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CORRECTION TO "THE UNIFORM DIMENSION OF THE LEVEL SETS OF A BROWNIAN SHEET"

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Professor Donald Geman has kindly drawn my attention to certain problems with the proofs of two of the lemmas in [1]. Lemma 5 should be preceded by the following clarifying sentences. "We say that such a distribution function satisfies a *Hölder condition* of order γ at each point of A if there exists a finite $M > 0$ such that the increment $G(\langle \mathbf{t}, \mathbf{t} + \mathbf{k} \rangle)$ of G over $\prod_{i=1}^{N}[t_i, t_i + k_i]$ is bounded by $M|k_1 \cdots k_N|\gamma$ for every $\mathbf{t}, \mathbf{t} + \mathbf{k} \in A$ and k_i small enough. Note that in Lemma 7 we use the term Hölder condition with its more standard meaning." In Lemma 5 of [1] the condition $\beta \leqslant N - 1 + \gamma$ should read $\beta < N\gamma$. The source of the error lies in an algebraic mistake in going from the first to the second of the inequalities appearing at the end of the proof. A correct proof, leading to $\beta < N\gamma$, can be obtained by substituting $N\gamma$ for $N - 1 + \gamma$ throughout the proof given.

Unfortunately, this error has implications for the proof of Lemma 6, where Lemma 5 is used in conjunction with Lemma 4, (line 5 of the proof). The relevant condition of Lemma 4, viz $\gamma < \frac{1}{2}$, could however be strengthened to $\gamma < 1 - (2N)^{-1}$ using effectively the same proof, if the inequalities of Tran (1976) used there were correct. Such a strengthening would enable the final proof of the theorem to stand as it currently does.

Recently it was pointed out in [4] that there is a flaw in the proof of the inequalities of Tran, which unfortunately invalidates our use of them in [1]. In a recent paper, [2], we have derived a Hölder-type condition for the local time of a Brownian sheet, and this can be used in the place of Lemma 4 of [1] to provide a full and correct version of the theorem presented there.

Far more general information about the local times of Gaussian random fields than that obtained in [1] is now available in the wide ranging survey [3]. In particular, in [3] it is shown that if $B(\mathbf{t})$ is Lèvy's N-parameter Brownian motion, so that $B(\mathbf{t})$ is a zero mean Gaussian field with covariance function

$$E\{B(\mathbf{s})B(\mathbf{t})\} = \tfrac{1}{2}\{|\mathbf{s}| + |\mathbf{t}| - |\mathbf{t} - \mathbf{s}|\},$$

then the local time of B satisfies the same Hölder conditions as does the N-parameter Brownian sheet. Using this fact, the methods developed in [1] for the Brownian sheet suffice to establish that the level sets of B also satisfy the dimension result of the theorem of [1].

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