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ON RANDOM INDICES AND LIMIT DISTRIBUTIONS

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This note complements a recent article by Thomas (1972). We prove a conjecture of Thomas which, in a way, makes his results complete. The proof uses the fact that for the sequences of random variables involved, the weak convergence property is conserved when taking certain transformations of the random variables.

For brevity we adopt all the conventions and assumptions of Thomas' paper. So (2.5) will mean: formula (2.5) in Thomas (1972). We are going to prove the conjecture on page 1723 which reads as follows.

THEOREM. *Assume that the characteristic function of the random variable X is not identically zero in some non-degenerate real interval and that the norming constants are of type* 3. *If* $\{(X_{N_m} - b_m)/a_m\}$ *converges in distribution to a random variable* Y, *then* $\{G_m(\cdot)\}$ *converges weakly to a distribution function* $G(\cdot)$ *satisfying* (2.5).

PROOF. By assumption there is a monotone function f such that (2.1)

$$\lim_{n\to\infty} nf(a_n x + b_n) = \gamma e^{\delta x}$$

for all real x where $\delta \neq 0$, $\gamma > 0$. Define $Y_n = 1/f(X_n)$ and $Z_n = (X_n - b_n)/a_n$. By assumption $\{Z_n\}$ converges weakly to X and as $n \to \infty$ we have for all x

$$P\left\{\frac{Y_n}{n} \leqq x\right\} = P\left\{\frac{1}{nf(a_n Z_n + b_n)} \leqq x\right\} \to P\{\gamma^{-1}e^{-\delta X} \leqq x\}.$$

We also define $R_m = X_{N_m} - b_m)/a_m$; by assumption $\{R_m\}$ converges weakly to Y and as $m \to \infty$ we have for all x

$$P\left\{\frac{Y_{N_m}}{m} \leqq x\right\} = P\left\{\frac{1}{mf(a_m R_m + b_m)} \leqq x\right\} \to P\{\gamma^{-1}e^{-\delta Y} \leqq x\}.$$

Now for the sequence $\{Y_n\}$ Theorem 2.2 applies and the result of the theorem follows. □

REFERENCE

THOMAS, D. L. (1972). On limiting distributions of a random number of dependent random variables. *Ann. Math. Statist.* **43** 1719–1726.

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