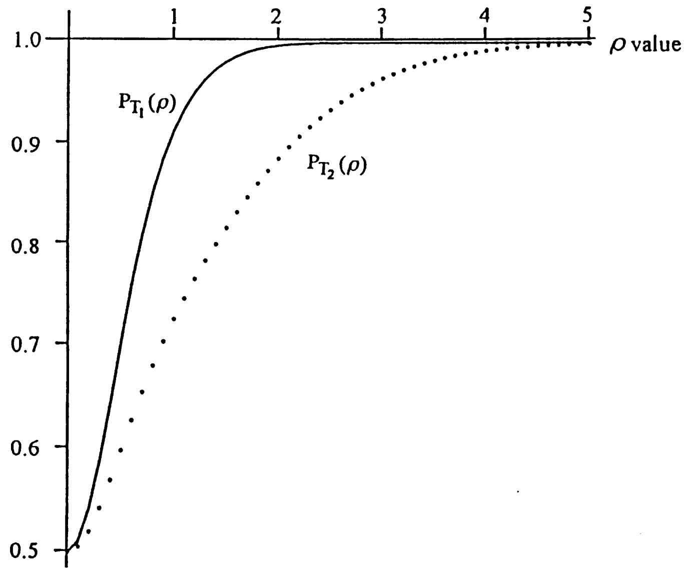

FIG. 1. *Searching probabilities of $P_{T_1}(\rho)$ and $P_{T_2}(\rho)$.*

Acknowledgment. The authors thank the referee for useful comments and suggestions.

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