where $\alpha_k = N$, $\alpha_0 = 0$. This is the distribution of cell frequencies in a p by k contingency table with all totals fixed when there is independence (see [1], p. 278) and thus, for large values of m_{ij} at least, the usual chi-square test can be used.

REFERENCE

[1] A. M. Mood, *Introduction to the Theory of Statistics*, McGraw-Hill Book Co., 1950.

AN OMISSION IN NORTON'S LIST OF 7 × 7 SQUARES

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1. In a previous paper the value 16,942,080 for the number of reduced 7 × 7 squares was obtained by the author by an exhaustive method, subject to a strict control ([4], Section 20). This number exceeds Norton's ([2], Table on p. 290) by 14,112. An attempt was made in Section 21 of [4] to show that this discrepancy in the total number does not affect Norton's conjecture ([2], p. 291) that the 146 species represent the whole of the universe of 7 × 7 Latin squares. However, R. A. Fisher has informed the author that the discrepancy cannot be explained away in this manner. It has therefore to be attributed to a gap in Norton's list.

2. Now, a 147th species containing 14,112 squares can arise only from an automorph type through an operator of the order 5^k. It is easy to construct a matrix Q corresponding to such an operator as, for example, $T = (34567)^3$. Here the cycle (34567) signifies a permutation [1] of columns, a permutation of rows and a substitution of elements.

The first two rows of Q are respectively identical with the first two columns and define the substitution (12) (34567). In the remaining 5 × 5 squares, it is necessary that the elements of the broken diagonals follow in the natural cyclic order, except the numbers 1 and 2, which each form a broken diagonal.

The square is given below:

$$Q = \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 1 & 4 & 5 & 6 & 7 & 3 \\ 3 & 4 & 5 & 7 & 1 & 2 & 6 \\ 4 & 5 & 7 & 6 & 3 & 1 & 2 \\ 5 & 6 & 2 & 3 & 7 & 4 & 1 \\ 6 & 7 & 1 & 2 & 4 & 3 & 5 \\ 7 & 3 & 6 & 1 & 2 & 5 & 4. \end{array}$$

3. On replacing each row of Q by the conjugate permutation and rotating the figure through an angle of 180° about the diagonal, we obtain the square

$$
R = \begin{array}{ccccccc}
1 & 2 & 5 & 6 & 7 & 3 & 4 \\
2 & 1 & 6 & 7 & 3 & 4 & 5 \\
3 & 7 & 1 & 5 & 4 & 6 & 2 \\
4 & 3 & 2 & 1 & 6 & 5 & 7 \\
5 & 4 & 3 & 2 & 1 & 7 & 6 \\
6 & 5 & 7 & 4 & 2 & 1 & 3 \\
7 & 6 & 4 & 3 & 5 & 2 & 1.
\end{array}
$$

It is easy to verify the equality

$$R(12\cdot 36475)(12\cdot 34567) = Q,$$

in which the first factor is a permutation of columns and the second a substitution of numbers.

Thus the number of reduced squares produced by Q is

$$6(7!)^3/(3\cdot 5\cdot 7!\,6!) = 14{,}112,$$

which is precisely the difference mentioned in Section 1.

4. The reversal of the unique intercalate 12 in Q gives a square S isomorphic with Q, and on interchanging rows and columns and the numbers 1 and 2 in S, we obtain Q again. Thus S is one of the 14,112 squares considered in Section 3 and does not give a new species. Therefore, Norton's conjecture ([2], p. 291) "that they can be enumerated by an exhaustive reversal of intercalates" is not borne out, at least for species with one intercalate. This assumption was founded on its truth for 6×6 squares; but it is to be expected that the classification of $n \times n$ squares would become more complicated with increasing n.

5. On the contrary, the conclusion of S. G. Ghurye [3] is confirmed, for the square Q possesses a different "$I - A$" from those of other species.

REFERENCES

[1] T. B. Sprague, "A new algebra, by means of which permutations can be transformed in a variety of ways, and their properties investigated," *Trans. Roy. Soc. Edinburgh*, Vol. 37 (1892), pp. 399–411.

[2] H. W. Norton, "The 7 × 7 squares," *Annals of Eugenics*, Vol. 9 (1939), pp. 269–307.

[3] S. G. Ghurye, "A characteristic of species of 7 × 7 Latin squares," *Annals of Eugenics*, Vol. 14 (1948), p. 133.

[4] A. Sade, *Enumération des carrés latins. Application au 7ème ordre. Conjectures pour les ordres supérieurs*, privately published.