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KERSTAN'S METHOD FOR COMPOUND POISSON APPROXIMATION

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We consider the approximation of the distribution of the sum of independent but not necessarily identically distributed random variables by a compound Poisson distribution and also by a finite signed measure of higher accuracy. Using Kerstan's method, some new bounds for the total variation distance are presented. Recently, several authors had difficulties applying Stein's method to the problem given. For instance, Barbour, Chen and Loh used this method in the case of random variables on the nonnegative integers. Under additional assumptions, they obtained some bounds for the total variation distance containing an undesirable log term. In the present paper, we shall show that Kerstan's approach works without such restrictions and yields bounds without log terms.

1. Introduction. In this paper, we consider the sum S_n of $n \in \mathbf{N} = \{1, 2, \dots\}$ independent random variables $X_1, \dots, X_n$ with values in $\mathbf{R}$ and

$$p_i = P(X_i \neq 0), \qquad Q_i(B) = P(X_i \in B | X_i \neq 0), \qquad i \in \{1, \dots, n\},$$

for Borel-measurable sets $B \subseteq \mathbf{R} \setminus \{0\}$, and $\lambda = \sum_{i=1}^n p_i > 0$. We shall always assume that the condition

$$Q_i = \sum_{r=1}^{\infty} q_{i,r}\, U_r, \qquad i \in \{1, \dots, n\}, \tag{A}$$

is valid, where, for all i and r, $q_{i,r} \in [0, 1]$ such that $\sum_{r=1}^{\infty} q_{i,r} = 1$ and the U_r are probability measures concentrated on $\mathbf{R} \setminus \{0\}$. Clearly, condition (A) can trivially be fulfilled if we let $U_r = Q_r$ for all $r \in \{1, \dots, n\}$ and if we let $q_{i,r} = \delta_{i,r}$ be the Kronecker symbol. An important nontrivial example follows.

EXAMPLE 1. If $\mathbf{R} \setminus \{0\}$ can be decomposed into pairwise disjoint measurable sets A_r, $r \in \mathbf{N}$, and if, for fixed $r \in \mathbf{N}$, the conditional distributions $P(X_i \in \cdot | X_i \in A_r)$ are independent of i and denoted by U_r, then condition (A) is valid with $q_{i,r} = P(X_i \in A_r | X_i \neq 0)$ for all i and r. One of the simplest nontrivial examples is the one with U_r being the Dirac measure ε_{x_r} at point $x_r \in \mathbf{R} \setminus \{0\}$, $r \in \mathbf{N}$.

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Later we give an additional example, where all the Q_i are possibly different exponential distributions (see Example 2). Sometimes we shall assume the finiteness of

$$\mu_i = \sum_{r=1}^{\infty} r q_{i,r}, \qquad i \in \{1, \dots, n\}.$$

Further, we set

$$q_r = \frac{1}{\lambda} \sum_{i=1}^{n} p_i q_{i,r}, \qquad r \in \mathbf{N},$$

$$\nu_i = \sum_{r=1}^{\infty} \frac{q_{i,r}^2}{q_r}, \qquad i \in \{1, \dots, n\}.$$

Here we define, for $r \in \mathbf{N}$, $q_{i,r}^2/q_r = 0$ whenever $q_r = 0$. Note that, if $p_i > 0$, then $\nu_i \leq \lambda/p_i$ is finite.

An important interpretation of the above setting comes from risk theory and is called the individual model: here we consider a portfolio with n policies, producing the nonnegative risks $X_1, \dots, X_n$, the so-called individual claim amounts. The probability that risk i produces a claim is denoted by p_i, and Q_i is the conditional distribution of the claim in risk i, given that a claim occurs in risk i. Further, S_n is the aggregate claim in the individual model. Frequently, the distribution $\mathcal{L}(S_n)$ of S_n is quite involved and should be approximated by a simpler distribution. Since the p_i are often assumed to be small, the compound Poisson distribution

$$\mathrm{CPo}(\lambda, Q) = \sum_{k=0}^{\infty} \pi(k, \lambda) Q^{*k},$$

with $\pi(k, \lambda) = e^{-\lambda}\lambda^k/k!$, $k \in \mathbf{Z}_+ = \{0, 1, 2, \dots\}$, is a convenient candidate, where

$$Q = \frac{1}{\lambda} \sum_{i=1}^{n} p_i Q_i = \sum_{r=1}^{\infty} q_r U_r,$$

Q^{*k}, $k \in \mathbf{N}$, denotes the k-fold convolution of Q with itself, and $Q^{*0} = \varepsilon_0$. In the terminology of risk theory, $\mathrm{CPo}(\lambda, Q)$ is called the aggregate claims distribution in the collective model. It is well known that $\mathrm{CPo}(\lambda, Q)$ can be obtained as the distribution of a sum S_n', which is derived from S_n by replacing X_i with a random sum $\sum_{j=1}^{N_i} Y_{i,j}$, where all $N_i, Y_{i,j}$ are independent, N_i is Poisson distributed with unit mean and $Y_{i,j}$ has the same distribution as X_i. For general works on mathematical risk theory, we refer the reader, for example, to the books by Gerber (1979) and Hipp and Michel (1990). Note that, below, we do not need to assume that the X_i are nonnegative.

In what follows, we are concerned with the accuracy of the approximation of $\mathcal{L}(S_n)$ by $\mathrm{CPo}(\lambda, Q)$ and also by a related finite signed measure of second order. As measures of accuracy, we consider the total variation distance and the Kolmogorov metric

$$d_{\mathrm{TV}}(R_1,\ R_2) = \sup_A |R_1(A) - R_2(A)|,$$
$$d_{\mathrm{KM}}(R_1,\ R_2) = \sup_{x\in\mathbf{R}} |F_1(x) - F_2(x)|$$

between two finite signed measures R_1 and R_2 on $\mathbf{R}$ with distribution functions F_1 and F_2. Here the $\sup_A$ is taken over all Borel-measurable sets $A \subseteq \mathbf{R}$. In the case that R_i is the distribution of a random variable Z_i, we also write $d_{\mathrm{TV}}(Z_1, Z_2)$ and $d_{\mathrm{KM}}(Z_1, Z_2)$ for the respective distances between R_1 and R_2. We often use the abbreviation

$$d_\tau = d_{\mathrm{TV}}\big(\mathcal{L}(S_n), \mathrm{CPo}(\lambda, Q)\big), \qquad d_\kappa = d_{\mathrm{KM}}\big(\mathcal{L}(S_n), \mathrm{CPo}(\lambda, Q)\big).$$

As mentioned above, we always suppose that condition (A) is valid. Furthermore, we sometimes suppose that some of the following conditions are true:

(B) $\quad U_r = U^{*r} \quad$ for all $r \in \mathbf{N}$, $\quad U = U_1$,

(C) $\quad U_r = \varepsilon_r = \varepsilon_1^{*r} \quad$ for all $r \in \mathbf{N}$,

(D) $\quad rq_r \geq (r+1)q_{r+1} \quad$ for all $r \in \mathbf{N}$.

We use a method originally due to Kerstan (1964), which was refined in Daley and Vere-Jones [(1988), pages 297–299], Witte (1990) and Roos [(1996), Kapitel 8 and (1999a)]. In the latter paper, it was used to remove an undesirable log-term, which appeared in Barbour's (1988) upper bound for the total variation distance between the generalized multinomial distribution and a multivariate Poisson law with independent components and the same mean vector. The present paper shows further advantages of Kerstan's approach over Stein's method in the compound Poisson approximation. However, it should be mentioned that, in contrast to Kerstan's approach, Stein's method remains easily applicable in the case of dependent random variables.

The paper is structured as follows. Section 2 is devoted to a review of known results under different conditions. In Section 3, we present our first- and second-order results, the proofs of which are contained in Section 4.

2. Review of some known results.

2.1. *The Khintchine–Doeblin inequality.* The first inequality known for the total variation distance is generally valid and is essentially due to Khintchine (1933) and Doeblin (1939) [see also Le Cam (1960), page 1183]. It reads as

$$d_\tau \leq \sum_{i=1}^{n} p_i^2 =: \lambda_2. \tag{1}$$

If we consider large n, this bound is often greater than 1 and therefore useless, since d_τ is always bounded by 1. However, it turns out that, in special cases, the order can be improved (see below).

2.2. *The case* $Q_1 = \cdots = Q_n = \varepsilon_1$. If $Q_1 = \cdots = Q_n = \varepsilon_1$ is the Dirac measure at point 1, bounds of better order are possible [see, e.g., Prohorov (1953) for the binomial case, Le Cam (1960), Kerstan (1964), Barbour and Hall (1984), Deheuvels and Pfeifer (1986), Witte (1990) and Roos (1999b, 2001)]. One of the most remarkable results is due to Barbour and Hall. By using Stein's method, they proved in their Theorems 1 and 2 that, in the case above,

$$\frac{\lambda_2}{32}\min\left\{\frac{1}{\lambda}, 1\right\} \le d_\tau \le \lambda_2 \frac{1-e^{-\lambda}}{\lambda} \le \lambda_2 \min\left\{\frac{1}{\lambda}, 1\right\}, \tag{2}$$

where d_τ and $\lambda_2 \min\{\lambda^{-1}, 1\}$ are of the same order. It is easy to show that, generally, the constants of the upper bound in (2) cannot be reduced. But under further assumptions, more can be done: in Roos [(1999b), formula (32)], we gave the following sharpening of a result of Deheuvels and Pfeifer (1986):

$$\left|\frac{\sqrt{2\pi e}}{\theta} d_\tau - 1\right| \le C_1 \min\left\{1, \frac{1}{\sqrt{\lambda}} + \theta\right\}, \qquad \theta = \frac{\lambda_2}{\lambda}, \tag{3}$$

where $C_1 \in (0, \infty)$ is an absolute constant. In the binomial case, this was shown by Prohorov [(1953), Theorem 2] [see also Barbour, Holst and Janson (1992), page 2]. In particular, here we have $d_\tau \sim \theta/\sqrt{2\pi e}$ as $\lambda \to \infty$ and $\theta \to 0$. In other words, if $\lambda \ge a > 0$ is large and $\theta \le \varepsilon \le 1$ is small, then $d_\tau \le C_2(a, \varepsilon)\theta$ with $C_2(a, \varepsilon)$ being nearly equal to $1/\sqrt{2\pi e}$, which, in turn, is asymptotically optimal for $a \to \infty$ and $\varepsilon \to 0$. There arose the problem of an inequality containing a sharp constant if we only assume that θ is small. In Roos [(2001), formula (10)], we gave such a bound, which reads as follows:

$$d_\tau \le \left(\frac{3}{4e} + \frac{7\sqrt{\theta}(3 - 2\sqrt{\theta})}{6(1 - \sqrt{\theta})^2}\right)\theta. \tag{4}$$

Further, we showed that the "$\le$" in (4) can be replaced with "$\sim$" if $\theta \to 0$ and $\lambda \to 1$, which indicates that the constant $3/(4e)$ in (4) is best possible. In particular, if we assume only small $\theta \le \varepsilon \le 1$, we have an inequality of the form $d_\tau \le C_3(\varepsilon)\theta$ with $C_3(\varepsilon)$ being nearly equal to $3/(4e)$, which, in turn, is the asymptotical optimal constant for $\varepsilon \to 0$. Note that, comparing the constants in (3) and (4), we see that $1/\sqrt{2\pi e} \approx 0.24$ does not differ much from $3/(4e) \approx 0.28$. It should be mentioned that, for the Kolmogorov metric and other distances, results similar to (3) and (4) can be found in the papers cited above.

2.3. *Results under further conditions.* From a simple observation made by Le Cam [(1965), page 187] and later used by Michel [(1987), page 167], it follows that the upper bound in (2) remains valid in the case $Q_1 = \cdots = Q_n \neq \varepsilon_1$. But if the Q_i are arbitrary and possibly unequal, an upper bound for d_τ independent of the Q_i cannot have order better than $\min\{\lambda_2, 1\}$ [see Le Cam (1965), page 188, and Zaitsev (1989), Remark 1.1]. Barbour and Utev [(1999), Example 1.4] observed that this statement even holds in the presence of condition (C). However, for the weaker Kolmogorov metric, more can be done, as shown by Le Cam (1965) and Zaitsev (1983). We only cite Zaitsev's result: in the general case, the inequality

$$d_\kappa \le C_4 p_{\max} \tag{5}$$

holds, where $C_4 \in (0, \infty)$ is an absolute constant and $p_{\max} = \max_{1\le i\le n} p_i$. Note that, since $\theta = \lambda_2/\lambda \le p_{\max}$, the upper bound in (5) has weaker order than θ. To get bounds for d_τ of better order than $\min\{\lambda_2, 1\}$, Barbour, Chen and Loh [(1992), Theorem 5] considered the case that conditions (C) and (D) are satisfied. In this context, we have $q_r = Q(\{r\})$ for $r \in \mathbf{N}$, $Q_1, \dots, Q_n$ are concentrated on $\mathbf{N}$ and μ_i is the mean of Q_i. In particular, their results [see also Barbour and Utev (1999), Proposition 1.5] imply that, in the case above, the inequality

$$d_\tau \le H(\lambda, Q) \sum_{i=1}^n p_i^2 \mu_i^2 \tag{6}$$

holds and $H(\lambda, Q)$ is a quantity, which was derived with the help of Stein's method and satisfies

$$H(\lambda, Q) \le \min\left\{1, \frac{1}{\lambda(q_1 - 2q_2)}\left(\frac{1}{4\lambda\,(q_1 - 2q_2)} + \log^+\bigl(2\lambda(q_1 - 2q_2)\bigr)\right)\right\}. \tag{7}$$

Therefore, if conditions (C) and (D) hold and if the Q_i have bounded means and $q_1 - 2q_2$ is bounded away from 0, then d_τ is of order $O(\lambda_2 \min\{\lambda^{-1}(1 + \log^+\lambda), 1\})$. Barbour and Xia [(1999), Theorem 2.5; see also Barbour and Chryssaphinou (2001), formula (2.21)] have shown that, if condition (C) is valid and if

$$\vartheta = \sum_{r=2}^\infty \frac{r(r-1)q_r}{\mu} < \frac{1}{2}, \qquad \mu = \sum_{r=1}^\infty r q_r,$$

then

$$H(\lambda, Q) \le \frac{1}{(1 - 2\vartheta)\lambda\mu}. \tag{8}$$

In Theorem 2 of the present paper, we will show substantial improvements of such inequalities under weaker assumptions. In particular, this theorem implies that, in the presence of condition (B), we have

$$d_\tau = O\left(\sum_{i=1}^n p_i^2 \min\left\{\frac{\mu_i^2}{\lambda q_1}, \frac{\nu_i}{\lambda}, 1\right\}\right) \tag{9}$$

and that, if additionally condition (D) is valid, the "q_1" in the denominator of the right-hand side of (9) can be removed. Note that $(1-2\vartheta)\mu \le 2q_1 - \mu \le q_1$, which shows that, in fact, the order in (9) is better than that of (6) together with (8). Barbour and Utev [(1999), page 93] pointed out that, if condition (D) is not satisfied, the best inequality known would be [see also Barbour, Chen and Loh (1992), Theorem 4]

$$H(\lambda, Q) \le e^{\lambda} \min\left\{\frac{1}{\lambda q_1}, 1\right\}; \tag{10}$$

note that the right-hand side of (10) is bounded away from 0, which shows that, under the present assumptions, the method of Barbour, Chen and Loh (1992) leads to unsatisfying results. For the Kolmogorov metric, Barbour and Xia [(2000), Proposition 1.1] proved that, if both conditions (C) and (D) are satisfied, then

$$d_\kappa \le \min\left\{\frac{1}{\lambda q_1 + 1}, \frac{1}{2}\right\} \sum_{i=1}^{n} p_i^2 \mu_i^2. \tag{11}$$

Since $d_\kappa \le d_\tau$, our Theorem 2 leads to a result better than (11) if we take into account only the order of the bounds.

See Barbour and Utev (1998, 1999) for refinements of the method given in Barbour, Chen and Loh (1992) and for other somewhat complicated estimates of d_τ and d_κ. Further contributions on compound Poisson approximation, came, for example, from Hipp (1985), Čekanavičius (1997, 1998) and Vellaisamy and Chaudhuri (1999).

3. Main results. Recall that, as mentioned above, we always assume that condition (A) holds.

3.1. *First-order results.* The following theorems concern the total variation distance d_τ. Some notation is necessary. For $x \in (0, \infty)$ and $z \in \mathbf{C}$, let

$$g(z) = 2\frac{e^z}{z^2}(e^{-z} - 1 + z),$$

$$\alpha_1(x) = \sum_{i=1}^{n} g(2p_i) p_i^2 \min\left\{\frac{x\nu_i}{\lambda}, 1\right\},$$

$$\beta_1 = \sum_{i=1}^{n} p_i^2 \min\left\{\frac{\nu_i}{\lambda}, 1\right\}.$$

The following bounds are direct consequences of Theorem 1 in Roos (1999a) and are valid without assuming further constraints.

THEOREM 1. *We have*

$$d_\tau \leq \frac{\alpha_1(2^{-3/2})}{1 - 2e\alpha_1(2^{-3/2})} \qquad \text{if } \alpha_1(2^{-3/2}) < \frac{1}{2e}, \tag{12}$$

$$d_\tau \leq 8.8\beta_1. \tag{13}$$

REMARK 1. (a) Theorem 1 also holds in more general situations, where, for example, the U_r are multidimensional (or even more general) measures. All that we need for a proof is the finite operator norm applicable to convolutions of operators; see Le Cam (1965).

(b) For practical usage, (12) is often sharper than (13), which, in turn, has theoretical value.

(c) We have

$$1 \leq \max_{1 \leq i \leq n} g(2p_i) \leq g(2p_{\max}) \leq g(2) \leq 4.1946 \quad \text{and} \quad \lim_{\varepsilon \downarrow 0} g(\varepsilon) = 1.$$

(d) Unfortunately, the Khintchine–Doeblin bound (1) is not contained in the bounds of Theorem 1. From (13), it only follows that $d_\tau \leq c\lambda_2$ with $c = 8.8$. But if we consider the case of small λ_2, it follows from (c) that (12) contains the bound with $c \approx 1$.

The next theorem requires further notation: For $x \in (0, \infty)$, let

$$\alpha_2(x) = \sum_{i=1}^{n} g(2p_i) p_i^2 \min\left\{\frac{x\mu_i^2}{\lambda}, \frac{\nu_i}{2^{3/2}\lambda}, 1\right\},$$

$$\beta_2(x) = \sum_{i=1}^{n} p_i^2 \min\left\{\frac{x\mu_i^2}{\lambda}, \frac{\nu_i}{\lambda}, 1\right\}.$$

THEOREM 2. *Let* $c_1 = 3/(4e)$. *In the presence of condition* (B), *we have*

$$d_\tau \leq \frac{\alpha_2(c_1 q_1^{-1})}{1 - 2e\alpha_2(c_1 q_1^{-1})} \qquad \text{if } \alpha_2(c_1 q_1^{-1}) < \frac{1}{2e}, \tag{14}$$

$$d_\tau \leq 8.8\beta_2(q_1^{-1}). \tag{15}$$

In the presence of (B) *and* (D),

$$d_\tau \leq \frac{\alpha_2(e^{-1})}{1 - 2e\alpha_2(e^{-1})} \qquad \text{if } \alpha_2(e^{-1}) < \frac{1}{2e}, \tag{16}$$

$$d_\tau \leq 9.2\beta_2(1). \tag{17}$$

REMARK 2. (a) Generally, in (14), the constant c_1 cannot be reduced. Indeed, if condition (C) holds, $Q_1 = \cdots = Q_n = \varepsilon_1$, $p_{\max} \to 0$ and $\lambda \to 1$, then we obtain

$\alpha_2(cq_1^{-1})/(1-2e\alpha_2(cq_1^{-1})) \sim c\lambda_2$ for $c \in (0, c_1]$, and, in view of Roos [(2001), Theorem 2], we see that $d_\tau \sim c_1\lambda_2$ (see also Section 2.2).

(b) The inequalities of Theorem 2 are substantial improvements of the bounds, which can be derived from the results of Barbour, Chen and Loh (1992) and Barbour and Xia (1999); see (6)–(8).

EXAMPLE 2. For $i \in \{1, \dots, n\}$, let $Q_i = \mathcal{E}(t_i)$ be an exponential distribution with parameter $t_i \in (0, \infty)$. Let $a > t_{\max} = \max_{1 \le i \le n} t_i$ be arbitrary. Then it is easily shown that conditions (A) and (B) are satisfied with

$$q_{i,r} = (1-b_i)^{r-1} b_i, \qquad b_i = \frac{t_i}{a}, \qquad U_r = (\mathcal{E}(a))^{*r}.$$

Indeed, the characteristic functions of Q_i and $\sum_{r=1}^{\infty} q_{i,r}(\mathcal{E}(a))^{*r}$ coincide. Using (15), it now follows that, if $t_{\max}$ is bounded and $t_{\min} = \min_{1 \le i \le n} t_i$ is bounded away from 0, then $d_\tau = O(\lambda_2 \min\{\lambda^{-1}, 1\})$. In particular, by letting $a \downarrow t_{\max}$, we obtain

$$d_\tau \le 8.8 \sum_{i=1}^n p_i^2 \min\left\{\frac{t_{\max}^3}{t_i^2 \sum_{j=1}^n p_j t_j}, 1\right\} \le 8.8\lambda_2 \min\left\{\frac{1}{\lambda}\left(\frac{t_{\max}}{t_{\min}}\right)^3, 1\right\}. \tag{18}$$

If $2t_{\min} > t_{\max}$, then, as is easily shown, condition (D) is satisfied and (17) leads to

$$d_\tau \le 9.2 \sum_{i=1}^n p_i^2 \min\left\{\frac{t_{\max}^2}{\lambda t_i^2}, 1\right\} \le 9.2\lambda_2 \min\left\{\frac{1}{\lambda}\left(\frac{t_{\max}}{t_{\min}}\right)^2, 1\right\},$$

which is, under the present assumptions, of the same order as (18). Note that, in this example, it is difficult to evaluate the ν_i, so that we did not use them in the bounds above.

3.2. *Second-order results.* The following theorems are devoted to the second-order approximation of $\mathcal{L}(S_n)$ by the finite signed measure

$$\mathrm{CPo}_2(\lambda, Q) = \left(\varepsilon_0 - \tfrac{1}{2}\sum_{i=1}^n p_i^2 (Q_i - \varepsilon_0)^{*2}\right) * \mathrm{CPo}(\lambda, Q). \tag{19}$$

We shall give bounds for

$$d'_\tau := d_{\mathrm{TV}}\big(\mathcal{L}(S_n), \mathrm{CPo}_2(\lambda, Q)\big).$$

Let

$$h(z) = \frac{3(g(z)-1)}{2z}, \qquad z \in \mathbf{C},$$

$$\gamma_1 = \sum_{i=1}^n h(2p_i) p_i^3 \min\left\{0.46\left(\frac{\nu_i}{\lambda}\right)^{3/2}, 1\right\}.$$

The next theorem is generally valid and follows from Theorem 2 in Roos (1999a).

THEOREM 3. *We have*

$$d'_\tau \le \frac{4}{3}\gamma_1 + (\alpha_1(1))^2\left(1 + \frac{0.82\alpha_1(1)}{1 - 2^{-1/2}\alpha_1(1)e}\right),$$

where we assume that $\alpha_1(1) < 2^{1/2}e^{-1}$.

For $x \in (0, \infty)$, let

$$\gamma_2(x) = \sum_{i=1}^{n} h(2p_i)p_i^3 \min\left\{\frac{x\mu_i^3}{\lambda^{3/2}}, 0.46\left(\frac{\nu_i}{\lambda}\right)^{3/2}, 1\right\}.$$

THEOREM 4. *If condition* (B) *is valid, then*

$$d'_\tau \le \frac{4}{3}\gamma_2(0.31q_1^{-3/2}) + \frac{4(\alpha_2(c_1q_1^{-1}))^2}{1 - 2e\alpha_2(c_1q_1^{-1})}, \tag{20}$$

where we suppose that $\alpha_2(c_1q_1^{-1}) < 1/(2e)$ *and* c_1 *is defined in Theorem* 2. *In the presence of* (B) *and* (D),

$$d'_\tau \le \frac{4}{3}\gamma_2(0.41) + \frac{4.2(\alpha_2(e^{-1}))^2}{1 - 2e\alpha_2(e^{-1})} \qquad \text{if } \alpha_2(e^{-1}) < \frac{1}{2e}. \tag{21}$$

The estimates in Section 4.2 and an inequality in Roos [(1999a), page 132] yield some bounds for the total variation norm (for a definition, see Section 4.1) of the finite signed measure $\mathrm{CPo}_2(\lambda, Q)$:

LEMMA 1. *In the general case*,

$$\|\mathrm{CPo}_2(\lambda, Q)\| \le 1 + 2\beta_1.$$

If condition (B) *is valid*, *then*

$$\|\mathrm{CPo}_2(\lambda, Q)\| \le 1 + 2\beta_2(q_1^{-1}).$$

If conditions (B) *and* (C) *are satisfied*, *then*

$$\|\mathrm{CPo}_2(\lambda, Q)\| \le 1 + 2\beta_2(1).$$

REMARK 3. With the help of Lemma 1, one can remove the singularities in the upper bounds of Theorems 3 and 4. For example, in the context of Theorem 3, an absolute constant $c \in (0, \infty)$ can easily be found such that $d'_\tau \le (4/3)\gamma_1 + c(\alpha_1(1))^2$. Similar statements are possible concerning the bounds in Theorem 4. Unfortunately, the constants involved are quite large, so that we do not state them explicitly. Note that Čekanavičius [(1998), proof of Corollary 3.1] has shown that, in the general case,

$$d'_\tau \le 2\lambda_2^2 + \tfrac{8}{3}\sum_{i=1}^n p_i^3.$$

It is easily seen that a bound of this order is also contained in the estimates of Theorems 3 and 4.

Using the bounds for d'_τ and a separate consideration of the leading term in Kerstan's expansion (see below), it is possible to derive a further interesting first-order result.

THEOREM 5. *In the presence of* (B) *and* (D),

$$d_\tau \le d'_\tau + \sum_{i=1}^n p_i^2 \min\left\{\mu_i^2 \mathrm{E}\binom{Y+2}{2}^{-1/2}, \mu_i^2 e^{-\min\{\lambda, 2q_1^{-1}\}}, \frac{\nu_i}{2^{3/2}\lambda}, 1\right\}, \tag{22}$$

where Y is a random variable with distribution $\mathrm{CPo}(\lambda, \sum_{r=1}^\infty q_r \varepsilon_r)$.

The bound in (22) can be further estimated, by using the following simple inequalities:

$$\mathrm{E}\binom{Y+2}{2}^{-1/2} \le \sqrt{2}\,\mathrm{E}\frac{1}{1+Y} \le \sqrt{2}\,\frac{1-e^{-\lambda}}{\lambda}. \tag{23}$$

4. Proofs.

4.1. *Kerstan's method.* We use the following notation. Products and powers of finite signed measures on $\mathbf{R}$ are understood in the sense of convolution. For a finite signed measure R on $\mathbf{R}$ with Hahn–Jordan decomposition $R = R_+ - R_-$, we define the total variation norm of R by $\|R\| = (R_+ + R_-)(\mathbf{R})$. We shall repeatedly use the easy inequality

$$\|R_1 R_2\| \le \|R_1\|\,\|R_2\|$$

for two finite signed measures R_1 and R_2. The proofs of our results are based on a slight modification of Kerstan's expansion [cf. Roos (1999a)], which reads as

$$\begin{aligned}\mathcal{L}(S_n) - \mathrm{CPo}(\lambda, Q) &= \left(\prod_{i=1}^n (L_i + \varepsilon_0) - \varepsilon_0\right)\mathrm{CPo}(\lambda, Q)\\ &= \sum_{j=1}^n \sum_{1\le i(1)<\cdots<i(j)\le n} \prod_{s=1}^j \left(L_{i(s)}\,\mathrm{CPo}\left(\frac{\lambda}{j}, Q\right)\right),\end{aligned} \tag{24}$$

where, for $i \in \{1, \dots, n\}$,

$$L_i = \big(\varepsilon_0 + p_i(Q_i - \varepsilon_0)\big)\exp\big(-p_i(Q_i - \varepsilon_0)\big) - \varepsilon_0 \tag{25}$$
$$= -\frac{p_i^2}{2}(Q_i - \varepsilon_0)^2 g\big(p_i(\varepsilon_0 - Q_i)\big).$$

Applying the polynomial theorem, we are now led to

$$d_\tau = \frac{1}{2}\|\mathcal{L}(S_n) - \mathrm{CPo}(\lambda, Q)\|$$
$$\leq \frac{1}{2}\sum_{j=1}^{n}\sum_{1\leq i(1)<\cdots<i(j)\leq n}\prod_{s=1}^{j}\left\|L_{i(s)}\,\mathrm{CPo}\Big(\frac{\lambda}{j}, Q\Big)\right\| \tag{26}$$
$$\leq \frac{1}{2}\sum_{j=1}^{n}\frac{1}{j!}\left(\sum_{i=1}^{n}\left\|L_i\,\mathrm{CPo}\Big(\frac{\lambda}{j}, Q\Big)\right\|\right)^j.$$

In what follows, we are concerned with suitable norm estimates.

4.2. *Norm estimates.*

LEMMA 2. *Let $t \in (0, \infty)$, $i \in \{1, \dots, n\}$, and c_1 be as in Theorem* 2. *Then, in the presence of condition* (B), *we have*

$$\|(U - \varepsilon_0)\,\mathrm{CPo}(t, Q)\| \leq \sqrt{\frac{2}{tq_1e}}, \tag{27}$$

$$\|(U - \varepsilon_0)^2\,\mathrm{CPo}(t, Q)\| \leq \frac{3}{tq_1e}, \tag{28}$$

$$\|(U - \varepsilon_0)^3\,\mathrm{CPo}(t, Q)\| \leq \frac{3}{\sqrt{2}}\Big(\frac{3}{tq_1e}\Big)^{3/2}, \tag{29}$$

$$\|(Q_i - \varepsilon_0)^2\,\mathrm{CPo}(t, Q)\| \leq \frac{\sqrt{2}\nu_i}{t}, \tag{30}$$

$$\|(Q_i - \varepsilon_0)^3\,\mathrm{CPo}(t, Q)\| \leq \frac{1}{\sqrt{2}}\Big(\frac{3\nu_i}{t}\Big)^{3/2}, \tag{31}$$

$$\|L_i\,\mathrm{CPo}(t, Q)\| \leq 2g(2p_i)p_i^2\min\left\{\frac{c_1\mu_i^2}{tq_1}, \frac{\nu_i}{2^{3/2}t}, 1\right\}. \tag{32}$$

If, additionally, $t \geq p_i \sup_{r\in\mathbf{N}} q_{i,r}/q_r$, then

$$\|L_i\,\mathrm{CPo}(t, Q)\| \leq 2p_i^2. \tag{33}$$

PROOF. Inequalities (27) and (28) follow from Roos [(2001), formulas (27) and (29)] and the observation that, for $q_1 < 1$,

$$\mathrm{CPo}(t, Q) = \mathrm{CPo}(tq_1, U)\,\mathrm{CPo}\big(t(1-q_1), \tilde{Q}\big), \qquad \tilde{Q} = \sum_{m=2}^{\infty} \frac{q_m}{1-q_1} U^m.$$

The proof of (29) is easily done with the help of (27), (28) and the inequality

$$\|(U-\varepsilon_0)^3\,\mathrm{CPo}(t, Q)\| \le \left\|(U-\varepsilon_0)\,\mathrm{CPo}\Big(\frac{t}{3}, Q\Big)\right\| \left\|(U-\varepsilon_0)^2\,\mathrm{CPo}\Big(\frac{2t}{3}, Q\Big)\right\|.$$

The bounds (30) and (31) follow from Roos [(1999a), formulas (19) and (26)] and the simple fact that, for $j \in \mathbf{Z}_+$,

$$\|(Q_i-\varepsilon_0)^j\,\mathrm{CPo}(t, Q)\| = \lim_{k\to\infty}\left\|\left(\sum_{r=1}^{k} q_{i,r}(U_r-\varepsilon_0)\right)^j \mathrm{CPo}\big(t^{(k)}, Q^{(k)}\big)\right\|,$$

where

$$t^{(k)} = t\sum_{r=1}^{k} q_r, \qquad Q^{(k)} = \sum_{r=1}^{k} \frac{tq_r}{t^{(k)}} U_r. \tag{34}$$

Inequality (32) is an immediate consequence of (25), (28), (30) and the representation

$$Q_i - \varepsilon_0 = \left(\sum_{k=0}^{\infty}\sum_{r=k+1}^{\infty} q_{i,r} U^k\right)(U-\varepsilon_0). \tag{35}$$

The proof of (33) is similar to the one of (23) in Roos (1999a) and is therefore omitted. □

REMARK 4. (a) If $U = Q = \varepsilon_1$, $\mathrm{CPo}(t, Q)$ is a Poisson distribution with mean t and it follows from Proposition 4 of Roos (1999b) that, in this context, we have, for $t \to \infty$ and fixed $j \in \mathbf{Z}_+$,

$$\|(U-\varepsilon_0)^j\,\mathrm{CPo}(t, Q)\| \sim \frac{1}{t^{j/2}} \int_{\mathbf{R}} \left|\frac{1}{\sqrt{2\pi}} \frac{d^j}{dx^j} e^{-x^2/2}\right| dx.$$

In particular, here the norm term is of order $O(t^{-j/2})$ for $t \to \infty$. But in the general case, this need not be true for $j \in \mathbf{N}$. For example, if $U = \varepsilon_1$ and $Q = \varepsilon_{j+1}$, we easily get $\|(U-\varepsilon_0)^j\,\mathrm{CPo}(t, Q)\| = 2^j$. This explains why the "q_1" appears in the denominator of some upper bounds given in Lemma 2.

(b) From (33), it follows that $\|L_i\,\mathrm{CPo}(\lambda, Q)\| \le 2p_i^2$.

In the presence of conditions (B) and (D), we shall give norm estimates better than (27)–(29). For the proof, we need the following lemma.

LEMMA 3. *Let $t \in (0,\infty)$. If conditions* (B) *and* (D) *are satisfied, then, for all $\eta \in (0,1)$, there exists a distribution R_η on $\mathbf{R}$ such that the decomposition*

$$\mathrm{CPo}(t, Q) = \mathrm{CPo}\big(t, Q(\eta)\big) R_\eta, \qquad Q(\eta) = \sum_{r=1}^{\infty} q_r \big(\varepsilon_0 + \eta(U - \varepsilon_0)\big)^r$$

is valid.

This lemma is essentially due to Steutel and van Harn [(1979), Lemma 1.2 and Theorem 2.2], who investigated the case $U = \varepsilon_1$. Under this assumption, the above property of $\mathrm{CPo}(t, Q)$ is called discrete self-decomposability. The generalization of the statement by Steutel and van Harn to arbitrary distributions U is trivial.

LEMMA 4. *Let $t \in (0,\infty)$ and $j \in \mathbf{Z}_+$. Further, let Y denote a random variable with distribution $\mathcal{L}(Y) = \mathrm{CPo}(t, \sum_{r=1}^{\infty} q_r \varepsilon_r)$. We assume that* (B) *and* (D) *are satisfied. Then*

$$\|(U - \varepsilon_0)^j \,\mathrm{CPo}(t, Q)\| \le 2^j \mathrm{E} \binom{Y + j}{j}^{-1/2}. \tag{36}$$

PROOF. From Lemma 3, we obtain, for all $\eta \in (0,1)$,

$$\begin{aligned}
&\|(U - \varepsilon_0)^j \,\mathrm{CPo}(t,\ Q)\| \\
&\quad \le \big\|(U - \varepsilon_0)^j \,\mathrm{CPo}\big(t, Q(\eta)\big)\big\| \\
&\quad \le \mathrm{E}\big\|(U - \varepsilon_0)^j \big(\varepsilon_0 + \eta(U - \varepsilon_0)\big)^Y\big\| \\
&\quad \le \frac{1}{(\eta(1-\eta))^{j/2}} \mathrm{E} \binom{Y + j}{j}^{-1/2},
\end{aligned}$$

where the latter inequality follows from Roos [(2000), formula (37)]. The proof of (36) is completed by letting $\eta = 1/2$. □

REMARK 5. From (36) and (23), we derive an inequality better than (28): if (B) and (D) are valid and $t \in (0,\infty)$, then

$$\|(U - \varepsilon_0)^2 \,\mathrm{CPo}(t, Q)\| \le 4\sqrt{2}\,\frac{1 - e^{-t}}{t}. \tag{37}$$

In particular, here the "q_1" does not appear in the denominator. In what follows, we show that (37) can be further improved. We need the following lemma due to Steutel and van Harn [(1979), Lemma 1.2 and Theorem 2.3].

LEMMA 5. *Let $t \in (0,\infty)$. If the conditions* (C) *and* (D) *are satisfied, then $\mathrm{CPo}(t, Q)$ is discrete unimodal. This means that, if $a_m = \mathrm{CPo}(t, Q)(\{m\})$ for $m \ge -1$, then the sequence $(a_{m-1} - a_m)_{m \in \mathbf{Z}_+}$ changes sign at most once.*

LEMMA 6. *Let the conditions of Lemma* 4 *be valid and* $i \in \{1, \dots, n\}$. *Then*:

$$\|(U - \varepsilon_0)^{j+1} \operatorname{CPo}(t, Q)\| \le \left(\frac{2(j+1)}{te}\right)^{(j+1)/2}, \tag{38}$$

$$\|L_i \operatorname{CPo}(t, Q)\| \le 2g(2p_i)p_i^2 \frac{\mu_i^2}{te}. \tag{39}$$

Further, we have

$$\|(U - \varepsilon_0)^{j+1} \operatorname{CPo}(t, Q)\| \le 2^{j+1} \exp\left(-\min\left\{t, \frac{j+1}{q_1}\right\}\right). \tag{40}$$

PROOF. Due to a simple coupling argument, we may assume that condition (C) is valid. By Lemma 5, it follows that, here,

$$\begin{aligned}
&\|(U - \varepsilon_0)^{j+1} \operatorname{CPo}(t, Q)\| \\
&\quad \le \left\|(U - \varepsilon_0) \operatorname{CPo}\left(\frac{t}{j+1}, Q\right)\right\|^{j+1} \\
&\quad = 2^{j+1}\left(\sup_{m \in \mathbf{Z}_+} \operatorname{CPo}\left(\frac{t}{j+1}, Q\right)(\{m\})\right)^{j+1} \\
&\quad \le \left(\frac{2(j+1)}{te}\right)^{(j+1)/2},
\end{aligned}$$

where the latter bound is a consequence of

$$\sup_{n \in \mathbf{Z}_+} \pi\left(n, \frac{t}{j+1}\right) \le \left(\frac{j+1}{2te}\right)^{1/2}, \qquad \sup_{m \in \mathbf{Z}_+} \sum_{n=0}^{\infty} Q^n(\{m\}) \le 1. \tag{41}$$

The second inequality in (41) follows from the fact that, if $T_1, T_2, \dots$ are independent and identically distributed random variables with $\mathcal{L}(T_1) = Q$, then, according to (C), we may assume that $T_i \ge 1$, $i \in \mathbf{N}$, and therefore, for $m \in \mathbf{Z}_+$,

$$\sum_{n=0}^{\infty} Q^n(\{m\}) = \sum_{n=0}^{\infty} P\left(\sum_{i=1}^{n} T_i = m\right) = P\left(\bigcup_{n=0}^{\infty}\left\{\sum_{i=1}^{n} T_i = m\right\}\right) \le 1.$$

Hence, (38) is shown. The bound (39) follows from (25), (35) and (38). To prove (40), it suffices to consider only the case $j = 0$ and $0 < tq_1 \le 1$, since

$$\|(U - \varepsilon_0)^{j+1} \operatorname{CPo}(t, Q)\| \le \left\|(U - \varepsilon_0) \operatorname{CPo}\left(\min\left\{\frac{t}{j+1}, \frac{1}{q_1}\right\}, Q\right)\right\|^{j+1}.$$

Let $\varphi_1(z) = \exp(t(\sum_{r=1}^{\infty} q_r z^r - 1))$ and $\varphi_2(z) = (z-1)\varphi_1(z)$, $|z| \le 1$, denote the generating functions of $\operatorname{CPo}(t, Q)$ and $(U - \varepsilon_0) \operatorname{CPo}(t, Q)$, respectively. In view of

$$\frac{d}{dz}\varphi_2(z) = \varphi_1(z)\left(1 - tq_1 + \sum_{r=1}^{\infty}(rq_r - (r+1)q_{r+1})z^r\right),$$

we see that, since $tq_1 \le 1$, the power series expansion of the derivative of $\varphi_2(z)$ has only nonnegative coefficients. Therefore, in this case, we have $\varphi_2(z) = -e^{-t} + \sum_{m=1}^{\infty} B_m z^m$, where the B_m are all nonnegative. This yields

$$\|(U - \varepsilon_0)\,\mathrm{CPo}(t, Q)\| = \varphi_2(1) + 2e^{-t} = 2e^{-t}$$

and completes the proof of (40). □

PROOF OF LEMMA 1. The assertions easily follow from (19), (28), (30), (35), (38) and an inequality in Roos [(1999a), page 132]. □

4.3. *Proofs of the theorems.*

PROOF OF THEOREM 2. Using (26), (32) and Stirling's formula, (14) is easily shown. To prove (15), we use (26), (32) and (33) and obtain

$$d_\tau \le \min\{f_1(g(2)\beta_2(q_1^{-1})), 1\},$$

where

$$f_1(x) = \frac{x}{2^{3/2}} + x^2 + \frac{1}{2}\sum_{j=3}^{\infty}\frac{1}{j!}\left(\frac{jx}{\sqrt{2}}\right)^j, \qquad x \ge 0.$$

If $x_1 \in (0, \infty)$ denotes the unique positive solution of $f_1(x) = 1$, then we obtain $d_\tau \le g(2)\beta_2(q_1^{-1})/x_1$. Numerical computations yield $0.477 < x_1 < 0.478$ giving (15). The proofs of (16) and (17) are analogously shown by using (26), (32), (33) and (39). In particular, in the presence of (B) and (D), we obtain (17) in the following way:

$$d_\tau \le \min\{f_2(g(2)\beta_2(1)), 1\} \le \frac{g(2)}{x_2}\beta_2(1) \le 9.2\beta_2(1),$$

where

$$f_2(x) = \frac{x}{e} + x^2 + \frac{1}{2}\sum_{j=3}^{\infty}\frac{1}{j!}\left(\frac{2jx}{e}\right)^j, \qquad x \ge 0,$$

and $x_2 \in (0.459, 0.460)$ is the unique positive solution of $f_2(x) = 1$. This completes the proof of the theorem. □

PROOF OF THEOREM 4. To prove the second-order results, we use the equality

$$\begin{aligned} \mathcal{L}(S_n) - \mathrm{CPo}_2(\lambda, Q) &= \frac{1}{3}\sum_{i=1}^{n} p_i^3 h(p_i(\varepsilon_0 - Q_i))(Q_i - \varepsilon_0)^3\,\mathrm{CPo}(\lambda, Q) \\ &\quad + \sum_{j=2}^{n}\sum_{1\le i(1)<\cdots<i(j)\le n}\prod_{s=1}^{j}\left(L_{i(s)}\,\mathrm{CPo}\left(\frac{\lambda}{j}, Q\right)\right), \end{aligned} \tag{42}$$

which follows from (19) and (24). Using (42), we get

$$d'_\tau \le \frac{1}{6}\sum_{i=1}^{n} h(2p_i)p_i^3 \|(Q_i-\varepsilon_0)^3 \operatorname{CPo}(\lambda, Q)\| + \frac{1}{2}\sum_{j=2}^{\infty}\frac{1}{j!}\biggl(\sum_{i=1}^{n}\biggl\|L_i \operatorname{CPo}\Bigl(\frac{\lambda}{j}, Q\Bigr)\biggr\|\biggr)^j.$$

Considering the summand for $j=2$ separately, (20) is immediately shown by using (29), (31), (32) and (35). Inequality (21) is analogously shown by using (35), (31), (32), (38), (39) and (42). Theorem 4 is proved. □

Theorem 5 can easily be proved by using (19), (30), (35), (36) and (40).

PROOF OF THEOREM 1. First, let us mention that, for arbitrary $k \in \mathbf{N}$,

$$d_\tau \le d_{\mathrm{TV}}(S_n, S_n^{(k)}) + d_{\mathrm{TV}}\bigl(\mathcal{L}(S_n^{(k)}), \operatorname{CPo}(\lambda^{(k)}, Q^{(k)})\bigr) + d_{\mathrm{TV}}\bigl(\operatorname{CPo}(\lambda^{(k)}, Q^{(k)}), \operatorname{CPo}(\lambda, Q)\bigr),$$

where $\lambda^{(k)}$ and $Q^{(k)}$ are defined as in (34) (with $t=\lambda$) and $S_n^{(k)}$ is the sum of independent random variables $X_1^{(k)}, \dots, X_n^{(k)}$ with distributions

$$\mathcal{L}(X_i^{(k)}) = \varepsilon_0 + \sum_{r=1}^{k} p_i q_{i,r}(U_r - \varepsilon_0), \qquad i \in \{1,\dots,n\}.$$

It is not difficult to show that

$$d_{\mathrm{TV}}(S_n, S_n^{(k)}) \overset{k\to\infty}{\longrightarrow} 0, \qquad d_{\mathrm{TV}}\bigl(\operatorname{CPo}(\lambda^{(k)}, Q^{(k)}), \operatorname{CPo}(\lambda, Q)\bigr) \overset{k\to\infty}{\longrightarrow} 0.$$

Further, from Roos [(1999a), Theorem 1] and a simple coupling argument, it follows that

$$\lim_{k\to\infty} d_{\mathrm{TV}}\bigl(\mathcal{L}(S_n^{(k)}), \operatorname{CPo}(\lambda^{(k)}, Q^{(k)})\bigr) \le \frac{\alpha_1(2^{-3/2})}{1-2e\alpha_1(2^{-3/2})},$$

$$\lim_{k\to\infty} d_{\mathrm{TV}}\bigl(\mathcal{L}(S_n^{(k)}), \operatorname{CPo}(\lambda^{(k)}, Q^{(k)})\bigr) \le 8.8\beta_1,$$

where, for the first inequality, we assume that $\alpha_1(2^{-3/2}) < 1/(2e)$. This leads to (12) and (13). □

The proof of Theorem 3 is similar to the preceeding one and is therefore omitted. Note that here we used Theorem 2 of Roos (1999a).

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