The Annals of Probability
1980, Vol. 8, No. 4, 841–850

OPTIMAL PREDICTION OF CATASTROPHES WITH APPLICATIONS TO GAUSSIAN PROCESSES[1]

BY JACQUES DE MARÉ

Chalmers University of Technology

An alarm system is optimal if it detects catastrophes with a certain high probability and simultaneously gives a minimum number of false alarms. In a general context an optimal alarm system is derived and then the method is applied to Gaussian processes.

0. Introduction. In some applications critical events occur when a time dependent random quantity $\{X(t) : t \in R\}$ exceeds a high level u. An alarm system is a system which predicts in advance when such an event will take place. A usual approach when designing such a system is to construct a predictor process $\{\hat{X}(t) : t \in R\}$ where the value $\hat{X}(t)$ is the conditional expectation of $X(t)$ given the information available t_0 time units before the time point t (for some fixed constant $t_0 > 0$).

In a paper Lindgren (1975a) pointed out that such a predictor need not be optimal when prediction occurs from a random time point, e.g., when the prediction takes place at the time when the predictor process $\hat{X}$ upcrosses an alarm level.

On the ISI session in Warsaw Lindgren (1975b) argued that an optimal alarm system should be judged from its ability of giving a minimum number of false alarms and undetected catastrophes rather than predicting the actual path of X.

Formal definitions of the concepts of catastrophes, false alarms and optimal alarm systems are given in Section 1. In the subsequent sections an optimal alarm system is derived in a general context and then applied to Gaussian processes.

1. Definition of an optimal alarm system. As an applied example where alarm systems have been used, consider the recorded sea level height $x = \{x(t) : t = \cdots -1, 0, 1, 2, \cdots\}$ above the mean level. The problem is to forcast when the height will exceed some level u. In this situation a catastrophe occurs at time t if x has an upcrossing of u at t, i.e., $x(t-1) \leqslant u < x(t)$.

One approach to forcast a catastrophe t_0 time units ahead is to condense the information available in a prediction $\hat{x}(t)$ of $x(t)$ and give an alarm for a catastrophe at t if and only if $\hat{x}$ has an upcrossing of some specified alarm level $\hat{u}$ at time t.

When judging the performance of the alarm system it is not very interesting how close $\hat{x}$ is to x in the mean, but an important feature is the ability of the system to detect catastrophes without making too many false alarms. We say that a

Received February 21, 1978; revised February 29, 1979.

[1]Research reported in this paper has been supported by the Swedish Natural Research Council.

AMS 1970 *subject classifications.* Primary 62M20; secondary 60G35, 60G15.

Key words and phrases. Prediction, Palm measure, Gaussian processes, horizontal window conditioning, model processes, level crossings.

catastrophe at time t is detected t_0 time units ahead if there is an alarm at time $t - t_0$. On the other hand we say that an alarm at time $t - t_0$ is correct if a catastrophe actually takes place at time t. An alarm system has a high ability of detecting catastrophes if the proportion of detected catastrophes out of all catastrophes is close to one. If, at the same time, the proportion of correct alarms out of all alarms is as high as possible then the alarm system is said to be optimal.

Let $\tau_1, \tau_2, \cdots, \tau_n$ be the time points of the upcrossings of x in $[0, T]$ and let $\hat{\tau}_1, \hat{\tau}_2, \cdots, \hat{\tau}_{\hat{n}}$ be those of $\hat{x}$. Then the proportion of detected catastrophes is

$$P_T = \#\{\nu : \exists\mu \text{ with } \hat{\tau}_\mu = \tau_\nu\}/n$$

and the proportion of correct alarms is

$$\hat{P}_T = \#\{\mu : \exists\nu \text{ with } \tau_\nu = \hat{\tau}_\mu\}/\hat{n}.$$

If the processes have a suitable ergodic behaviour when T tends to infinity these quantities converge,

$$P_T \to \frac{\text{Prob}\{\text{alarm \& catastrophe}\}}{\text{Prob}\{\text{catastrophe}\}} = \text{Prob}\{\text{alarm}\|\text{catastrophe}\}$$

$$\hat{P}_T \to \frac{\text{Prob}\{\text{alarm \& catastrophe}\}}{\text{Prob}\{\text{alarm}\}} = \text{Prob}\{\text{catastrophe}\|\text{alarm}\}.$$

The event that x has an upcrossing of u at t is denoted by

$$C_t = \{x(t-1) \leqslant u < x(t)\}$$

and the event that $\hat{x}$ has an upcrossing of $\hat{u}$ at t by

$$A_t = \{\hat{x}(t-1) \leqslant \hat{u} < x(t)\}.$$

In this approach the problem is to find that predictor process $\hat{x}$ and alarm level $\hat{u}$ which maximize the conditional probability, $P(C_t \| A_t)$, that an alarm is correct under the constraint that the detection level $P(A_t \| C_t)$ is equal to some prescribed value.

It may happen that not even the best predictor process $\hat{x}$ gives a satisfactory alarm system because of the rather rigid condition that the upcrossing of the $\hat{x}$-process must occur exactly at the same time as the upcrossing of the x-process.

A better alarm system can be obtained by using a wider class of events than level crossings at time t in the predictor process. The most general class of events is all events we know of at time $t - t_0$. The best alarm system is obtained if we choose in this larger class that event A_t which on the given detection level $P(A_t \| C_t)$ maximizes the conditional probability of correct alarm $P(C_t \| A_t)$. We shall here pursue this idea in a rather general setting.

Let X be a stochastic process in one-dimensional continuous or discrete time. For each time point t, let C_t be a measurable set of sample functions. Then we will say that a catastrophe occurs at time t if $X \in C_t$. When X is real-valued, examples of C_t are

$$C_t = \{x : x(t-\varepsilon) \leqslant u < x(t+\varepsilon)\}$$

and

$$C_t = \{x : x(t) \leqslant u < x(t+s) \text{ for some } s, 0 < s < \varepsilon\}.$$

Here typically ε is a small positive number and u a high level.

The problem is to predict t_0 time units in advance if C_t will occur or not. Let our information available at time $t - t_0$ be condensed in the value at time t of a stochastic process Y. This process can be finite-dimensional, e.g.,

$$Y(t) = (X(t - t_0), X'(t - t_0))$$

(if X is differentiable) or infinite-dimensional, e.g.,

$$Y(t) = \{X(s) : s \leqslant t - t_0\}.$$

The σ-algebra generated by $Y(t)$ is denoted by

$$\mathcal{F}_t = \sigma[Y(t)]$$

and define an alarm system $\mathcal{A}$ as a family of t-indexed sets A_t which are $\mathcal{F}_t$-measurable, i.e.,

$$\mathcal{A} = \{A_t\} \text{ where } A_t \in \mathcal{F}_t \text{ for each } t.$$

We say that there is an alarm for a catastrophe at time t if $Y \in A_t$. If $Y \in A_t$ but $X \notin C_t$ we will say that the alarm is false and on the other hand if $X \in C_t$ but $Y \notin A_t$ we will say that there is an undetected catastrophe at time t. This means that the catastrophe at time t is not detected t_0 time units in advance.

An alarm system $\mathcal{A} = \{A_t\}$ is optimal if for each t

$$P(C_t \| A_t) = \sup\{P(C_t \| B_t) : B_t \in \mathcal{F}_t \text{ and } P(B_t \| C_t) = P(A_t \| C_t)\}.$$

Hence an alarm system is optimal if it gives a maximal probability of a catastrophe when alarming out of all systems with the same ability of detecting a catastrophe.

2. Optimal choice of alarm systems. The formal similarity to power considerations in test theory and to the signal detection problem indicates that some likelihood ratio quantity should be crucial in the problem of designing an optimal alarm system. The following theorem prescribes which likelihood ratio is of interest. The stochastic process which carries the information is denoted by Y as in Section 1 where the concepts of a catastrophe, C_t, and of an alarm system, $\mathcal{A} = \{A_t\}$, are defined. Use the notation $P_{Y(t)}(\cdot \| C_t)$ for the marginal distribution of $Y(t)$ conditioned on C_t. Let the function

$$p = \frac{dP_{Y(t)}(\cdot \| C_t^*)}{dP_{Y(t)}(\cdot \| C_t)} \leqslant \infty$$

denote the density of the marginal distribution of $Y(t)$ conditioned on C_t^* the complement of C_t. The density is calculated with respect to $P_{Y(t)}(\cdot \| C_t)$.

THEOREM 2.1. *The alarm system $\mathcal{A} = \{A_t\}$ defined by*

$$A_t = \{y : p[y(t)] \leqslant k\} \text{ with } k < \infty$$

is optimal, i.e.,

$$P(C_t \| A_t) = \sup\{P(C_t \| B_t) : B_t \in \mathfrak{F}_t \text{ and } P(B_t \| C_t) = P(A_t \| C_t)\}.$$

PROOF. Take any event $B_t \in \mathfrak{F}_t$ with $P(B_t \| C_t) = P(A_t \| C_t)$. Then

$$P(C_t \| B_t) \leqslant P(C_t \| A_t) \Leftrightarrow P(C_t^* \| B_t) \geqslant P(C_t^* \| A_t)$$
$$\Leftrightarrow P(B_t \| C_t^*) \geqslant P(A_t \| C_t^*).$$

The last inequality follows from Bayes' theorem

$$P(C_t^* \| B_t) = \frac{P(B_t \| C_t^*)P(C_t^*)}{P(B_t \| C_t^*)P(C_t^*) + P(B_t \| C_t)P(C_t)}$$
$$\geqslant \frac{P(A_t \| C_t^*)P(C_t^*)}{P(A_t \| C_t^*)P(C_t^*) + P(B_t \| C_t)P(C_t)} = P(C_t^* \| A_t)$$

and this inequality is true if and only if $P(B_t \| C_t^*) \geqslant P(A_t \| C_t^*)$. (We have $P(B_t \| C_t) = P(A_t \| C_t)$).

To prove $P(B_t \| C_t^*) \geqslant P(A_t \| C_t^*)$, note that

$$P(A_t \| C_t) = \int_{A_t} dP(\cdot \| C_t) = \int_{B_t} dP(\cdot \| C_t) = P(B_t \| C_t)$$

implies

$$\int_{A_t \cap B_t^*} dP(\cdot \| C_t) = \int_{A_t^* \cap B_t} dP(\cdot \| C_t)$$

and hence

$$\int_{\{y(t) : y \in A_t \cap B_t^*\}} p \, dP_{Y(t)}(\cdot \| C_t) \leqslant \int_{\{y(t) : y \in A_t^* \cap B_t\}} p \, dP_{Y(t)}(\cdot \| C_t).$$

Thus

$$P(A_t \| C_t^*) = \int_{\{y(t) : y \in A_t\}} p \, dP_{Y(t)}(\cdot \| C_t)$$
$$\leqslant \int_{\{y(t) : y \in B_t\}} p \, dP_{Y(t)}(\cdot \| C_t) \leqslant P(B_t \| C_t^*)$$

which concludes the proof.

REMARK 2.2. The first part of the proof implies that it is the same to maximize $P(C_t \| B_t)$ over all possible $B_t \in \mathfrak{F}_t$ as to minimize $P(B_t \| C_t^*)$. But in the degenerate case when C_t has probability zero, the latter quantity alone is of interest and then $P(B_t \| C_t^*) = P(B_t)$. When C_t has probability zero we will say that an alarm system is optimal if

$$P(A_t) = \inf\{P(B_t) : B_t \in \mathfrak{F}_t \text{ and } P(B_t \| C_t) = P(A_t \| C_t)\}.$$

In this case we prefer that alarm system which has minimal probability to give an alarm out of all systems with the same detection ability.

3. Applications to Gaussian process. In this section the attention will be focused on stationary and continuous real-valued Gaussian processes. In the stationary case it is natural to consider a family of catastrophes which is shift invariant. Hence

$$C_t = \{x : x(\cdot - t) \in C\}$$

for a suitable set C. A further simplification is to consider only the degenerate case when

$$C = \{x : x \text{ has an upcrossing of the level } u \text{ at zero}\}$$
$$= \{x : \forall \varepsilon > 0 \exists s_1, s_2 \in]0, \varepsilon[\text{ with } x(-s_1) < u < x(s_2)\}.$$

Here, as in the introduction, we think of u as a high level. For this family of catastrophes and these kinds of processes it is possible to give an explicit description of the probabilities when conditioning on a catastrophe at time t, (cf. Slepian (1962), Lindgren (1972), Geman and Horowitz (1973), and de Maré (1977)).

EXAMPLE 3.1. Let X be a Gaussian Markov process with mean zero and covariance function r,

$$r(t) = \exp(-|t|), \qquad t \in R.$$

By the (strong) Markov property it is natural to define Y through

$$Y(t) = X(t - t_0), \qquad t \in R, (t_0 > 0).$$

Then the conditional distribution of $Y(t)$ given a catastrophe (a u-upcrossing) at time t is given through

$$Y(t) =_{\mathcal{L}} u \exp(-|t_0|) + \eta.$$

The stochastic variable η is Gaussian with zero mean and variance $\Lambda = 1 - \exp(-2|t_0|)$, (cf. de Maré (1977)). Now $P(C_t) = 0$ which implies that $P(\cdot \| C_t^*) = P(\cdot)$, and hence

$$p[y(t)] = \frac{dP_{Y(t)}(\cdot \| C_t^*)}{dP_{Y(t)}(\cdot \| C_t)}[y(t)]$$
$$= \Lambda^{\frac{1}{2}} \frac{\exp\left[-\frac{1}{2}y^2(t)\right]}{\exp\left\{-\frac{1}{2}[y(t) - u\exp(-|t_0|)]^2/\Lambda\right\}}.$$

Since $\Lambda = 1 - \exp(-2|t_0|)$ the following alarm region is obtained,

$$A_t = \{y : [y(t) - u\exp(|t_0|)]^2 \leqslant K\}.$$

If $u = 4$ then X will exceed the catastrophe level $3 \cdot 10^{-3}\%$ of the time. Since $Y(t)$ is Gaussian conditioned on C_t^* as well as on C_t, though with different means and standard deviations, one obtains directly the following conditional probabilities

$$\begin{cases} P(A_t \| C_t) = 93\% \\ P(A_t \| C_t^*) = 2.3\% \end{cases}$$

for $t_0 = -\ln 0.75 = 0.3$ and $k = 100/9 = 11.1$. For an interpretation of $P(A_t \| C_t^*)$ see Remark 2.2 and Figure 3.2.

After this introductory example assume that (X, Y) is a stationary Gaussian process where the first component X is real-valued with mean zero, and differentiable. Choose time-scale and level-scale so that

$$EX^2(0) = EX'^2(0) = 1.$$

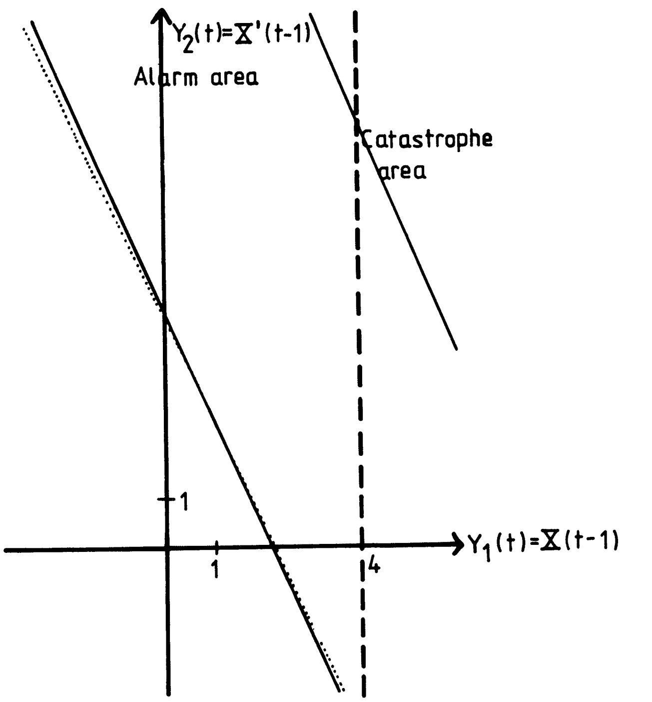

FIG. 3.2. *Alarm occurs when X enters the alarm area. With a probability of 93%, X is found in the alarm area $t_0 = 0.3$ time units before X enters the catastrophe area.*

The second component Y which carries the information is supposed to be stationarily correlated with X and the covariance matrix of $(X(t), X'(t), Y(t))$ is denoted by

$$\Sigma = \begin{bmatrix} 1 & 0 & \Sigma_{13} \\ 0 & 1 & \Sigma_{23} \\ \Sigma_{31} & \Sigma_{32} & \Sigma_{33} \end{bmatrix}. \tag{3.3}$$

Then it is possible to calculate the conditional density of $Y(t)$,

$$p = \frac{dP_{Y(t)}(\cdot \| C_t^*)}{dP_{Y(t)}(\cdot \| C_t)}. \tag{3.4}$$

It should be noted that conditioning on C_t is the same as conditioning on a u-upcrossing in the X-process. This means that the process at the time points when C_t occurs is steeper than usual which gives the derivative a Rayleigh distribution conditioned on C_t (cf. Slepian (1962), Lindgren (1975a, b)). This phenomenon does not take place in Example 3.1 since the process is nondifferentiable.

In the nondifferentiable case both $P_{Y(t)}(\cdot \| C_t)$ and $P_{Y(t)}(\cdot \| C_t^*)$ are Gaussian and hence either equivalent or orthogonal but when X is differentiable $P_{Y(t)}(\cdot \| C_t)$

is not Gaussian in general. In this case the third possibility that $P_{Y(t)}(\cdot \,\|\, C_t)$ and $P_{Y(t)}(\cdot \,\|\, C_t)$ are neither orthogonal nor equivalent, may occur.

THEOREM 3.5. *Let (X, Y) be a finite-dimensional and stationary Gaussian process with a real-valued first component which is differentiable in quadratic mean. Let C_t denote a u-upcrossing in the X-process at time t and $\mathcal{G}_t = \sigma(Y(t))$ be the σ-algebra generated by $Y(t)$. Then $P_{Y(t)}(\cdot \,\|\, C_t)$ and $P_{Y(t)}(\cdot \,\|\, C_t^*)$ are equivalent if and only if no nonzero linear function of $X(t)$ and $X'(t)$ is $\mathcal{G}_t$-measurable.*

PROOF. It is no loss of generality to assume that $Y(t)$ has a nonsingular distribution with covariance matrix Σ_{33} since otherwise the number of components in $Y(t)$ can be reduced until a nonsingular distribution is obtained.

We start by proving that when no nonzero linear function of $X(t)$ and $X'(t)$ is $\mathcal{G}_t$-measurable, the conditional distributions $P_{Y(t)}(\cdot \,\|\, C_t)$ and $P_{Y(t)}(\cdot \,\|\, C_t^*)$ are equivalent. This is because Σ in (3.3) is then invertible. Assuming zero mean, the following formula is obtained (cf. (3.4) page 230 in Lindgren (1975b))

$$(3.6)\quad p[y(t)] = \left(\frac{\det \Lambda}{\det \Sigma_{33}}\right)^{\frac{1}{2}} \frac{\exp\left[-\frac{1}{2}\|y(t)\|^2_{\Sigma_{33}^{-1}}\right]}{\int_{z=0}^{\infty} \exp\left[-\frac{1}{2}\|y_u(t) - z\Sigma_{23}\|^2_{\Lambda^{-1}}\right]\cdot z \exp\left(-\frac{1}{2}z^2\right) dz}$$

with $y_u(t) = y(t) - u\Sigma_{13}$ and $\Lambda = \Sigma_{33} - \Sigma_{32}\Sigma_{23} - \Sigma_{31}\Sigma_{13}$ (and, e.g., $\|y(t)\|^2_{\Sigma_{33}^{-1}} = y(t)\Sigma_{33}^{-1}y^T(t)$). Hence the "if"-part is established.

Conversely, assume that $\eta = aX(t) + bX'(t)$ is $\mathcal{G}_t$-measurable. Then conditioned on C_t^*, the random variable η is Gaussian distributed but conditioned on C_t it follows a one-sided Rayleigh distribution on a subset of the real line and hence the two conditional distributions are equivalent only if $a = b = 0$. This concludes the proof of the theorem.

COROLLARY 3.7. *Assume that no nonzero linear function of $X(t)$ and $X'(t)$ is $\mathcal{G}_t$-measurable and use the notations in formula* (3.6). *Then an alarm system $\mathcal{Q} = \{A_t\}$ is optimal in the sense of Remark* 2.2 *if*

$$A_t = \left\{y : \|y_u(t)\|^2_{\Lambda^{-1} - LL^T} - \|y(t)\|^2_{\Sigma_{33}^{-1}} - 2 \ln \psi\{L[y_u(t)]\} \leq K\right\}$$

(for some constant K).

Here

$$L[y_u(t)] = y_u(t)\cdot L = y_u(t)\cdot \Lambda^{-1}\Sigma_{32}/\gamma$$

with

$$\gamma = \left(1 + \Sigma_{23}\Lambda^{-1}\Sigma_{32}\right)^{\frac{1}{2}}$$

and

$$\psi(x) = \int_{-\infty}^{x}\left[\int_{-\infty}^{y} e^{-z^2/2}\, dz\right] dy, \qquad x \in R.$$

PROOF. Carry out the integration in formula (3.6) and obtain

$$p[y(t)] = \gamma^2 \cdot \left(\frac{\det \Lambda}{\det \Sigma_{33}}\right)^{\frac{1}{2}} \frac{\exp\left[-\frac{1}{2}\|y(t)\|^2_{\Sigma_{33}^{-1}}\right]}{\exp\left[-\frac{1}{2}\|y_u(t)\|^2_{\Lambda^{-1}-LL^T}\right]\cdot\psi\{L[y_u(t)]\}}.$$

COROLLARY 3.8. *The theorem holds even when Y is infinite-dimensional.*

PROOF. Write $Y(t) = (Y_1(t), Y_2(t))$ with

$$Y_1(t) = E[(X(t), X'(t))\|\mathcal{F}_t] \text{ and } Y_2(t) \perp Y_1(t).$$

Then the distribution of $Y_2(t)$ is unaffected by conditioning on $(X(t), X'(t))$ and the infinite-dimensional problem is reduced to the finite-dimensional case.

EXAMPLE 3.9. Let X be a stationary solution of the stochastic differential equation

$$X'' + aX' + bX = B'$$

where B is Brownian motion. Then it is natural to consider

$$Y(t) = (X(t-t_0), X'(t-t_0)).$$

Slepian (1962) derives the following explicit expression for the conditional distribution of $Y(t)$ given a catastrophe (a u-upcrossing) at time t

$$\begin{cases} Y_1(t) = X(t-t_0) =_{\mathcal{L}} u\cdot r(t_0) + \zeta\cdot r'(t_0) + \eta_1 \\ Y_2(t) = X'(t-t_0) =_{\mathcal{L}} -u\cdot r'(t_0) - \zeta\cdot r''(t_0) + \eta_2 \end{cases}$$

where ζ and $\eta = (\eta_1, \eta_2)$ are independent. The distribution of ζ is Rayleigh and that of η is bivariate Gaussian with mean zero and covariance

$$\Lambda = \begin{bmatrix} 1 - r^2(t_0) - r'^2(t_0) & r(t_0)r'(t_0) + r'(t_0)r''(t_0) \\ r(t_0)r'(t_0) + r'(t_0)r''(t_0) & 1 - r'^2(t_0) - r''^2(t_0) \end{bmatrix}.$$

To obtain a numerical example we specialize further,

$$t_0 = 1.0,\ r(t_0) = .75,\ r'(t_0) = -.35,\ r''(t_0) = 0$$

which yields

$$\Lambda = \begin{bmatrix} .31 & -.27 \\ -.27 & .88 \end{bmatrix}.$$

The alarm region $A_t = \{y : (y_1(t), y_2(t)) \in D\}$, where from Corollary 3.8

$$D = \{(\eta_1 + .75u, \eta_2 + .35u) : 1.4\eta_1^2 + 1.7\eta_1\eta_2 + .4\eta_2^2 - 1.5u\eta_1 - .7u\eta_2 - .7u^2$$
$$< K + 2\ln\psi(-1.2\eta_1 - .4\eta_2)\}.$$

If we, as in Example 3.1, choose $u = 4$ the process visits the catastrophe area $3\cdot 10^{-3}\%$ of the time. The mean number of catastrophes per time unit is $0.5\cdot 10^{-4}$ and if we again accept

$$P(A\|C_t^*) = 2.3\%$$

we now obtain the detection probability

$$P(A_t\|C_t) = 96\%.$$

We have here evaluated an alarm system which spends a minimum proportion of its time in an alarm region out of all systems with the detection probability of 96%. In Figure 3.10 it is also indicated what will happen if the alarm system is based on the usual mean square predictor

$$\hat{X}(t) = r(t_0)X(t - t_0) - r'(t_0)X'(t - t_0).$$

It is noted that in this particular example the mean square predictor has an almost optimal performance.

4. Conclusions. In Section 2 it is emphasized that the problem of constructing alarm systems is analogous to the problem of testing hypotheses. This view leads to an optimal alarm system which is based on an alarm region with a curved boundary. However, in an example this region has an almost linear boundary at that part of the plane where the probability is concentrated.

A natural question to ask is when this will happen and another question in this direction is when the boundary of the alarm region is well approximated by a level curve of the mean square predictor.

The probability tools which are needed in order to calculate the alarm regions are mostly developed for stationary processes. However, in a variety of applications the processes of interest have a nonstationary component. Lindgren (1979) outlines a development of a model process for a stochastic process which is the sum of a stationary Gaussian process and an almost periodic deterministic function. The model process gives an explicit description of the behaviour after an upcrossing.

To develop the alarm system we use a model process for the information process Y given a catastrophe in the X-process. But to examine the property of the X-process after the entry of the Y-process in the alarm region, a model is needed

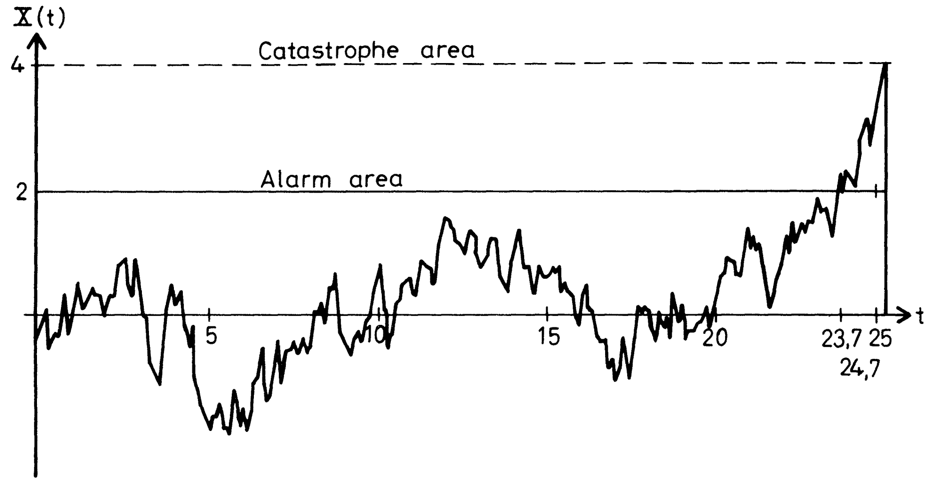

FIG. 3.10. *Alarm occurs when $Y(t)$ belongs to the alarm area and a catastrophe when $X(t)$ enters the catastrophe area. The dotted line indicates where the mean square predictor $\hat{X}(t) = 1.7$, ($\hat{X}(t) = .75X(t - 1) + .35X'(t - 1)$).*

conditioned on the event that Y hits the boundary of the alarm region. This problem is discussed in Lindgren (1980).

A quantity which is of interest is the waiting time between an alarm and a catastrophe. Usually this waiting time is substantially larger than the time t_0, because the system is designed to grant that a catastrophe will not happen earlier than t_0 time units after alarm.

REFERENCES

[1] Geman, D. and Horowitz, J. (1973). Occupation times for smooth stationary processes. *Ann. Probability* **1** 131–137.

[2] Lindgren, G. (1972). Wave-length and amplitude for a stationary Gaussian process after a high maximum. *Z. Wahrscheinlichkeitstheorie und Verw. Gebiete* **23** 293–326.

[3] Lindgren, G. (1975a). Prediction from a random time point. *Ann. Probability* **3** 412–423.

[4] Lindgren, G. (1975b). Prediction of catastrophes and high level crossings. *Bull Internat. Statist. Inst.* **46** 225–240.

[5] Lindgren, G. (1979). Prediction of level crossings for normal processes containing deterministic components. *Advances in Appl. Probabilities* **11**.

[6] Lindgren, G. (1980). Model processes in non-linear prediction, with application to`detection and alarm. *Ann. Probability* **8** 775–792.

[7] de Maré, J. (1977). The behaviour of a non-differentiable stationary Gaussian process after a level crossing. *Stochastic Processes Appl.* **6** 77–86.

[8] Slepian, D. (1962). On the zeros of Gaussian noise. In *Time Series Analysis*. (M. Rosenblatt, ed.) 104–115 Wiley, New York.

Department of Mathematics
Chalmers University of Technology
S-412 96 Göteborg
Sweden