A TABLE FOR COMPUTING TRIVARIATE NORMAL PROBABILITIES

By George P. Steck

Sandia Corporation

1. Introduction. For convenience in the following discussion let X, Y, and Z be random variables with a trivariate normal distribution such that $EX = EY = EZ = 0$, $EX^2 = EY^2 = EZ^2 = 1$, $EXY = \rho_{12}$, $EXZ = \rho_{13}$, $EYZ = \rho_{23}$, let $C(h, k, m; \rho_{12}, \rho_{13}, \rho_{23})$ denote the probability that $X \leqq h$, $Y \leqq k$, $Z \leqq m$, and let $D(h, k, m; \rho_{12}, \rho_{13}, \rho_{23})$ denote the probability that $X \geqq h$, $Y \geqq k$, $Z \geqq m$. Several tables have been prepared from which certain particular values of the trivariate normal integral can be obtained. A tabulation of the area of hyperspherical simplices is given by H. Ruben [1]. The function Ruben has tabulated as $\bar{u}_n(x)$ is, for the case $n = 3$, equal to $C(0, 0, 0; 1/x, 1/x, 1/x)$ and the tabulation is for $x = 2(1)11$. This probability can be computed directly, however, as a special case of the well-known formula (for example, see [2]).

$$C(0, 0, 0; \rho_{12}, \rho_{13}, \rho_{23}) = D(0, 0, 0; \rho_{12}, \rho_{13}, \rho_{23})$$

$$= \frac{1}{4\pi}(2\pi - \arccos \rho_{12} - \arccos \rho_{13} - \arccos \rho_{23}) \tag{1.1}$$

Short tabulations of $C(h, h, h; 1/2, 1/2, 1/2)$ have been published by D. Teichroew [3] for $h\sqrt{2} = 0(.01)6.09$ and by P. N. Somerville [11] for $h = 0(.1)2(.5)3$. In addition to these published tables, there are some unpublished tables [4] giving $C(h, h, h; \rho, \rho, \rho)$ for $\rho = 1/(1 + \sqrt{3})$ and $\frac{1}{4}$, $h = 0(.1)3(.5)8$ and for $\rho = 0(.1)0.9$, $h = 0(.2)1$.

Methods for computing $D(h, k, m; \rho_{12}, \rho_{13}, \rho_{23})$ have been given by M. G. Kendall [5], R. L. Plackett [6], and S. C. Das [7]. The method of Kendall is to express the trivariate normal density as the inverse of its characteristic function obtaining $D(h, k, m; \rho_{12}, \rho_{13}, \rho_{23})$ as a six-dimensional integral. The part of the integral involving the ρ_{ij} is expanded in a power series and the result integrated term by term. The resulting series coverges slowly, however, when the ρ_{ij} are large. Plackett's method, on the other hand, is to consider $D(h, k, m; \rho_{12}, \rho_{13}, \rho_{23})$ as a function of the ρ_{ij} and write it as a line integral from $(\rho_{12}, \rho_{13}, \overset{*}{\rho}_{23})$ to $(\rho_{12}, \rho_{13}, \rho_{23})$ where $\overset{*}{\rho}_{23}$ is chosen to give a degenerate trivariate normal density so that $D(h, k, m; \rho_{12}, \rho_{13}, \overset{*}{\rho}_{23})$ becomes a bivariate normal integral. The result of this procedure is that $D(h, k, m; \rho_{12}, \rho_{13}, \rho_{23})$ can be expressed as a sum of lower dimensional normal integrals and an integral which must be evaluated by numerical integration.

The method of Das reduces the trivariate integral to a single integral which is then evaluated numerically provided the correlations are such that their product is positive and each is numerically greater than the product of the other two.

In this paper $C(h, k, m; \rho_{12}, \rho_{13}, \rho_{23})$ is expressed in terms of the univariate

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normal integral, the T-function, which is tabulated by D. B. Owen [8, 9], and the function $S(h, a, b)$ which is tabulated here. Although the reduction of $C(h, k, m; \rho_{12}, \rho_{13}, \rho_{23})$ is given in terms of the T-function, it is also possible to give it in terms of the V-function tabulated by C. Nicholson [12] and by the National Bureau of Standards [13], or the L-function tabulated by Karl Pearson [14] and by the National Bureau of Standards [13]. The V and L-functions are related to the T-function by the expressions

$$V(h, ah) = \frac{\arctan a}{2\pi} - T(h, a), \tag{1.2}$$

$$\begin{aligned} L(h, k; \rho) &= \frac{1}{2\pi(1-\rho^2)^{\frac{1}{2}}} \int_h^\infty \int_k^\infty \\ &\qquad \exp\left[-\tfrac{1}{2}(x^2 + y^2 - 2\rho xy)/(1-\rho^2)\right] dx\, dy \\ &= 1 - \tfrac{1}{2}[G(h) + G(k) + \delta_{hk}] - T\left(h, \frac{k - \rho h}{h\sqrt{1-\rho^2}}\right) \\ &\qquad - T\left(k, \frac{h - \rho k}{k\sqrt{1-\rho^2}}\right), \end{aligned} \tag{1.3}$$

where (this is the same δ defined equivalently by (2.3))

$$\delta_{hk} = \begin{cases} & \text{if } h < 0 \text{ or } k < 0 \text{ but not both,} \\ & \text{otherwise.} \end{cases}$$

For $h > 0$, $a > 0$, $b > 0$, $S(h, a, b) - (1/4\pi)\arctan(b/(1 + a^2 + a^2b^2)^{1/2})$ is the probability that three independent, standardized, normal variables will lie in the region between the planes $x = 0$, $x - bz = 0$, $y = 0$, and $y = h$ and beyond (in the sense that $z \geqq ay$) the plane $z - ay = 0$, i.e., will lie in the truncated infinite wedge shown in Figure 1.

2. Summary of formulas. The fundamental formulas for $C(h, k, m; \rho_{12}, \rho_{13}, \rho_{23})$ are:

Case (i): $h \geqq 0, \quad k \geqq 0, \quad m \geqq 0 \quad$ or $\quad h \leqq 0, \quad k \leqq 0, \quad m \leqq 0,$

$$\begin{aligned} C(h, k, m; \rho_{12}, \rho_{13}, \rho_{23}) &= \tfrac{1}{2}[(1 - \delta_{a_1c_1})G(h) + (1 - \delta_{a_2c_2})G(k) \\ &\quad + (1 - \delta_{a_3c_3})G(m)] - \tfrac{1}{2}[T(h, a_1) + T(h, c_1) + T(k, a_2) + T(k, c_2) \\ &\quad + T(m, a_3) + T(m, c_3)] - [S(h, a_1, b_1) + S(h, c_1, d_1) \\ &\quad + S(k, a_2, b_2) + S(k, c_2, d_2) + S(m, a_3, b_3) + S(m, c_3, d_3)], \end{aligned} \tag{2.1}$$

Case (ii): $h \geqq 0, \quad k \geqq 0, \quad m < 0 \quad$ or $\quad h \leqq 0, \quad k \leqq 0, \quad m > 0,$

$$\begin{aligned} C(h, k, m; \rho_{12}, \rho_{13}, \rho_{23}) &= \tfrac{1}{2}[G(h) + G(k) - \delta_{hk}] - T(h, a_1) - T(k, c_2) \\ &\quad - C(h, k, -m; \rho_{12}, -\rho_{13}, -\rho_{23}), \end{aligned} \tag{2.2}$$

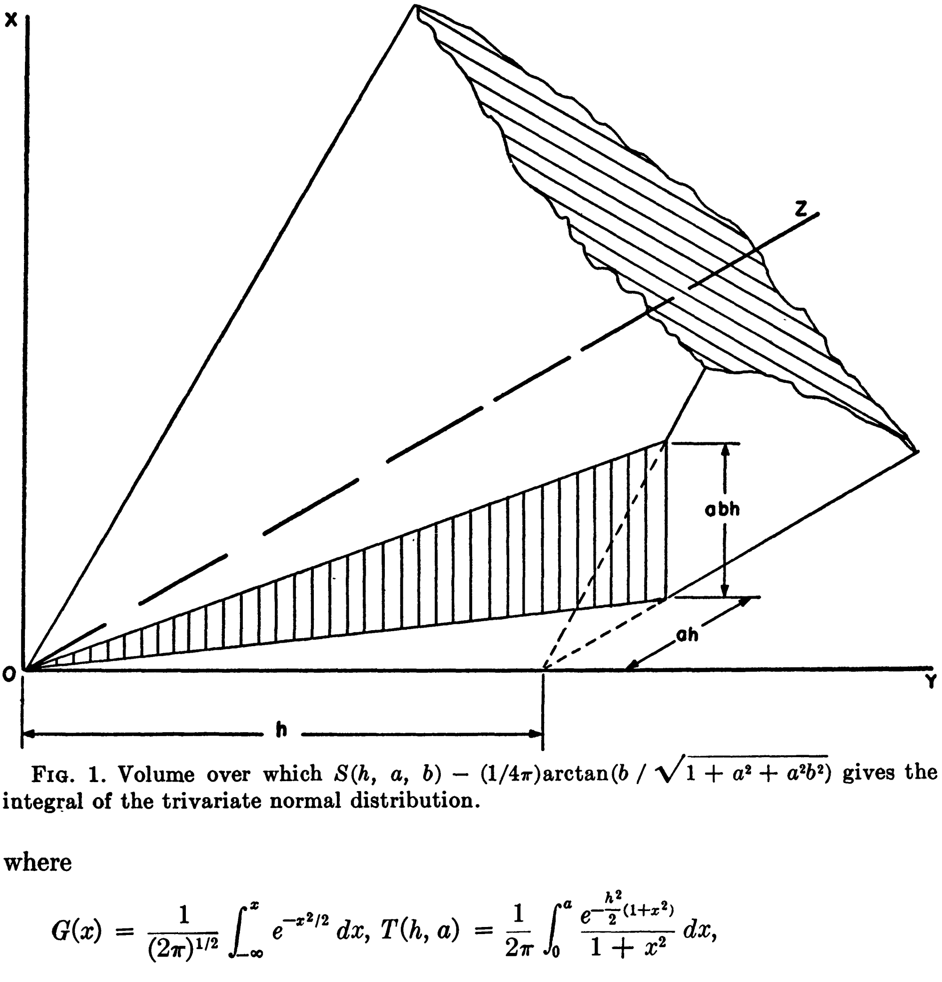

FIG. 1. Volume over which $S(h, a, b) - (1/4\pi)\arctan(b / \sqrt{1 + a^2 + a^2b^2})$ gives the integral of the trivariate normal distribution.

where

$$G(x) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{x} e^{-x^2/2}\, dx, \quad T(h, a) = \frac{1}{2\pi} \int_0^a \frac{e^{-\frac{h^2}{2}(1+x^2)}}{1 + x^2}\, dx,$$

$$a_1 = \frac{k - h\rho_{12}}{h(1 - \rho_{12}^2)^{1/2}}, \quad a_2 = \frac{m - k\rho_{23}}{k(1 - \rho_{23}^2)^{1/2}}, \quad a_3 = \frac{h - m\rho_{13}}{m(1 - \rho_{13}^2)^{1/2}},$$

$$c_1 = \frac{m - h\rho_{13}}{h(1 - \rho_{13}^2)^{1/2}}, \quad c_2 = \frac{h - k\rho_{12}}{k(1 - \rho_{12}^2)^{1/2}}, \quad c_3 = \frac{k - m\rho_{23}}{m(1 - \rho_{23}^2)^{1/2}},$$

$$b_1 = \frac{(1 - \rho_{12}^2)(m - h\rho_{13}) - (\rho_{23} - \rho_{12}\rho_{13})(k - h\rho_{12})}{(k - h\rho_{12})\Delta^{1/2}},$$

$$(2.3) \quad d_1 = \frac{(1 - \rho_{13}^2)(k - h\rho_{12}) - (\rho_{23} - \rho_{12}\rho_{13})(m - h\rho_{13})}{(m - h\rho_{13})\Delta^{1/2}},$$

$$b_2 = \frac{(1 - \rho_{23}^2)(h - k\rho_{12}) - (\rho_{13} - \rho_{12}\rho_{23})(m - k\rho_{23})}{(m - k\rho_{23})\Delta^{1/2}},$$

$$d_2 = \frac{(1 - \rho_{12}^2)(m - k\rho_{23}) - (\rho_{13} - \rho_{12}\rho_{23})(h - k\rho_{12})}{(h - k\rho_{12})\Delta^{1/2}},$$

$$b_3 = \frac{(1 - \rho_{13}^2)(k - m\rho_{23}) - (\rho_{12} - \rho_{13}\,\rho_{23})(h - m\rho_{13})}{(h - m\rho_{13})\Delta^{1/2}},$$

$$d_3 = \frac{(1 - \rho_{23}^2)(h - m\rho_{13}) - (\rho_{12} - \rho_{13}\,\rho_{23})(k - m\rho_{23})}{(k - m\rho_{23})\Delta^{1/2}},$$

$$\Delta = 1 - \rho_{12}^2 - \rho_{13}^2 - \rho_{23}^2 + 2\rho_{12}\,\rho_{13}\,\rho_{23},$$

$$\delta_{xy} = \begin{cases} 0 \text{ if } (\operatorname{sgn} x)(\operatorname{sgn} y) = 1 \\ +1 \text{ otherwise} \end{cases},$$

and

$$\operatorname{sgn} x = \begin{cases} 1\; x \geqq 0 \\ -1\; x < 0 \end{cases}$$

The S-function is tabulated for $0 < b \leqq 1$, but it is possible to obtain values for $1 < b < \infty$ by use of one of the following formulas, $a > 0$, $b > 0$:

$$S(h, a, b) = [G(h) - \tfrac{1}{2}]\, T(ah, b) - [G(hab) - \tfrac{1}{2}]\, T(ah, 1/a) + S(hab, 1/b, 1/a), \tag{2.4}$$

$$S(h, a, b) = (\tfrac{1}{4})G(h) + [G(hab) - \tfrac{1}{2}]\, T(h, a) - S(hab, 1/ab, a) - S(h, ab, 1/b). \tag{2.5}$$

if $a > 1$, $b > 1$ then (2.4) should be used, and if $0 < a \leqq 1$, $b > 1$ then (2.5) should be used. Values for negative h, a, or b may be obtained by using

$$S(-h, a, b) = S(\infty, a, b) - S(h, a, b), \tag{2.6}$$

$$S(h, -a, b) = S(h, a, b), \tag{2.7}$$

$$S(h, a, -b) = -S(h, a, b). \tag{2.8}$$

Note that (2.4) and (2.5) require both a and b to be positive and hence when a or b is negative (2.7) or (2.8) should be applied before (2.4) or (2.5).

Other useful formulas are:

$$\begin{aligned} &S(0, a, b) = \tfrac{1}{2}\, S(\infty, a, b), \qquad S(h, 0, b) = \frac{1}{2\pi}\, G(h) \arctan b, \\ &S(h, a, 0) = 0, \qquad S(\infty, a, b) = \frac{1}{2\pi} \arctan\left[\frac{b}{(1 + a^2 + a^2b^2)^{1/2}}\right], \\ &S(h, \infty, b) = 0, \\ &S(h, a, \infty) = \begin{cases} \tfrac{1}{2}[\tfrac{1}{2}G(h) + T(h, |a|)] - \dfrac{\arctan |a|}{2\pi}, & h \geqq 0 \\ \tfrac{1}{2}[\tfrac{1}{2}G(h) - T(h, |a|)], & h < 0. \end{cases} \end{aligned} \tag{2.9}$$

Equations (2.1) and (2.2) can be easily rewritten in terms of the V-function by use of (1.2); however, in order to reduce the computation it should be noted that

$$\arctan a_1 + \arctan c_2 = \arctan \left(\sqrt{1 - \rho_{12}^2}/\rho_{12}\right).$$

Similar expressions hold for the pairs (a_2, c_3) and (a_3, c_1).

Rewriting equations (2.1) and (2.2) in terms of the L-function gives

Case (i): $h \geqq 0, \quad k \geqq 0, \quad m \geqq 0 \quad \text{or} \quad h \leqq 0, \quad k \leqq 0, \quad m \leqq 0,$

$$\begin{aligned} C(h, k, m; \rho_{12}, \rho_{13}, \rho_{23}) &= (1 - \tfrac{1}{2}\delta_{a_1c_1})G(h) + (1 - \tfrac{1}{2}\delta_{a_2c_2})G(k) \\ &\quad + (1 - \tfrac{1}{2}\delta_{a_3c_3})G(m) + \tfrac{1}{4}(\delta_{hk} + \delta_{hm} + \delta_{km}) \\ &\quad + \tfrac{1}{2}[L(h, k; \rho_{12}) + L(h, m; \rho_{13}) + L(k, m; \rho_{23}) - 3] \\ &\quad - [S(h, a_1, b_1) + S(h, c_1, d_1) + S(k, a_2, b_2) + S(k, c_2, d_2) \\ &\quad + S(m, a_3, b_3) + S(m, c_3, d_3)], \end{aligned} \tag{2.1$'$}$$

Case (ii): $h \geqq 0, \quad k \geqq 0, \quad m < 0 \quad \text{or} \quad h \leqq 0, \quad k \leqq 0, \quad m > 0,$

$$\begin{aligned} C(h, k, m; \rho_{12}, \rho_{13}, \rho_{23}) &= L(h, k; \rho_{12}) + G(h) + G(k) - 1 \\ &\quad - C(h, k, -m; \rho_{12}, -\rho_{13}, -\rho_{23}). \end{aligned} \tag{2.2$'$}$$

3. Derivation of the relationship between the trivariate normal integral and the tabulated function. The density function for the standardized trivariate normal distribution is

$$\begin{aligned} f(x, y, z; \rho_{12}, \rho_{13}, \rho_{23}) &= \left(\frac{1}{2\pi}\right)^{3/2} \frac{1}{\Delta^{1/2}} \exp\left[-\tfrac{1}{2}(A_{11}x^2 + A_{22}y^2\right. \\ &\quad \left. + A_{33}z^2 + 2A_{12}xy + 2A_{13}xz + 2A_{23}yz)\right], \end{aligned} \tag{3.1}$$

where

$$\begin{aligned} A_{11} &= \frac{1 - \rho_{23}^2}{\Delta}, \quad A_{22} = \frac{1 - \rho_{13}^2}{\Delta}, \quad A_{33} = \frac{1 - \rho_{12}^2}{\Delta}, \\ A_{12} &= \frac{\rho_{13}\rho_{23} - \rho_{12}}{\Delta}, \quad A_{13} = \frac{\rho_{12}\rho_{23} - \rho_{13}}{\Delta}, \quad A_{23} = \frac{\rho_{12}\rho_{13} - \rho_{23}}{\Delta}, \end{aligned} \tag{3.2}$$

and

$$\Delta = 1 - \rho_{12}^2 - \rho_{13}^2 - \rho_{23}^2 + 2\rho_{12}\rho_{13}\rho_{23}.$$

The definition of $C(h, k, m; \rho_{12}, \rho_{13}, \rho_{23})$ given earlier is equivalent to

$$C(h, k, m; \rho_{12}, \rho_{13}, \rho_{23}) = \int_{-\infty}^{h} \int_{-\infty}^{k} \int_{-\infty}^{m} f(x, y, z; \rho_{12}, \rho_{13}, \rho_{23})\, dx\, dy\, dz. \tag{3.3}$$

Let $G(x)$, $T(h, a)$ be as defined in (2.3). It will be convenient to have an alternative form of $T(h, a)$. This is given in [8] and is

$$T(h, a) = \frac{\arctan a}{2\pi} + \tfrac{1}{2}G(h) - \tfrac{1}{4} - \int_0^h G(ax)G'(x)\, dx. \tag{3.4}$$

Also, from [9],

$$(3.5)\quad T(h, a) = \tfrac{1}{2}[G(h) + G(ah)] - G(h)G(ah) - T(ah, 1/a),\ a > 0.$$

Finally, let

$$(3.6)\qquad S(h, a, b) = \int_{-\infty}^{h} T(as, b)G'(s)\, ds.$$

It will also be convenient to have an alternative form of (3.6). If the T-function is replaced by its integral representation as given by (2.3) and the order of integrations reversed, the result is

$$(3.7)\qquad S(h, a, b) = \frac{b}{2\pi}\int_0^1 \frac{G[h(1 + a^2 + a^2b^2y^2)^{1/2}]}{(1 + b^2y^2)(1 + a^2 + a^2b^2y^2)^{1/2}}\, dy.$$

Integration of (3.6) by parts gives (2.4), and substituting (3.5) into (3.6) and integrating gives (2.5).

The relation between $C(h, k, m; \rho_{12}, \rho_{13}, \rho_{23})$ and the S-function can be shown as follows. If h, k, and m are all nonnegative (or nonpositive) and if $0/0$ is taken as one, then it can be shown that

$$(3.8)\qquad \begin{aligned} P(X \leqq h, Y \leqq k, Z \leqq m) &= P\left(X \leqq h, Y \leqq \frac{k}{h}X, Z \leqq \frac{m}{h}X\right) \\ &+ P\left(X \leqq \frac{h}{k}Y, Y \leqq k, Z \leqq \frac{m}{k}Y\right) + P\left(X \leqq \frac{h}{m}Z, Y \leqq \frac{k}{m}Z, Z \leqq m\right). \end{aligned}$$

Since these three probabilities are all similar in form, it is sufficient to consider only the last. Let the conditional probability, given $Z = s$, that $X \leqq hs/m$ and $Y \leqq ks/m$ be denoted by $A(s)$; then

$$(3.9)\quad A(s) = B\left(\frac{h - m\rho_{13}}{m(1 - \rho_{13}^2)^{1/2}}\, s,\ \frac{k - m\rho_{23}}{m(1 - \rho_{23}^2)^{1/2}}\, s;\ \frac{\rho_{12} - \rho_{13}\rho_{23}}{((1 - \rho_{13}^2)(1 - \rho_{23}^2))^{1/2}}\right).$$

where

$$(3.10)\quad B(h, k; \rho) = \frac{1}{2\pi(1 - \rho^2)^{1/2}}\int_{-\infty}^{h}\int_{-\infty}^{k} \cdot \exp\left[-\tfrac{1}{2}(x^2 - 2\rho xy + y^2)/(1 - \rho^2)\right] dx\, dy.$$

Therefore,

$$(3.11)\qquad P\left(X \leqq \frac{h}{m}Z, Y \leqq \frac{k}{m}Z, Z \leqq m\right) = \int_{-\infty}^{m} A(s)G'(s)\, ds.$$

However, it is shown in [9] that

$$(3.12)\qquad \begin{aligned} B(h, k; \rho) = \tfrac{1}{2}[G(h) + G(k)] &- T\left(h, \frac{k - \rho h}{h(1 - \rho^2)^{1/2}}\right). \\ &- T\left(k, \frac{h - \rho k}{k(1 - \rho^2)^{1/2}}\right) - \tfrac{1}{2}\delta_{hk} \end{aligned}$$

where δ_{hk} has already been defined by (2.3); therefore, expressing (3.9) in the form of (3.12), substituting in (3.11), and noting (3.4) it follows that

$$
\begin{aligned}
P\left(X \leqq \frac{h}{m} Z, Y \leqq \frac{k}{m} Z, Z \leqq m\right) &= \tfrac{1}{2}(1 - \delta_{a_3c_3})G(m) \\
&\quad - \tfrac{1}{2}[T(m, a_3) + T(m, c_3)] - \int_{-\infty}^{m} G'(s)T(a_3 s, b_3)\, ds \\
&\quad - \int_{-\infty}^{m} G'(s)T(c_3 s, d_3)\, ds,
\end{aligned}
\tag{3.13}
$$

where a_3, b_3, d_3, and $\delta_{a_3c_3}$ have already been defined by (2.3). The integrals on the right side of (3.13) are, noting (3.6), $S(m, a_3, b_3)$ and $S(m, c_3, d_3)$. Thus

$$
\begin{aligned}
P\left(X \leqq \frac{h}{m} Z, Y \leqq \frac{k}{m} Z, Z \leqq m\right) &= \tfrac{1}{2}(1 - \delta_{a_3c_3})G(m) \\
&\quad - \tfrac{1}{2}[T(m, a_3) + T(m, c_3)] - [S(m, a_3, b_3) + S(m, c_3, d_3)].
\end{aligned}
\tag{3.14}
$$

The other two probabilities on the right side of (3.8) can be obtained from (3.14) by replacing m, a_3, b_3, c_3, d_3 by h, a_1, b_1, c_1, d_1 and k, a_2, b_2, c_2, d_2, respectively. Summing the expressions for these three probabilities gives (2.1). Equation (2.2) follows by noting that if h, k, and m are nonnegative or nonpositive, then

$$
\begin{aligned}
&C(h, k, m; \rho_{12}, \rho_{13}, \rho_{23}) \\
&\quad = \int_{-\infty}^{h}\int_{-\infty}^{k}\int_{-\infty}^{m} f(x, y, z; \rho_{12}, \rho_{13}, \rho_{23})\, dx\, dy\, dz \\
&\quad = \int_{-\infty}^{h}\int_{-\infty}^{k}\int_{-m}^{\infty} f(x, y, z; \rho_{12}, -\rho_{13}, -\rho_{23})\, dx\, dy\, dz \\
&\quad = \int_{-\infty}^{h}\int_{-\infty}^{k}\left(\int_{-\infty}^{\infty} - \int_{-\infty}^{-m}\right) f(x, y, z; \rho_{12}, -\rho_{13}, -\rho_{23})\, dx\, dy\, dz \\
&\quad = B(h, k; \rho_{12}) - C(h, k, -m; \rho_{12}, -\rho_{13}, -\rho_{23}).
\end{aligned}
$$

The reader can verify that the familiar expression

$$
C(0, 0, 0; \rho_{12}, \rho_{13}, \rho_{23}) = \frac{1}{4\pi}(2\pi - \arccos \rho_{12} - \arccos \rho_{13} - \arccos \rho_{23})
$$

holds when $h = k = m = 0$ is substituted in (2.1).

If the G-function in the integrand of (3.7) is expanded in a Taylor series to three terms with remainder about the point $h(1 + a^2 + a^2(b/2)^2)^{1/2}$, then the following limited expansion can be shown to hold for $S(h, a, b)$.

$$
\begin{aligned}
S(h, a, b) &= \frac{1}{2\pi} G(h(1 + a^2 + a^2(b/2)^2)^{1/2}) \arctan (b/(1 + a^2 + a^2b^2)^{1/2}) \\
&\quad + \frac{h}{2\pi} G'(h(1 + a^2 + a^2(b/2)^2)^{1/2}) \cdot \Delta_1(a, b) \\
&\quad + \frac{h^2}{2!\,2\pi} G''(h(1 + a^2 + a^2(b/2)^2)^{1/2}) \cdot \Delta_2(a, b) \\
&\quad + \frac{\theta h^3}{3!\,2\pi} \sup_{0 \leqq \xi \leqq 1} G'''(h(1 + a^2 + a^2\xi^2)^{1/2}) \cdot \Delta_3(a, b),
\end{aligned}
\tag{3.15}
$$

where $| \theta | \leqq 1$, and

$$\Delta_1(a, b) = \arctan b - \{1 + a^2 + a^2(b/2)^2\}^{1/2} \arctan(b/(1 + a^2 + a^2b^2)^{1/2}),$$

$$\begin{aligned}\Delta_2(a, b) = {} & \{2 + a^2 + a^2(b/2)^2\}\arctan(b/(1 + a^2 + a^2b^2)^{1/2}) \\ & - 2\{1 + a^2 + a^2(b/2)^2\}^{1/2} \arctan b \\ & + a\{\log(ab + (1 + a^2 + a^2b^2)^{1/2}) - \tfrac{1}{2}\log(1 + a^2)\},\end{aligned}$$

$$\begin{aligned}\Delta_3(a, b) = {} & a^2b + \{4 + 3a^2 + 3a^2(b/2)^2\}\arctan b \\ & -\{1 + a^2 + a^2(b/2)^2\}^{3/2}\arctan(b/(1 + a^2 + a^2b^2)^{1/2}) \\ & - 3\{1 + a^2 + a^2(b/2)^2\}^{1/2}\{\arctan(b/(1 + a^2 + a^2b^2)^{1/2}) \\ & + a[\log(ab + (1 + a^2 + a^2b^2)^{1/2}) - \tfrac{1}{2}\log(1 + a^2)]\}.\end{aligned}$$

If the first term of the series is used, the maximum error is one in the fourth decimal place for $h \leqq 2$ and one in the fifth decimal place for $h > 2$, and if the first three terms of this series are used, the maximum error encountered will be less than six in the sixth decimal place (note from (2.9) that the arc-tangent terms in the series can be read from the $h = \infty$ entries in the table).

4. Description of the table. The values of $S(h, a, b)$ given in the table were computed using a seven-point Gaussian quadrature formula on (3.7). The G-function in the integrand of (3.7) was approximated by a formula of C. Hastings (see [10], p. 187). A check of the computations was made for selected parameter values first by using an eight-point Gaussian quadrature formula with an improved method for evaluating $G(x)$ and second by using a sixteen-point Gaussian quadrature formula with the same improved method for evaluating $G(x)$. These two checks agreed with each other to nine decimal places and differed from the initially computed values by at most one in the eighth decimal place. These checks indicate that the tabulated values may occasionally be off by as much as 0.6 in the seventh decimal place because of rounding errors. Any number in the table whose last nonzero digit is a five is followed by a plus or minus sign to indicate that the number should be rounded up or down, respectively, when dropping the five.

The range of parameter values for which the S-function is tabulated was chosen so that outside the table $S(h, a, b)$ may be approximated by the first term of (3.15) with an error not exceeding five in the fifth decimal place.

The accuracy of linear interpolation in the table was checked empirically in the following way. Let Δh, Δa, Δb denote the intervals of tabulation on h, a, b, respectively. The check was performed by computing $S(h + \frac{1}{2}\Delta h, a + \frac{1}{2}\Delta a, b + \frac{1}{2}\Delta b)$ for a systematic selection of h, a, and b. Even though the errors found in this way are not necessarily the maximum errors in the various incremental cubes, it is felt that they are a reasonable approximation to these maximum errors. The errors found varied from about one to thirty in the fifth decimal place, which indicates that linear interpolation anywhere in the table should give an error of less than four or five in the fourth decimal place.

5. A numerical example. In [6] Plackett applies his reduction method to the computation of

$$\begin{aligned} D(-1.2,\ -1.0,\ 0.5;\ 0.7,\ 0.2,\ -0.4) &= C(1.2,\ 1.0,\ -0.5;\ 0.7,\ 0.2,\ -0.4) \\ &= B(1.2, 1.0; 0.7) - C(1.2, 1.0, 0.5; 0.7, -0.2, 0.4). \end{aligned}$$

The numerical values of the constants defined by (2.3) are:

$$\begin{array}{lll} a_1 = 0.1867040 & b_1 = 4.0873367 & h = 1.2 \\ a_2 = 0.1091089 & b_2 = 10.5175180 & k = 1.0 \\ a_3 = 2.6536139 & b_3 = -0.4252646 & m = 0.5 \\ c_1 = 0.6293828 & c_2 = 0.7001401 & c_3 = 1.7457432 \\ d_1 = -0.7470863 & d_2 = 1.3079477 & d_3 = 1.3146897, \end{array}$$

and, therefore, by (2.1)

$$\begin{aligned} C(1.2,\ 1.0,\ 0.5;\ 0.7,\ -0.2,\ 0.4) = {} & \tfrac{1}{2}[G(1.2) + G(1) + G(0.5)] \\ & - \tfrac{1}{2}[T_1(1.2, 0.1867040) + T_2(1.2, 0.6293828) \\ & + T_3(1, 0.1091089) + T_4(1, 0.7001401) \\ & + T_5(0.5, 2.6536139) + T_6(0.5, 1.7457432)] \\ & - [S_1(1.2, 0.1867040, 4.0873367) + S_2(1.2, 0.6293828, -0.7470863) \\ & + S_3(1, 0.1091089, 10.5175180) + S_4(1, 0.7001401, 1.3079477) \\ & + S_5(0.5, 2.6536139, -0.4252646) + S_6(0.5, 1.7457432, 1.3146897)]. \end{aligned}$$

Tables of the G-function give $\frac{1}{2}[G(1.2) + G(1) + G(0.5)] = 1.2088688$, and the tables in [9] or [10] give

$$-\tfrac{1}{2}\sum T_i = -0.2025741, \qquad B(1.2, 1.0; 0.7) = 0.7940171.$$

Applying (2.5) to compute S_1, S_2, and S_3 and (2.4) to compute S_6, one finds

$$\begin{array}{lll} S_1 = 0.1808805, & S_2 = -0.0783075, & S_3 = 0.1927877, \\ S_4 = 0.1016940, & S_5 = -0.0204185, & S_6 = 0.0562510, \end{array}$$

and $\sum S_i = -0.4328872$, giving $C(1.2,\ 1.0,\ 0.5;\ 0.7,\ -0.2,\ 0.4) = 0.5734075$, and $D(-1.2,\ -1.0,\ 0.5;\ 0.7,\ 0.2,\ -0.4) = 0.2206096$.

If the bivariate probability $P(X > -1.0,\ Y > 0.5;\ \rho = -0.559714)$, incorrectly computed by Plackett as 0.587191, is given its correct value of 0.204267, then Plackett's answer is

$$D(-1.2, -1.0, 0.5; 0.7, 0.2, -0.4) = 0.220610,$$

and the answers agree to six decimal places.

6. Extension of method to higher dimensions. Equation (3.8) can be generalized to any number of dimensions giving

$$
\begin{aligned}
P(X_1 \leqq u_1, X_2 \leqq u_2, \cdots, X_n \leqq u_n) \\
&= P\left(X_1 \leqq u_1, X_2 \leqq \frac{u_2}{u_1} X_1, \cdots, X_n \leqq \frac{u_n}{u_1} X_1\right) \\
&\quad + P\left(X_1 \leqq \frac{u_1}{u_2} X_2, X_2 \leqq u_2, \cdots, X_n \leqq \frac{u_n}{u_2} X_2\right) \\
&\quad + \cdots \\
&\quad + P\left(X_1 \leqq \frac{u_1}{u_n} X_n, X_2 \leqq \frac{u_2}{u_n} X_n, \cdots, X_n \leqq u_n\right).
\end{aligned}
\tag{6.1}
$$

provided all the u_i's are nonnegative (or nonpositive) and $0/0$ is taken as one. Each term on the right side of (6.1) is expressible as an integral of a lower dimensional probability, for example,

$$
\begin{aligned}
P\left(X_1 \leqq \frac{u_1}{u_n} X_n, X_2 \leqq \frac{u_2}{u_n} X_n, \cdots, X_n \leqq u_n\right) \\
= \int_{-\infty}^{u_n} P\left(X_1 \leqq \frac{u_1}{u_n} s, \cdots, X_{n-1} \leqq \frac{u_{n-1}}{u_n} s \,\middle|\, X_n = s\right) G'(s)\, ds.
\end{aligned}
\tag{6.2}
$$

Since the three-dimensional normal distribution can be tabulated as a function of three variables, it follows by mathematical induction, using (6.1) and (6.2), that the n-dimensional normal distribution can be tabulated as a function of n variables.

As an example, consider the case $n = 4$. If $EX_i = 0$, $EX_i^2 = 1$, and $EX_iX_j = \rho_{ij}$ then the probability in the integrand of (6.2) can be expressed as

$$
\begin{aligned}
P\left(X_1 \leqq \frac{u_1}{u_4} s, X_2 \leqq \frac{u_2}{u_4} s, X_3 \leqq \frac{u_3}{u_4} s \,\middle|\, X_4 = s\right) \\
= C(\alpha_{41}\, s, \alpha_{42}\, s, \alpha_{43}\, s; \overset{*}{\rho}_{12}, \overset{*}{\rho}_{13}, \overset{*}{\rho}_{23}),
\end{aligned}
\tag{6.3}
$$

where

$$
\alpha_{4i} = \frac{u_i - u_4 \rho_{i4}}{u_4(1 - \rho_{i4}^2)^{1/2}} \qquad \overset{*}{\rho}_{ij} = \frac{\rho_{ij} - \rho_{i4}\rho_{j4}}{[(1 - \rho_{i4}^2)(1 - \rho_{j4}^2)]^{1/2}}.
$$

Therefore, (6.2) can be written as

$$
\begin{aligned}
P\left(X_1 \leqq \frac{u_1}{u_n} X_n, X_2 \leqq \frac{u_2}{u_n} X_n, \cdots, X_n \leqq u_n\right) \\
= \int_{-\infty}^{u_n} C(\alpha_{41}\, s, \alpha_{42}\, s, \alpha_{43}\, s; \overset{*}{\rho}_{12}, \overset{*}{\rho}_{13}, \overset{*}{\rho}_{23}) G'(s)\, ds.
\end{aligned}
$$

If the integrand of (6.4) is expressed by (2.10) and the result integrated, it is apparent that the left side of (6.1) can be expressed in terms of the G-, T-, and S-functions and integrals of the form

$$
R(h, a, b, c) = \int_{-\infty}^{h} S(as, b, c) G'(s)\, ds.
$$

TABLE

	m\b	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
a = 0	0.0	.0079314	.0157082	.0231934	.0302797	.0368959	.0430052	.0486000	.0536942	.0583156	.0625000
	0.1	.0085632	.0169595-	.0250409	.0326917	.0398349	.0464308	.0524713	.0579712	.0629607	.0674785-
	0.2	.0091887	.0181983	.0268700	.0350797	.0427446	.0498224	.0563041	.0622057	.0675597	.0724075-
	0.3	.0098018	.0194126	.0286629	.0374204	.0455968	.0531468	.0600610	.0663565	.0720677	.0772389
	0.4	.0103968	.0205910	.0304029	.0396920	.0483648	.0563731	.0637070	.0703847	.0764426	.0819277
	0.5	.0109685+	.0217233	.0320747	.0418746	.0510243	.0594730	.0672102	.0742550+	.0806461	.0864328
	0.6	.0115123	.0228004	.0336651	.0439508	.0535542	.0624218	.0705426	.0779368	.0846447	.0907184
	0.7	.0120245+	.0238148	.0351629	.0459063	.0559369	.0651990	.0736812	.0814043	.0884106	.0947545+
	0.8	.0125021	.0247607	.0365595-	.0477296	.0581586	.0677886	.0766077	.0846375+	.0919222	.0985181
	0.9	.0129431	.0256340	.0378488	.0494129	.0602097	.0701793	.0793094	.0876224	.0951640	.1019925-
	1.0	.0133460	.0264321	.0390273	.0509514	.0620843	.0723644	.0817788	.0903506	.0981270	.1051681
	1.1	.0137107	.0271543	.0400937	.0523436	.0637808	.0743417	.0840133	.0928194	.1008083	.1080417
	1.2	.0140374	.0278014	.0410491	.0535909	.0653006	.0761132	.0860153	.0950312	.1032104	.1106163
	1.3	.0143272	.0283754	.0418965+	.0546973	.0666487	.0776846	.0877911	.0969931	.1053412	.1129000
	1.4	.0145817	.0288794	.0426408	.0556689	.0678326	.0790645+	.0893505+	.0987160	.1072124	.1149054
	1.5	.0148030	.0293176	.0432878	.0565137	.0688620	.0802643	.0907064	.1002141	.1088394	.1166491
	∞	.0158628	.0314165-	.0463868	.0605595-	.0737918	.0860104	.0972001	.1073884	.1166311	.1250000
a = 0.1	0.0	.0078919	.0156293	.0230749	.0301218	.0366986	.0427685-	.0483239	.0533787	.0579608	.0621060
	0.1	.0085237	.0168805-	.0249224	.0325337	.0396375-	.0461940	.0521951	.0576557	.0626059	.0670844
	0.2	.0091491	.0181192	.0267513	.0349214	.0425469	.0495851	.0560274	.0618896	.0672042	.0720127
	0.3	.0097620	.0193331	.0285436	.0372614	.0453981	.0529084	.0597830	.0660388	.0717105	.0768422
	0.4	.0103567	.0205108	.0302825+	.0395315+	.0481642	.0561325+	.0634264	.0700641	.0760821	.0815273
	0.5	.0109278	.0216419	.0319526	.0417118	.0508208	.0592289	.0669255-	.0739298	.0802803	.0860266
	0.6	.0114708	.0227173	.0335405-	.0437847	.0533466	.0621727	.0702522	.0776049	.0842715-	.0903039
	0.7	.0119819	.0237296	.0350350+	.0457358	.0557238	.0649435-	.0733831	.0810637	.0880277	.0943292
	0.8	.0124582	.0246728	.0364276	.0475538	.0579388	.0675250+	.0763002	.0842862	.0915271	.0980793
	0.9	.0128975-	.0255428	.0377121	.0492307	.0599819	.0699061	.0789907	.0872584	.0947546	.1015378
	1.0	.0132987	.0263373	.0388851	.0507619	.0618475+	.0720803	.0814474	.0899720	.0977012	.1046952
	1.1	.0136613	.0270555+	.0399455+	.0521461	.0635339	.0740456	.0836679	.0924247	.1003644	.1075488
	1.2	.0139859	.0276983	.0408944	.0533848	.0650430	.0758041	.0856547	.0946193	.1027472	.1101018
	1.3	.0142734	.0282677	.0417351	.0544821	.0663798	.0773619	.0874147	.0965631	.1048576	.1123629
	1.4	.0145256	.0287671	.0424724	.0554444	.0675521	.0787280	.0889580	.0982676	.1067081	.1143452
	1.5	.0147446	.0292007	.0431125+	.0562799	.0685699	.0799139	.0902977	.0997470	.1083142	.1160658
	∞	.0157838	.0312585+	.0461499	.0602437	.0733971	.0855369	.0966478	.1067575-	.1159217	.1242121
a = 0.2	0.0	.0077769	.0153993	.0227302	.0296626	.0361251	.0420812	.0475234	.0524654	.0569355-	.0609693
	0.1	.0084086	.0166505-	.0245775+	.0320743	.0390638	.0455066	.0513943	.0567421	.0615802	.0659473
	0.2	.0090338	.0178887	.0264058	.0344611	.0419721	.0488964	.0552251	.0609743	.0661766	.0708734
	0.3	.0096462	.0191014	.0281964	.0367988	.0448204	.0522162	.0589766	.0651189	.0706776	.0756971
	0.4	.0102397	.0202769	.0299320	.0390645-	.0475810	.0554336	.0626123	.0691353	.07[illegible]93	.0803712
	0.5	.0108091	.0214046	.0315968	.0412378	.0502289	.0585196	.066[illegible]2	.0729871	.07[illegible]18	.0848531
	0.6	.0113496	.0224751	.0331773	.0433009	.0527424	.0614486	.069[illegible]	.0766425-	.0[illegible]08	.0891057
	0.7	.0118575+	.0234808	.0346621	.0452390	.0551034	.0641999	.072[illegible]	.0800754	.0[illegible]79	.0930987
	0.8	.0123297	.0244160	.0360426	.0470409	.0572984	.0667574	.075[illegible]	.0832658	.0[illegible]13	.0968087
	0.9	.0127643	.0252766	.0373130	.0486988	.0593178	.0691100	.078[illegible]	.0862001	.[illegible]61	.1002199
	1.0	.0131601	.0260602	.0384697	.0502084	.0611563	.0712518	.080[illegible]	.0888706	.[illegible]42	.1033234
	1.1	.0135168	.0267665+	.0395122	.0515688	.0628129	.0731814	.082[illegible]	.0912757	.[illegible]40	.1061177
	1.2	.0138349	.0273965+	.0404420	.0527819	.0642901	.0749017	.084[illegible]	.0934193	.1013995-	.1086071
	1.3	.0141158	.0279526	.0412627	.0538526	.0655936	.0764195+	.086[illegible]	.0953100	.1034502	.1108019
	1.4	.0143612	.0284384	.0419795+	.0547877	.0667319	.0777448	.087[illegible]	.0969602	.1052396	.1127166
	1.5	.0145733	.0288584	.0425992	.0555960	.0677157	.0788899	.089[illegible]+	.0983855-	.1067848	.1143696
	∞	.0155537	.0307986	.0454604	.0593251	.0722502	.0841624	.0950467	.1049309	.1138709	.1219386

TABLE

	m \ b	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
a = 0.3	0.0	.0075958	.0150376	.0221884	.0289417	.0352265-	.0410064	.0462742	.0510441	.0553443	.0592109
	0.1	.0082275+	.0162886	.0240356	.0313533	.0381650-	.0444315-	.0501448	.0553204	.0599886	.0641885-
	0.2	.0088524	.0175261	.0258628	.0337387	.0410715-	.0478191	.0539730	.0595497	.0645817	.0691109
	0.3	.0094637	.0187369	.0276505-	.0360724	.0439149	.0511330	.0577177	.0636865-	.0690740	.0739249
	0.4	.0100555-	.0199088	.0293806	.0383308	.0466664	.0543395+	.0613407	.0676883	.0734192	.0785807
	0.5	.0106220	.0210306	.0310367	.0404926	.0492998	.0574081	.0648073	.0715169	.0775758	.0830337
	0.6	.0111584	.0220930	.0326049	.0425393	.0517928	.0603127	.0680881	.0751397	.0815081	.0872456
	0.7	.0116609	.0230880	.0340737	.0444560	.0541271	.0630319	.0711590	.0785299	.0851871	.0911853
	0.8	.0121266	.0240100	.0354344	.0462315-	.0562891	.0655498	.0740019	.0816676	.0885913	.0948296
	0.9	.0125533	.0248549	.0366813	.0478582	.0582695+	.0678558	.0766049	.0845398	.0917063	.0981633
	1.0	.0129402	.0256209	.0378116	.0493325-	.0600639	.0699446	.0789620	.0871399	.0945254	.1011792
	1.1	.0132872	.0263078	.0388249	.0506540	.0616721	.0718161	.0810732	.0894679	.0970485-	.1038774
	1.2	.0135950+	.0269170	.0397236	.0518258	.0630976	.0734745-	.0829434	.0915293	.0992818	.1062646
	1.3	.0138651	.0274516	.0405120	.0528534	.0643474	.0749280	.0845819	.0933346	.1012367	.1083533
	1.4	.0140995+	.0279155-	.0411960	.0537449	.0654312	.0761880	.0860018	.0948982	.1029291	.1101606
	1.5	.0143008	.0283137	.0417832	.0545098	.0663609	.0772684	.0872186	.0962377	.1043781	.1117072
	∞	.0151917	.0300752	.0443769	.0578835-	.0704530	.0820128	.0925485-	.1020882	.1106887	.1184219
a = 0.4	0.0	.0073624	.0145715-	.0214912	.0280157	.0340745-	.0396321	.0446818	.0492383	.0533303	.0569943
	0.1	.0079940	.0158224	.0233382	.0304269	.0370126	.0430567	.0485519	.0535140	.0579740	.0619711
	0.2	.0086184	.0170589	.0251638	.0328102	.0399165+	.0464412	.0523765-	.0577392	.0625624	.0668884
	0.3	.0092284	.0182670	.0269474	.0351386	.0427532	.0497471	.0561117	.0618651	.0670425+	.0716889
	0.4	.0098175+	.0194337	.0286699	.0373868	.0454919	.0529383	.0597168	.0658465+	.0713648	.0763193
	0.5	.0103800	.0205475+	.0303140	.0395324	.0481053	.0559828	.0631554	.0696431	.0754853	.0807322
	0.6	.0109108	.0215985-	.0318652	.0415565-	.0505699	.0588534	.0663966	.0732206	.0793668	.0848877
	0.7	.0114059	.0225788	.0333117	.0434436	.0528673	.0615283	.0694159	.0765520	.0829797	.0887540
	0.8	.0118624	.0234825+	.0346452	.0451828	.0549839	.0639918	.0721954	.0796174	.0863027	.0923083
	0.9	.0122785+	.0243061	.0358601	.0467669	.0569111	.0662340	.0747241	.0824049	.0893228	.0955368
	1.0	.0126534	.0250480	.0369542	.0481930	.0586455-	.0682510	.0769976	.0849097	.0920349	.0984343
	1.1	.0129872	.0257085+	.0379282	.0494621	.0601882	.0700441	.0790177	.0871339	.0944417	.1010038
	1.2	.0132810	.0262899	.0387851	.0505783	.0615444	.0716196	.0807915+	.0890856	.0965521	.1032552
	1.3	.0135367	.0267957	.0395304	.0515486	.0627227	.0729877	.0823308	.0907780	.0983807	.1052044
	1.4	.0137566	.0272306	.0401710	.0523823	.0637346	.0741618	.0836508	.0922282	.0999464	.1068719
	1.5	.0139436	.0276002	.0407153	.0530904	.0645935+	.0751576	.0847695+	.0934563	.1012710	.1082813
	∞	.0147248	.0291430	.0429825-	.0560313	.0681490	.0792642	.0893636	.0984766	.1066607	.1139887
a = 0.5	0.0	.0070917	.0140314	.0206845-	.0269462	.0327475-	.0380537	.0428591	.0471792	.0510432	.0544882
	0.1	.0077232	.0152821	.0225312	.0293571	.0356851	.0414777	.0467285+	.0514541	.0556860	.0594641
	0.2	.0083469	.0165173	.0243548	.0317378	.0385857	.0448583	.0505485+	.0556740	.0602686	.0643748
	0.3	.0089552	.0177219	.0261332	.0340591	.0414137	.0481538	.0542716	.0597861	.0647331	.0691579
	0.4	.0095411	.0188821	.0278459	.0362943	.0441361	.0513254	.0578539	.0637415-	.0690261	.0737558
	0.5	.0100984	.0199857	.0294746	.0384195-	.0467239	.0543392	.0612567	.0674972	.0731007	.0781178
	0.6	.0106220	.0210222	.0310042	.0404147	.0491526	.0571667	.0644477	.0710174	.0769178	.0822020
	0.7	.0111077	.0219838	.0324227	.0422646	.0514034	.0597858	.0674020	.0742746	.0804475+	.0859761
	0.8	.0115528	.0228647	.0337219	.0439580	.0534630	.0621811	.0701021	.0772496	.0836691	.0894181
	0.9	.0119556	.0236616	.0348968	.0454890	.0553239	.0643439	.0725385+	.0799320	.0865714	.0925164
	1.0	.0123155-	.0243736	.0359462	.0468556	.0569841	.0662721	.0747089	.0823194	.0891522	.0952688
	1.1	.0126331	.0250018	.0368717	.0480603	.0584466	.0679694	.0766177	.0844171	.0914176	.0976822
	1.2	.0129099	.0255492	.0376778	.0491089	.0597186	.0694444	.0782749	.0862365+	.0933802	.0997708
	1.3	.0131482	.0260202	.0383711	.0500102	.0608111	.0707101	.0796955-	.0877943	.0950587	.1015548
	1.4	.0133508	.0264205	.0389600	.0507752	.0617376	.0717823	.0808976	.0891110	.0964757	.1030589
	1.5	.0135208	.0267565-	.0394539	.0514163	.0625134	.0726792	.0819020	.0902097	.0976565-	.1043106
	∞	.0141834	.0280628	.0413690	.0538924	.0654949	.0761074	.0857182	.0943583	.1020864	.1089764

TABLE

	m \ b	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
a = 0.6	0.0	.0067981	.0134463	.0198118	.0257918	.0313188	.0363597	.0409097	.0449854	.0486166	.0518406
	0.1	.0074295+	.0146968	.0216582	.0282023	.0342559	.0397830	.0447783	.0492594	.0532583	.0568153
	0.2	.0080524	.0159303	.0234794	.0305797	.0371525+	.0431588	.0485928	.0534729	.0578337	.0617181
	0.3	.0086586	.0171307	.0252515-	.0328926	.0399699	.0464415+	.0523010	.0575681	.0622792	.0664801
	0.4	.0092405-	.0182830	.0269522	.0351119	.0426725+	.0495894	.0558556	.0614918	.0665366	.0710386
	0.5	.0097916	.0193741	.0285623	.0372123	.0452293	.0525661	.0592152	.0651982	.0705558	.0753391
	0.6	.0103065+	.0203934	.0300659	.0391730	.0476149	.0553419	.0623460	.0686498	.0742959	.0793379
	0.7	.0107811	.0213326	.0314510	.0409782	.0498100	.0578943	.0652227	.0718186	.0777266	.0830024
	0.8	.0112126	.0221863	.0327094	.0426175+	.0518021	.0602089	.0678290	.0746868	.0808286	.0863123
	0.9	.0115996	.0229518	.0338374	.0440859	.0535851	.0622786	.0701572	.0772463	.0835934	.0892588
	1.0	.0119420	.0236289	.0348345+	.0453832	.0551589	.0641037	.0722080	.0794979	.0860227	.0918443
	1.1	.0122409	.0242198	.0357041	.0465136	.0565290	.0656908	.0739892	.0814509	.0881268	.0940803
	1.2	.0124983	.0247283	.0364522	.0474851	.0577053	.0670517	.0755145-	.0831210	.0899233	.0959866
	1.3	.0127170	.0251601	.0370869	.0483087	.0587013	.0682026	.0768025+	.0845291	.0914355-	.0975884
	1.4	.0129002	.0255218	.0376181	.0489974	.0595332	.0691624	.0778750-	.0856996	.0926903	.0989153
	1.5	.0130516	.0258207	.0380567	.0495653	.0602183	.0699517	.0787555-	.0866589	.0937169	.0999988
	∞	.0135962	.0268926	.0396236	.0515836	.0626377	.0727194	.0818195-	.0899707	.0972331	.1036813
a = 0.7	0.0	.0064941	.0128409	.0189104	.0246019	.0298500	.0346233	.0389184	.0427525+	.0461562	.0491670
	0.1	.0071254	.0140912	.0207563	.027[illegible]	.0327864	.0380458	.0427860	.0470254	.0507967	.0541403
	0.2	.0077473	.0153228	.0225747	.029[illegible]-	.0356783	.0414160	.0465938	.0512314	.0553636	.0590336
	0.3	.0083510	.0165182	.0243393	.031[illegible]	.0384833	.0446838	.0502847	.0553067	.0597867	.0637708
	0.4	.0089282	.0176612	.0260261	.033[illegible]	.0411627	.0478038	.0538069	.0591934	.0640026	.0682832
	0.5	.0094721	.0187379	.0276145+	.035[illegible]	.0436834	.0507373	.0571161	.0628424	.0679574	.0725124
	0.6	.0099770	.0197370	.0290880	.037[illegible]	.0460189	.0534531	.0601772	.0662146	.0716085-	.0764126
	0.7	.0104387	.0206504	.0304344	.039[illegible]+	.0481500-	.0559290	.0629649	.0692822	.0749259	.0799520
	0.8	.0108546	.0214731	.0316465-	.0412126	.0500652	.0581517	.0654646	.0720292	.0778924	.0831125-
	0.9	.0112238	.0222031	.0327212	.0426104	.0517603	.0601165+	.0676713	.0744508	.0805035-	.0858897
	1.0	.0115467	.0228412	.0336601	.0438302	.0532378	.0618269	.0695893	.0765522	.0827654	.0882914
	1.1	.0118249	.0233907	.0344680	.0448787	.0545063	.0632931	.0712308	.0783473	.0846941	.0903353
	1.2	.0120611	.0238569	.0351528	.0457665+	.0555788	.0645307	.0726139	.0798571	.0863129	.0920473
	1.3	.0122586	.0242466	.0357246	.0465069	.0564719	.0655595-	.0737614	.0811070	.0876504	.0934587
	1.4	.0124213	.0245675-	.0361950-	.0471152	.0572043	.0664016	.0746987	.0821259	.0887381	.0946040
	1.5	.0125534	.0248277	.0365762	.0476073	.0577959	.0670804	.0754527	.0829435+	.0896090	.0955188
	∞	.0129882	.0256819	.0378208	.0492038	.0597001	.0692466	.0778367	.0855050+	.0923124	.0983340
a = 0.8	0.0	.0061894	.0122347	.0180090	.0234144	.0283879	.0328998	.0369480	.0405506	.0437383	.0465488
	0.1	.0068205-	.0134846	.0198545-	.0258237	.0313235+	.0363214	.0408145+	.0448222	.0483774	.0515206
	0.2	.0074413	.0147141	.0216696	.0281930	.0342100	.0396850-	.0446147	.0490195+	.0529346	.0564030
	0.3	.0080421	.0159038	.0234256	.0304845+	.0370007	.0429356	.0482856	.0530720	.0573320	.0611118
	0.4	.0086141	.0170361	.0250965-	.0326640	.0396535+	.0460239	.0517708	.0569165-	.0615005+	.0655716
	0.5	.0091497	.0180963	.0266602	.0347027	.0421332	.0489083	.0550229	.0605004	.0653823	.0697199
	0.6	.0096432	.0190727	.0280997	.0365779	.0444123	.0515566	.0580055-	.0637832	.0689334	.0735097
	0.7	.0100904	.0199573	.0294029	.0382744	.0464719	.0539471	.0606941	.0667382	.0721249	.0769102
	0.8	.0104892	.0207456	.0305635+	.0397837	.0483021	.0560683	.0630762	.0693520	.0749428	.0799072
	0.9	.0108389	.0214367	.0315801	.0411042	.0499012	.0579189	.0651507	.0715239	.0773874	.0825017
	1.0	.0111406	.0220326	.0324559	.0422404	.0512751	.0595059	.0669264	.0733646	.0794710	.0847082
	1.1	.0113967	.0225380	.0331980	.0432018	.0524357	.0608441	.0684204	.0751937	.0812161	.0865518
	1.2	.0116105+	.0229597	.0338164	.0440020	.0533998	.0619533	.0696561	.0765379	.0826522	.0880652
	1.3	.0117862	.0233059	.0343234	.0446568	.0541873	.0628574	.0706606	.0776278	.0838137	.0892858
	1.4	.0119221	.0235853	.0347322	.0451839	.0548197	.0635817	.0714634	.0784965-	.0847368	.0902534
	1.5	.0120409	.0238072	.0350564	.0456011	.0553193	.0641523	.0720941	.0791770	.0854580	.0910071
	∞	.0123787	.0244694	.0360180	.0468289	.0567759	.0657996	.0738960	.0811011	.0874767	.0930977

TABLE

	m \ b	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
a = 0.9	0.0	.0058910	.0116416	.0171284	.0222566	.0255555+	.0312275-	.0350415+	.0384264	.0414131	.0440387
	0.1	.0065219	.0128912	.0189733	.0246651	[illegible]	[illegible]	[illegible]	.0426966	.0460505+	.0490086
	0.2	.0071415+	.0141182	.0207848	.0270296	.0327806	.0380012	[illegible]	.0468841	.0505966	.0538788
	0.3	.0077391	.0153014	.0225311	.0293082	.0355551	.0412355+	.0463468	.0509109	.0549652	.0585556
	0.4	.0083051	.0164219	.0241841	.0314638	.0381783	.0442332	[illegible]	.0547081	.0590807	.0629566
	0.5	.0088316	.0174637	.0257204	.0334659	.0406123	[illegible]	[illegible]	.0582196	.0628814	.0670153
	0.6	.0093125-	.0184150+	.0271222	.0352912	.0428201	[illegible]	[illegible]	.0614043	.0663227	.0706839
	0.7	.0097439	.0192680	.0283782	.0369248	.0448106	[illegible]	[illegible]	.0642367	.0693775-	.0739339
	0.8	.0101240	.0200191	.0294832	.0383603	[illegible]	.0540011	.0607115-	.0667074	.0720363	.0767562
	0.9	.0104528	.0206685+	.0304376	.0395984	.0483460	.0557301	[illegible]	.0688212	.0743054	.0791588
	1.0	.0107323	.0212199	.0312470	.0406468	[illegible]	.0571483	[illegible]	.0705949	.0762044	.0811641
	1.1	.0109654	.0216797	.0319210	.0415183	[illegible]	[illegible]	[illegible]	.0720549	.0777631	.0828053
	1.2	.0111565-	.0220561	.0324721	.0422296	.0512152	[illegible]	[illegible]	.0732337	.0790177	.0841223
	1.3	.0113102	.0223588	.0329145-	.0427993	[illegible]	[illegible]	.0675683	.0741674	.0800084	.0851591
	1.4	.0114318	.0225977	.0332632	.0432475-	.0524337	.0607667	.0682421	.0748930	.0807757	.0859595+
	1.5	.0115261	.0227830	.0335331	.0435935-	.0528162	[illegible]	.0687576	[illegible]	.0813588	.0865658
	∞	.0117819	.0232833	.0342569	.0445132	[illegible]	[illegible]	.0700830	[illegible]	.0828261	.0880774
a = 1.0	0.0	.0056037	.0110711	.0162824	[illegible]	.0256041	.0296305+	.0332256	[illegible]	.0392099	.0416667
	0.1	.0062344	.0123202	.0181267	[illegible]	.0285378	.0330498	.0370895-	[illegible]	.0438456	.0466346
	0.2	.0068526	.0135445+	.0199340	[illegible]	.0314113	.0363978	.0408715-	[illegible]	.0483793	.0514912
	0.3	.0074466	.0147206	.0216696	.0281770	.0341679	.0396077	.0444949	[illegible]	.0527159	.0561324
	0.4	.0080060	.0158278	.0233028	[illegible]	.0367583	.0426211	.0478928	[illegible]	.0567729	.0604687
	0.5	.0085224	.0168495+	.0248089	[illegible]	.0391423	.0453911	.0510119	[illegible]	.0604852	.0644297
	0.6	.0089897	.0177735+	.0261700	[illegible]	.0412916	.0478843	.0538145+	[illegible]	.0638079	.0679676
	0.7	.0094041	.0185926	.0273752	[illegible]	.0431893	.0500817	.0562796	[illegible]	.0667171	.0710577
	0.8	.0097644	.0193042	.0284212	[illegible]	.0448307	.0519783	.0584021	.0641299	.0692090	.0736974
	0.9	.0100714	.0199101	.0293106	.0381150-	.0462212	.0535811	.0601912	.0660802	.0712975-	.0759031
	1.0	.0103278	.0204157	.0300518	[illegible]	.0473751	.0549077	.0616675+	.0676847	.0730101	.0777061
	1.1	.0105378	.0208293	.0306571	.0398513	.0483130	[illegible]	.0628603	.0689767	.0743846	.0791482
	1.2	.0107063	.0211608	.0311415+	.0404778	.0490598	[illegible]	.0638039	.0699951	.0754642	.0802770
	1.3	.0108388	.0214213	.0315214	.0409656	.0496423	[illegible]	.0645347	.0707811	.0762943	.0811419
	1.4	.0109410	.0216219	.0318134	.0413395-	.0500873	.0580915+	.0650891	.0713750-	.0769192	.0817907
	1.5	.0110183	.0217733	.0320333	.0416202	.0504203	[illegible]	.0655008	.0718143	.0773798	.0822672
	∞	.0112074	.0221422	.0325647	.0422921	.0512082	[illegible]	.0664511	.0728168	.0784198	.0833333
a = 1.1	0.0	.0053304	.0105287	.0154790	.0200932	.0243158	.0281225-	.0315145+	.0345114	.0371435+	.0394469
	0.1	.0059609	.0117774	.0173227	.0225001	.0272484	.0315404	.0353769	.0387782	.0417772	.0444127
	0.2	.0065776	.0129987	.0191255+	.0248530	.0301143	.0348794	.0391483	.0429427	.0462974	.0492541
	0.3	.0071677	.0141669	.0208493	.0271015+	.0328513	.0380658	.0427443	.0469097	.0505989	.0538565-
	0.4	.0077199	.0152597	.0224610	.0292022	.0354059	.0410363	.0460925-	.0505981	.0545923	.0581224
	0.5	.0082254	.0162596	.0239345-	.0311208	.0377359	.0437417	.0491366	.0539454	.0582094	.0619783
	0.6	.0086780	.0171544	.0252517	.0328337	.0398129	.0461488	.0518394	.0569106	.0614059	.0653776
	0.7	.0090744	.0179375+	.0264033	.0343288	.0416222	.0482410	.0541829	.0594750+	.0641626	.0683006
	0.8	.0094140	.0186079	.0273877	.0356045-	.0431627	.0500178	.0561675+	.0616400	.0664827	.0707528
	0.9	.0096985+	.0191690	.0282105-	.0366686	.0444445-	.0514920	.0578089	.0634247	.0683887	.0727603
	1.0	.0099318	.0196285-	.0288830	.0375364	.0454868	.0526870	.0591349	.0648613	.0699172	.0743643
	1.1	.0101188	.0199964	.0294205+	.0382282	.0463152	.0536335+	.0601813	.0659906	.0711141	.0756155+
	1.2	.0102654	.0202846	.0298406	.0387674	.0469588	.0543661	.0609880	.0668576	.0720293	.0765686
	1.3	.0103779	.0205053	.0301617	.0391783	.0474474	.0549201	.0615955+	.0675078	.0727129	.0772777
	1.4	.0104623	.0206706	.0304016	.0394843	.0478101	.0553295+	.0620426	.0679842	.0732117	.0777931
	1.5	.0105242	.0207918	.0305770	.0397073	.0480731	.0556253	.0623640	.0683253	.0735673	.0781592
	∞	.0106608	.0210574	.0309581	.0401864	.0486316	.0562450-	.0630291	.0690227	.0742870	.0788938

TABLE

	m\b	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
a = 1.2	0.0	.0050726	.0100174	.0147226	.0191032	.0231063	.0267093	.0299142	.0327405+	.0352184	.0373829
	0.1	.0057028	.0112656	.0165655+	.0215091	.0260377	.0301258	.0337748	.0370054	.0398499	.0423462
	0.2	.0063179	.0124836	.0183634	.0238554	.0288953	.0334549	.0375348	.0411567	.0443553	.0471713
	0.3	.0069037	.0136432	.0200743	.0260868	.0316111	.0366157	.0411010	.0450898	.0486188	.0517314
	0.4	.0074481	.0147206	.0216627	.0281566	.0341270	.0395401	.0443955-	.0487171	.0525437	.0559217
	0.5	.0079419	.0156972	.0231013	.0300287	.0363993	.0421765-	.0473596	.0519736	.0560594	.0596659
	0.6	.0083790	.0165610	.0243723	.0316802	.0383999	.0444926	.0499572	.0548199	.0591235+	.0629198
	0.7	.0087566	.0173066	.0254678	.0331011	.0401172	.0464755+	.0521747	.0572421	.0617227	.0656707
	0.8	.0090749	.0179345-	.0263889	.0342932	.0415543	.0481299	.0540187	.0592492	.0638686	.0679335+
	0.9	.0093368	.0184504	.0271445+	.0352686	.0427268	.0494751	.0555126	.0608690	.0655936	.0697452
	1.0	.0095470	.0188642	.0277491	.0360471	.0436594	.0505411	.0566917	.0621422	.0669438	.0711575-
	1.1	.0097117	.0191879	.0282211	.0366529	.0443826	.0513646	.0575986	.0631171	.0679731	.0722296
	1.2	.0098377	.0194350-	.0285805+	.0371129	.0449295+	.0519845+	.0582783	.0638445+	.0687376	.0730225+
	1.3	.0099317	.0196191	.0288476	.0374534	.0453327	.0524395+	.0587748	.0643733	.0692910	.0735940
	1.4	.0100001	.0197529	.0290411	.0376992	.0456226	.0527651	.0591283	.0647481	.0696813	.0739956
	1.5	.0100487	.0198478	.0291780	.0378724	.0458258	.0529922	.0593737	.0650069	.0699498	.0742707
	∞	.0101452	.0200349	.0294451	.0382063	.0462126	.0534185+	.0598284	.0654811	.0704368	.0747658
a = 1.3	0.0	.0048308	.0095382	.0140142	.0181773	.0219768	.0253916	.0284245+	.0310950+	.0334325+	.0354713
	0.1	.0054608	.0107859	.0158564	.0205821	.0249068	.0288066	.0322834	.0353578	.0380617	.0404320
	0.2	.0060741	.0120003	.0176489	.0229212	.0277556	.0321249	.0360308	.0394949	.0425511	.0452393
	0.3	.0066553	.0131507	.0193460	.0251343	.0304483	.0352582	.0395650+	.0433914	.0467735+	.0497541
	0.4	.0071914	.0142114	.0209095+	.0271709	.0329230	.0381332	.0428021	.0469535-	.0506256	.0538638
	0.5	.0076728	.0151633	.0223111	.0289939	.0351341	.0406967	.0456818	.0501142	.0540344	.0574904
	0.6	.0080937	.0159947	.0235337	.0305812	.0370551	.0429181	.0481699	.0528367	.0569611	.0605938
	0.7	.0084518	.0167015+	.0245714	.0319256	.0386777	.0447887	.0502581	.0551135+	.0593996	.0631695+
	0.8	.0087485-	.0172864	.0254284	.0330330	.0400104	.0463197	.0519608	.0569625-	.0613715+	.0652439
	0.9	.0089877	.0177573	.0261171	.0339205+	.0410746	.0475376	.0533095+	.0584206	.0629198	.0668653
	1.0	.0091755+	.0181265+	.0266557	.0346123	.0419011	.0484793	.0543477	.0595378	.0641004	.0680960
	1.1	.0093190	.0184082	.0270655-	.0351369	.0425252	.0491872	.0551242	.0603693	.0649749	.0690035-
	1.2	.0094258	.0186174	.0273689	.0355238	.0429835-	.0497045+	.0556888	.0609707	.0656043	.0696537
	1.3	.0095031	.0187685+	.0275875+	.0358015-	.0433108	.0500720	.0560878	.0613936	.0660448	.0701067
	1.4	.0095576	.0188749	.0277408	.0359953	.0435381	.0503259	.0563620	.0616826	.0663444	.0704137
	∞	.0096616	.0190765-	.0280285-	.0363545+	.0439535+	.0507832	.0568491	.0621900	.0668650+	.0709426
a = 1.4	0.0	.0046050-	.0090908	.0133534	.0173144	.0209255+	.0241670	.0270421	.0295701	.0317800	.0337049
	0.1	.0052347	.0103379	.0151947	.0197181	.0238542	.0275802	.0308989	.0338307	.0364066	.0386628
	0.2	.0058460	.0115485-	.0169814	.0220495+	.0266933	.0308871	.0346329	.0379524	.0408788	.0434509
	0.3	.0064223	.0126890	.0186637	.0242429	.0293614	.0339909	.0381329	.0418099	.0450575-	.0479173
	0.4	.0069495+	.0137320	.0202008	.0262444	.0317923	.0368136	.0413092	.0453029	.0488323	.0519418
	0.5	.0074179	.0146579	.0215636	.0280158	.0339393	.0393007	.0441004	.0483634	.0521295+	.0554459
	0.6	.0078219	.0154557	.0227359	.0295366	.0357778	.0414240	.0464755+	.0509585+	.0549150+	.0583949
	0.7	.0081602	.0161229	.0237146	.0308029	.0373041	.0431806	.0484329	.0530884	.0571915+	.0607947
	0.8	.0084352	.0166646	.0245074	.0318258	.0385326	.0445888	.0499952	.0547808	.0589920	.0626837
	0.9	.0086522	.0170914	.0251306	.0326272	.0394913	.0456829	.0512033	.0560830	.0603703	.0641230
	1.0	.0088185-	.0174179	.0256060	.0332363	.0402167	.0465068	.0521084	.0570534	.0613923	.0651848
	1.1	.0089422	.0176603	.0259578	.0336853	.0407490	.0471082	.0527654	.0577540	.0621263	.0659437
	1.2	.0090315-	.0178350-	.0262105+	.0340065-	.0411278	.0475338	.0532278	.0582443	.0626372	.0664695-
	1.3	.0090941	.0179572	.0263867	.0342292	.0413892	.0478258	.0535431	.0585769	.0629820	.0668227
	1.4	.0091367	.0180401	.0265058	.0343792	.0415641	.0480200	.0537517	.0587956	.0632077	.0670530
	∞	.0092099	.0181816	.0267068	.0346288	.0418510	.0483340	.0540842	.0591403	.0635600	.0674097

TABLE

	m \ b	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
a = 1.5	0.0	.0043945-	.0086741	.0127383	.0165120	.0199490	.0230309	.0257612	.0281591	.0302527	.0320743
	0.1	.0050239	.0099206	.0145787	.0189145+	.0228762	.0264423	.0296159	.0324172	.0348766	.0370291
	0.2	.0056332	.0111270	.0163593	.0212377	.0257050+	.0297368	.0333355+	.0365225-	.0393303	.0417969
	0.3	.0062042	.0122570	.0180258	.0234101	.0283470	.0328094	.0367991	.0403386	.0434626	.0462118
	0.4	.0067221	.0132814	.0195350+	.0253745+	.0307317	.0355769	.0399115-	.0437589	.0471563	.0501470
	0.5	.0071769	.0141802	.0208572	.0270922	.0328120	.0379845-	.0426108	.0467155+	.0503380	.0535243
	0.6	.0075636	.0149434	.0219780	.0285447	.0345659	.0400075-	.0448704	.0491806	.0529797	.0563165+
	0.7	.0078817	.0155706	.0228971	.0297324	.0359951	.0416494	.0466964	.0511636	.0550946	.0585413
	0.8	.0081352	.0160694	.0236263	.0306716	.0371208	.0429368	.0481212	.0527029	.0567279	.0602506
	0.9	.0083306	.0164535-	.0241861	.0313899	.0379779	.0439121	.0491947	.0538564	.0579452	.0615178
	1.0	.0084765+	.0167396	.0246018	.0319211	.0386087	.0446260	.0499761	.0546911	.0588211	.0624248
	1.1	.0085820	.0169459	.0249005+	.0323012	.0390575-	.0462115+	.0505255-	.0552745+	.0594299	.0630521
	1.2	.0086557	.0170898	.0251082	.0325641	.0393662	.0454763	.0508988	.0556686	.0598388	.0634713
	1.3	.0087057	.0171871	.0252479	.0327400	.0395716	.0457044	.0511439	.0559258	.0601044	.0637424
	∞	.0087890	.0173481	.0254767	.0330241	.0398981	.0460617	.0515223	.0563181	.0605053	.0641485+
a = 1.6	0.0	.0041986	.0082864	.0121665+	.0157667	.0190429	.0219777	.0245751	.0268539	.0288415+	.0305692
	0.1	.0048277	.0095322	.0140059	.0181679	.0219684	.0253872	.0284275+	.0311094	.0334625+	.0355208
	0.2	.0054348	.0107342	.0157799	.0204824	.0247863	.0286685+	.0321319	.0351973	.0378967	.0402669
	0.3	.0060002	.0118531	.0174298	.0226326	.0274007	.0317081	.0355570	.0389696	.0419799	.0446276
	0.4	.0065084	.0128580	.0189098	.0245581	.0297370	.0344179	.0386025+	.0423141	.0455890	.0484696
	0.5	.0069491	.0137287	.0201900	.0262202	.0317483	.0367434	.0412070	.0451636	.0486518	.0517168
	0.6	.0073180	.0144566	.0212581	.0276031	.0334161	.0386644	.0433495-	.0474972	.0511484	.0543511
	0.7	.0076160	.0150437	.0221176	.0287121	.0347485+	.0401922	.0450451	.0493345+	.0531037	.0564035-
	0.8	.0078484	.0155005+	.0227845-	.0295696	.0357739	.0413621	.0463364	.0507259	.0545761	.0579403
	0.9	.0080231	.0158435+	.0232836	.0302086	.0365343	.0422248	.0472830	.0517397	.0556425-	.0590472
	1.0	.0081500-	.0160920	.0236439	.0306677	.0370776	.0428374	.0479510	.0524506	.0563859	.0598145+
	1.1	.0082389	.0162657	.0238947	.0309856	.0374517	.0432565+	.0484050-	.0529307	.0568850-	.0603270
	1.2	.0082990	.0163828	.0240631	.0311980	.0376999	.0435327	.0487022	.0532431	.0572078	.0606568
	1.3	.0083382	.0164589	.0241721	.0313346	.0378586	.0437082	.0488897	.0534389	.0574091	.0608617
	∞	.0083972	.0165727	.0243330	.0315334	.0380858	.0439554	.0491502	.0537078	.0576831	.0611384
a = 1.7	0.0	.0040164	.0079259	.0116351	.0150746	.0182022	.0210015+	.0234768	.0256465-	.0275373	.0291794
	0.1	.0046452	.0091710	.0134735+	.0174744	.0211260	.0244089	.0273268	.0298993	.0321551	.0341276
	0.2	.0052499	.0103684	.0152405+	.0197796	.0239323	.0276764	.0310150-	.0339686	.0365686	.0388506
	0.3	.0058095-	.0114756	.0168729	.0219065+	.0265176	.0306811	.0343996	.0376949	.0406003	.0431544
	0.4	.0063074	.0124601	.0183224	.0237916	.0288036	.0333309	.0373756	.0409606	.0441215+	.0468998
	0.5	.0067337	.0133019	.0195594	.0253964	.0307439	.0355721	.0398830	.0437006	.0470630	.0500144
	0.6	.0070846	.0139940	.0205743	.0267089	.0323249	.0373904	.0419076	.0459021	.0494141	.0524908
	0.7	.0073626	.0145412	.0213745-	.0277400	.0335614	.0388055+	.0434748	.0475965-	.0512133	.0543751
	0.8	.0075743	.0149572	.0219809	.0285183	.0344900	.0398622	.0446380	.0488463	.0525322	.0557482
	0.9	.0077295-	.0152614	.0224227	.0290825+	.0351595+	.0406194	.0454661	.0497303	.0534592	.0567075-
	1.0	.0078389	.0154753	.0227321	.0294756	.0356230	.0411401	.0460317	.0503300	.0540841	.0573505-
	1.1	.0079130	.0156198	.0229403	.0297386	.0359312	.0414839	.0464025-	.0507205+	.0544885+	.0577644
	1.2	.0079613	.0157138	.0230750+	.0299078	.0361281	.0417019	.0466359	.0509647	.0547399	.0580205-
	∞	.0080328	.0158517	.0232703	.0301493	.0364044	.0420030	.0469536	.0512930	.0550745+	.0583588

TABLE

	m\b	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
a = 1.8	0.0	[illegible]	.0075907	.0111413	.0144318	[illegible]	.0200963	.0224591	[illegible]	[illegible]	.0278948
	0.1	[illegible]	.0088351	.0129736	.0168302	[illegible]	.0235015-	.0263066	[illegible]	[illegible]	.0328393
	0.2	[illegible]	.0100276	.0147332	.0191255+	[illegible]	.0267543	.0299777	[illegible]	[illegible]	.0375381
	0.3	[illegible]	.0111225-	.0163522	.0212280	[illegible]	.0297225+	.0333200	[illegible]	[illegible]	.0417824
	0.4	[illegible]	.0120858	.0177700	.0230710	[illegible]	.0323102	.0362240	[illegible]	[illegible]	.0454282
	0.5	[illegible]	.0128930	.0189629	.0246174	[illegible]	.0344655-	.0386324	[illegible]	[illegible]	.0484083
	0.6	[illegible]	.0135541	.0199242	.0258592	[illegible]	.0361809	.0405393	[illegible]	[illegible]	.0507283
	0.7	[illegible]	.0140620	.0206660	.0268137	.0324308	.0374857	.0419811	[illegible]	[illegible]	.0524499
	0.8	[illegible]	.0144388	.0212144	.0275160	.0332668	.0384344	.0430225+	[illegible]	[illegible]	.0536692
	0.9	[illegible]	.0147066	.0216026	.0280105+	.0338518	.0390939	.0437414	.0478250+	[illegible]	.0544941
	1.0	.0075430	.0148890	.0218658	.0283439	.0342436	.0395324	.0442158	.0483261	.0519117	.0550279
	1.1	.0076041	.0150080	.0220368	.0285591	.0344947	.0398112	.0445151	.0486401	.0522357	.0553585+
	1.2	.0076425-	.0150825-	.0221431	.0286921	.0346486	.0399808	.0446959	.0488285-	.0524289	.0555548
	∞	.0076939	.0151813	.0222826	.0288637	.0348439	.0401925+	[illegible]	.0490574	.0526616	.0557896
a = 1.9	0.0	.0036892	.0072788	.0106820	.0138344	.0166973	.0192561	[illegible]	[illegible]	.0252135+	.0267059
	0.1	.0043172	.0085225-	.0125181	.0162312	.0196173	[illegible]	[illegible]	[illegible]	.0298247	.0316466
	0.2	.0049169	.0097098	.0142700	.0185162	.0223985-	[illegible]	[illegible]	[illegible]	.0341932	.0363199
	0.3	.0054639	.0107918	.0158643	.0205931	.0249215-	[illegible]	[illegible]	[illegible]	.0381150-	.0405024
	0.4	.0059403	.0117333	.0172500-	.0223929	.0271015-	[illegible]	[illegible]	[illegible]	.0414524	.0440460
	0.5	.0063366	.0125153	.0183979	.0238798	.0288958	[illegible]	[illegible]	[illegible]	.0441461	.0468906
	0.6	.0066514	.0131354	.0193056	.0250511	.0303028	[illegible]	[illegible]	[illegible]	.0462098	.0490564
	0.7	.0068902	.0136049	.0199905-	.0259308	.0313538	[illegible]	[illegible]	.0443736	.0477115+	.0506220
	0.8	.0070632	.0139442	.0204835-	.0265608	.0321019	[illegible]	[illegible]	.0453637	.0487506	.0516981
	0.9	.0071829	.0141782	.0208221	.0269911	.0326093	[illegible]	.0421059	.0460203	.0494348	.0524022
	1.0	.0072620	.0143324	.0210441	.0272713	.0329374	[illegible]	.0425000-	.0464350+	.0498640	.0528412
	1.1	.0073119	.0144294	.0211829	.0274454	.0331396	[illegible]	.0427389	.0466846	.0501207	.0531024
	∞	.0073784	.0145576	.0213640	.0276688	.0333946	[illegible]	.0430307	.0469856	.0504271	.0534118
a = 2.0	0.0	.0035423	.0069884	[illegible]	.0132786	.0160236	[illegible]	.0206391	.0225316	.0241775-	.0256041
	0.1	.0041699	.0082312	[illegible]	.0156738	.0189416	[illegible]	.0244810	.0267751	.0287849	.0305407
	0.2	.0047669	.0094131	[illegible]	.0179480	.0217092	[illegible]	.0281153	.0307831	.0331294	.0351874
	0.3	.0053072	.0104818	[illegible]	.0199984	.0241991	[illegible]	.0313676	.0343589	.0369930	.0393057
	0.4	.0057724	.0114008	[illegible]	.0217536	.0263239	[illegible]	.0341216	.0373736	.0402351	.0427449
	0.5	.0061533	.0121523	[illegible]	.0231803	.0280439	[illegible]	.0363287	.0397761	.0428039	.0454538
	0.6	.0064501	.0127366	[illegible]	.0242816	.0293648	[illegible]	.0380035-	.0415872	.0447273	.0474686
	0.7	.0066701	.0131687	[illegible]	.0250889	.0303273	[illegible]	.0392071	.0428794	.0460898	.0488858
	0.8	.0068251	.0134725+	[illegible]	.0256509	.0309929	[illegible]	.0400269	.0437526	.0470036	.0498298
	0.9	.0069292	.0136757	[illegible]	.0260225+	.0314297	[illegible]	.0405563	.0443121	.0475848	.0504262
	1.0	.0069955+	.0138049	[illegible]	.0262559	.0317019	[illegible]	.0408806	.0446521	.0479356	.0507841
	1.1	.0070358	.0138830	[illegible]	.0263951	.0319791	.0367516	.0410691	.0448483	.0481368	.0509882
	∞	.0070846	.0139768	[illegible]	.0265573	.0320471	.0369509	.0412781	.0450632	.0483549	.0512082
a = 2.2	0.0	.0032775+	.0064651	.0094846	.0122783	.0148118	.0170727	.0190656	.0208071	.0223201	.0236303
	0.1	.0039042	.0077062	.0113169	.0146700	.0177255-	[illegible]	.0229015-	.0250437	.0269197	.0285583
	0.2	.0044955-	.0088765+	.0130433	.0169211	.0204643	[illegible]	.0264956	.0290059	.0312129	.0331481
	0.3	.0050217	.0099171	.0145760	.0189155-	.0228845+	[illegible]	.0296514	.0324723	.0349543	[illegible]
	0.4	.0054634	.0107895-	.0158580	.0205785+	.0248951	[illegible]	.0322489	.0353104	.0380007	[illegible]
	0.5	.0058134	.0114792	.0168685+	.0218840	.0264654	[illegible]	.0342537	.0374865-	.0403207	[illegible]
	0.6	.0060748	.0119934	.0176190	.0228486	.0276188	[illegible]	.0357053	.0390501	.0419749	[illegible]
	0.7	.0062592	.0123548	.0181442	.0235197	.0284155+	[illegible]	.0366920	.0401042	.0430809	[illegible]
	0.8	.0063817	.0125944	.0184905-	.0239592	.0289332	[illegible]	.0373221	.0407714	.0437753	[illegible]
	0.9	.0064587	.0127442	.0187057	.0242303	.0292498	[illegible]	.0377003	.0411684	.0441854	[illegible]
	1.0	.0065042	.0128324	.0188317	.0243878	.0294319	[illegible]	.0379139	.0413908	.0444134	[illegible]
	∞	.0065550+	.0129302	.0189693	.0245567	.0296237	[illegible]	.0381313	.0416142	.0446402	[illegible]

TABLE

	m\b	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
a = 2.4	0.0	.0030462	.0060082	.0088128	.0114061	.0137561	.0158517	.0176973	.0193088	.0207077	[illegible]
	0.1	.0036720	.0072474	.0106422	.0137939	.0166650+	.0192411	.0215265-	.0235377	.0252988	[illegible]
	0.2	.0042569	.0084052	.0123498	.0160201	.0193727	.0223905+	.0250772	.0274505-	.0295366	[illegible]
	0.3	.0047681	.0094158	.0138377	.0179549	.0217189	.0251099	.0281309	.0308011	.0331490	[illegible]
	0.4	.0051857	.0102400	.0150478	.0195228	.0236116	.0272922	.0305676	.0334583	.0359954	[illegible]
	0.5	.0055046	.0108679	.0159665+	.0207074	.0250332	.0289205-	.0323725-	.0354116	.0380715-	[illegible]
	0.6	.0057323	.0113150+	.0166176	.0215419	.0260277	.0300504	.0336144	.0367440	.0394754	[illegible]
	0.7	.0058842	.0116124	.0170484	.0220902	.0266757	.0307802	.0344089	.0375884	.0403572	[illegible]
	0.8	.0059790	.0117971	.0173144	.0224262	.0270692	.0312189	.0348819	.0380863	.0408727	[illegible]
	0.9	.0060342	.0119044	.0174679	.0226183	.0272919	.0314646	.0351442	.0383599	.0411536	[illegible]
	∞	.0060925-	.0120164	.0176256	.0228121	.0275123	.0317033	.0353946	.0386176	.0414155-	[illegible]
a = 2.6	0.0	.0028431	.0056070	.0082232	.0106411	[illegible]	.0147823	[illegible]	[illegible]	.0192987	.0204231
	0.1	.0034678	.0068442	.0100495+	.0130248	[illegible]	.0181656	[illegible]	[illegible]	.0238805-	.0253312
	0.2	.0040460	.0079885-	.0117370	.0152241	[illegible]	.0212750+	[illegible]	[illegible]	.0280590	.0297943
	0.3	.0045414	.0089676	.0131777	.0170964	[illegible]	.0239017	[illegible]	[illegible]	.0315367	.0334883
	0.4	.0049343	.0097424	.0143143	.0185671	[illegible]	.0259420	[illegible]	[illegible]	.0341814	.0362766
	0.5	.0052226	.0103097	.0151428	.0196332	[illegible]	.0273996	[illegible]	[illegible]	.0360226	.0382008
	0.6	.0054185-	.0106938	.0157010	.0203464	[illegible]	.0283580	[illegible]	[illegible]	.0371986	.0394187
	0.7	.0055416	.0109343	.0160484	.0207869	[illegible]	.0289383	[illegible]	[illegible]	.0378894	.0401279
	0.8	.0056133	.0110737	.0162483	.0210379	[illegible]	.0292621	[illegible]	[illegible]	.0382634	.0405090
	0.9	.0056519	.0111485-	.0163546	.0211701	[illegible]	.0294287	[illegible]	[illegible]	.0384505+	.0406985-
	∞	.0056862	.0112140	.0164464	.0212821	[illegible]	.0295646	[illegible]	[illegible]	.0385974	.0408463
a = 2.8	0.0	.0026637	.0052528	.0077028	.0099662	.0120152	.0138400	.0154454	.0168454	.0180593	.0191087
	0.1	.0032873	.0064877	.0095258	.0123455+	.0149133	.0172167	.0192596	.0210572	.0226311	.0240057
	0.2	.0038584	.0076177	.0111918	.0145163	.0175520	.0202836	.0227144	.0248608	.0267468	.0283993
	0.3	.0043373	.0085639	.0125834	.0163234	.0197396	.0228141	.0255499	.0279648	.0300850-	.0319406
	0.4	.0047050+	.0092889	.0136458	.0176962	.0213914	.0247116	.0276601	.0302562	.0325288	.0345113
	0.5	.0049637	.0097972	.0143870	.0186478	.0225276	.0260054	.0290853	.0317887	.0341474	.0361975+
	0.6	.0051302	.0101233	.0148597	.0192499	[illegible]	.0268080	.0299600	.0327192	.0351199	.0372010
	0.7	.0052285-	.0103148	.0151352	.0195977	[illegible]	.0272615+	.0304486	.0332332	.0356517	.0377449
	0.8	.0052815+	.0104177	.0152821	.0197811	[illegible]	.0274951	.0306972	.0334921	.0359172	.0380144
	∞	.0053274	.0105056	.0154057	.0199325-	[illegible]	.0276801	.0308908	.0336907	.0361186	.0382174
a = 3.0	0.0	.0025044	.0049384	.0072410	.0093675+	[illegible]	.0130049	.0145112	.0158243	.0169624	.0179458
	0.1	.0031268	.0061709	.0090604	.0117420	[illegible]	.0163744	.0183171	.0200266	[illegible]	.0228310
	0.2	.0036903	.0072857	.0107037	.0138827	[illegible]	.0193963	.0217197	.0237709	[illegible]	.0271514
	0.3	.0041521	.0081979	.0120445-	.0156225-	[illegible]	.0218277	.0244409	.0267459	[illegible]	.0305364
	0.4	.0044948	.0088729	.0130326	.0168974	[illegible]	.0235834	.0263890	.0288566	[illegible]	.0328934
	0.5	.0047250+	.0093248	.0136904	.0177399	[illegible]	.0247221	.0276393	.0301965-	[illegible]	.0343583
	0.6	.0048650-	.0095985-	.0140860	.0182421	[illegible]	.0253860	.0283596	.0309595-	[illegible]	.0351750-
	0.7	.0049421	.0097484	.0143009	.0185121	[illegible]	.0257343	.0287329	.0313503	[illegible]	.0355853
	0.8	.0049805-	.0098227	.0144065-	.0186432	[illegible]	.0258990	.0289072	.0315308	[illegible]	.0357718
	∞	.0050088	.0098767	.0144820	.0187350+	[illegible]	.0260098	.0290225-	.0316486	[illegible]	.0358916
a = 3.2	0.0	.0023622	.0046577	.0068289	.0088334	.0106468	.0122605+	.0136789	.0149149	.0159858	.0169109
	0.1	.0029833	.0058876	.0086444	.0112029	.0135326	.0156224	.0174759	.0191071	.0205356	.0217835-
	0.2	.0035388	.0069865+	.0102639	.0133118	.0160942	.0185971	.0208238	[illegible]	.0245154	.0260273
	0.3	.0039831	.0078637	.0115525+	.0149825+	.0181128	.0209270	.0234281	[illegible]	.0275651	.0292534
	0.4	.0043009	.0084893	.0124672	.0161608	.0195255+	.0225432	.0252174	[illegible]	.0296180	.0314029
	0.5	.0045042	.0088880	.0130463	.0169008	.0204039	.0235370	.0263049	[illegible]	.0308367	.0326648
	0.6	.0046205+	.0091149	.0133735-	.0173146	.0208894	.0240794	.0268908	[illegible]	.0314776	.0333215-
	0.7	.0046800	.0092304	.0135384	.0175208	.0211281	.0243425+	.0271712	[illegible]	.0317773	.0336260
	∞	.0047243	.0093153	.0136577	.0176669	.0212936	.0245210	.0273579	[illegible]	.0319717	.0338217

TABLE

	m\b	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
a = 3.4	0.0	.0022346	.0044059	.0064592	.0083546	.0100686	.0115935-	.0129334	.0141006	.0151117	.0159847
	0.1	.0028544	.0056331	.0082708	.0107186	.0129477	.0149473	.0167210	.0182821	.0196494	.0208440
	0.2	.0034015-	.0067154	.0098653	.0127943	.0154680	.0178728	.0200117	.0218994	.0235568	.0250080
	0.3	.0038279	.0075569	.0111007	.0143947	.0173996	.0200995+	.0224975-	.0246095+	.0264594	.0280740
	0.4	.0041212	.0081339	.0119433	.0154784	.0186963	.0215797	.0241324	.0263723	.0283260	.0300239
	0.5	.0042994	.0084828	.0124491	.0161231	.0194592	.0224400	.0250705-	.0273708	.0293703	.0311019
	0.6	.0043949	.0086688	.0127165+	.0164600	.0198528	.0228778	.0255412	.0278652	.0298811	.0316237
	0.7	.0044401	.0087563	.0128410	.0166149	.0200311	.0230731	.0257483	.0280801	.0301007	.0318461
	∞	.0044691	.0088117	.0129184	.0167092	.0201372	.0231870	.0258669	.0282013	.0302233	.0319694
a = 3.6	0.0	.0021195+	.0041789	.0061261	.0079231	.0095478	.0109929	.0122623	.0133678	.0143251	.0151516
	0.1	.0027379	.0054033	.0079334	.0102815-	.0124199	.0143382	.0160400	.0175380	.0188502	.0199969
	0.2	.0032763	.0064682	.0095019	.0123227	.0148972	.0172124	.0192713	.0210879	.0225824	.0240781
	0.3	.0036846	.0072735+	.0106834	.0138519	.0167408	.0193351	.0216376	.0236641	.0254374	.0269839
	0.4	.0039541	.0078032	.0114559	.0148437	.0179252	.0206839	.0231238	.0252625+	.0271259	.0287434
	0.5	.0041090	.0081062	.0118943	.0154009	.0185824	.0214225+	.0239264	.0261138	.0280133	.0296569
	0.6	.0041866	.0082570	.0121103	.0156720	.0188977	.0217716	.0243000	.0265047	.0284157	.0300667
	∞	.0042390	.0083578	.0122522	.0158462	.0190957	.0219858	.0245246	.0267356	.0286503	.0303033
a = 3.8	0.0	.0020154	.0039734	.0058245+	.0075326	.0090766	.0104495+	.0116553	.0127052	.0136141	.0143987
	0.1	.0026322	.0051948	.0076273	.0098850+	.0119412	.0137859	.0154226	.0168635-	.0181259	.0192291
	0.2	.0031616	.0062416	.0091689	.0118904	.0143739	.0166070	.0185924	.0203437	.0218805+	.0232251
	0.3	.0035516	.0070105+	.0102961	.0133479	.0161292	.0186253	.0208391	.0227860	.0244882	.0259713
	0.4	.0037981	.0074945+	.0110010	.0142514	.0172057	.0198484	.0221834	.0242281	.0260078	.0275511
	0.5	.0039318	.0077557	.0113780	.0147291	.0177674	.0204774	.0228644	.0249480	.0267558	.0283190
	0.6	.0039940	.0078764	.0115505-	.0149447	.0180170	.0207524	.0231575+	.0252534	.0270692	.0286373
	∞	.0040307	.0079468	.0116491	.0150652	.0181532	.0208991	.0233107	.0254104	.0272283	.0287974
a = 4.0	0.0	.0019206	.0037865+	.0055504	.0071777	.0086484	.0099559	.0111040	.0121035-	.0129686	.0137152
	0.1	.0025359	.0050048	.0073485-	.0095238	.0115051	.0132829	.0148604	.0162493	.0174663	.0185302
	0.2	.0030560	.0060330	.0088622	.0114922	.0138920	.0160493	.0179669	.0196578	.0211412	.0224385+
	0.3	.0034276	.0067654	.0099350+	.0128781	.0155589	.0179633	.0200944	.0219670	.0236029	.0250269
	0.4	.0036520	.0072056	.0105752	.0136971	.0165326	.0190670	.0213043	.0232616	.0249636	.0264381
	0.5	.0037665-	.0074289	.0108969	.0141035+	.0170088	.0195983	.0218775+	.0238655-	.0255892	.0270786
	∞	.0038413	.0075731	.0111008	.0143554	.0172968	.0199118	.0222081	.0242069	.0259372	.0274304
a = 4.2	0.0	.0018342	.0036160	.0053002	.0068539	.0082578	.0095057	.0106013	.0115548	.0123801	.0130922
	0.1	.0024478	.0048310	.0070934	.0091934	.0111063	.0128228	.0143462	.0156876	.0168632	.0178910
	0.2	.0029583	.0058400	.0085784	.0111238	.0134459	.0155330	.0173877	.0190227	.0204564	.0217097
	0.3	.0033116	.0065359	.0095971	.0124383	.0150250+	.0173436	.0193972	.0212003	.0227741	.0241428
	0.4	.0035149	.0069345-	.0101758	.0131772	.0159015+	.0183346	.0204807	.0223567	.0239865-	.0253973
	0.5	.0036122	.0071240	.0104482	.0135203	.0163022	.0187800	.0209595-	.0228593	.0245056	.0259275+
	∞	.0036684	.0072320	.0106005-	.0137077	.0165155+	.0190113	.0212025+	.0231096	.0247602	.0261844
a = 4.4	0.0	.0017550	.0034598	.0050711	.0065573	.0079001	.0090934	.0101410	.0110527	.0118415+	.0125222
	0.1	.0023669	.0046714	[illegible]	.0088900	.0107400	.0124004	.0138740	.0151719	.0163095-	.0173041
	0.2	.0028675+	.0056607	.0083148	.0107814	.0130313	.0150530	.0168491	.0184318	.0198192	.0210314
	0.3	.0032026	.0063204	.0092707	.0120252	.0145235+	.0167616	.0187423	.0204801	.0219956	.0233125+
	0.4	.0033860	.0066796	.0098604	.0126888	.0153088	.0176471	.0197080	.0215080	.0230707	.0244224
	0.5	.0034681	.0068393	.0100214	.0129762	.0156432	.0180175-	.0201047	.0219231	.0234982	.0248581
	∞	.0035100	.0069196	.0101122	.0131146	.0158001	.0181869	.0202821	.0221053	.0236831	.0250444
a = 4.6	0.0	[illegible]	.0033162	[illegible]	.0062847	.0075714	.0087148	.0097183	.0105915-	.0113470	.0119988
	0.1	[illegible]	.0045242	[illegible]	.0086103	.0104025-	.0120110	.0134389	.0146966	.0157991	.0167631
	0.2	.0027829	.0054934	[illegible]	.0104619	.0126444	.0146049	.0163462	.0178800	.0192239	.0203976
	0.3	[illegible]	.0061174	[illegible]	.0115360	.0140511	.0162132	.0181254	.0198018	.0212625+	.0225308
	0.4	.0032645	.0064396	.0094471	.0122292	.0147513	.0170007	.0189819	.0207111	.0222112	.0235081
	0.5	.0033334	.0065732	[illegible]	.0124631	.0150283	.0173064	.0193081	.0210514	.0225607	.0238634
	∞	.0033844	.0066325-	.0097211	.0125695-	.0151428	.0174295+	.0194366	.0211829	.0226040	.0239976

TABLE

	m \ b	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
a = 4.8	0.0	.0016151	.0031838	.0046664	.0060335+	.0072685-	.0083658	.0093287	.0101665+	.0108914	.0115166
	0.1	.0022234	.0043881	.0064435-	.0083517	.0100903	.0116510	.0130365-	.0142570	.0153270	.0162628
	0.2	.0027036	.0053368	.0078385-	.0101627	.0122819	.0141851	.0158749	.0173628	.0186658	.0198033
	0.3	.0030029	.0059256	.0086981	.0112684	.0136048	.0156952	.0175428	.0191613	.0205704	.0217929
	0.4	.0031502	.0062134	.0091141	.0117962	.0142263	.0163924	.0182989	.0199619	.0214038	.0226496
	∞	.0032301	.0063677	.0093328	.0120670	.0145369	.0167315+	.0186575-	.0203331	.0217828	.0230332
a = 5.0	0.0	.0015530	.0030614	.0044869	.0058012	.0069884	.0080431	.0089687	.0097738	.0104703	.0110711
	0.1	.0021594	.0042619	.0062582	.0081117	.0098007	.0113169	.0126631	.0138491	.0148890	.0157985-
	0.2	.0026291	.0051897	.0076220	.0098815+	.0119412	.0137905-	.0154317	.0168763	.0181408	.0192439
	0.3	.0029111	.0057440	.0084307	.0109203	.0131824	.0152050+	.0169915-	.0185552	.0199158	.0210952
	0.4	.0030422	.0060000	.0087999	.0113878	.0137315-	.0158193	.0176559	.0192569	.0206444	.0218427
	∞	.0031060	.0061229	.0089737	.0116024	.0139768	.0160863	.0179373	.0195476	.0209407	.0221422
a = 5.5	0.0	.0014165+	.0027923	.0040923	.0052907	.0063730	.0073344	.0081778	.0089113	.0095458	.0100929
	0.1	.0020178	.0039826	.0058483	.0075809	.0091599	.0105778	.0118369	.0129465-	.0139195-	.0147706
	0.2	.0024608	.0048571	.0071327	.0092455+	.0111704	.0128971	.0144282	.0157743	.0169511	.0179763
	0.3	.0027013	.0053292	.0078197	.0101254	.0122178	.0140860	.0157335-	.0171732	.0184237	.0195060
	0.4	.0027975-	.0055164	.0080886	.0104639	.0126128	.0145248	.0162049	.0176679	.0189345+	.0200276
	∞	.0028331	.0055847	.0081845+	.0105315-	.0127460	.0146687	.0163555+	.0178226	.0190915+	.0201858
a = 6.0	0.0	.0013018	.0025661	.0037606	.0048617	.0058559	.0067389	.0075135-	.0081870	.0087694	.0092717
	0.1	.0018976	.0037454	.0055002	.0071300	.0086156	.0099498	.0111347	.0121791	.0130949	.0138961
	0.2	.0023133	.0045655-	.0067035-	.0086876	.0104938	.0121128	.0135468	.0148060	.0159053	.0168616
	0.3	.0025156	.0049621	.0072792	.0094225-	.0113652	.0130977	.0146233	.0159546	.0171094	.0181076
	∞	.0026036	.0051323	.0075212	.0097235-	.0117119	.0134778	.0150269	.0163740	.0175389	.0185433
a = 6.5	0.0	.0012041	.0023734	.0034781	.0044963	.0054156	.0062319	.0069479	.0075704	.0081087	.0085728
	0.1	.0017940	.0035410	.0052002	.0067413	.0081462	.0094080	.0105288	.0115166	.0123829	.0131407
	0.2	.0021821	.0043063	.0063219	.0081914	.0098920	.0114148	.0127622	.0139438	.0149741	.0158691
	0.3	.0023502	.0046351	.0067979	.0087969	.0106070	.0122194	.0136377	.0148739	.0159451	.0168700
	∞	.0024081	.0047468	.0069562	.0089927	.0108312	.0124639	.0138958	.0151409	.0162174	.0171456
a = 7.0	0.0	.0011198	.0022074	.0032347	.0041815+	.0050363	.0057952	.0064608	.0070395-	.0075398	.0079711
	0.1	.0017035+	.0033625-	.0049381	.0064018	.0077361	.0089346	.0099991	.0109373	.0117599	.0124793
	0.2	.0020643	.0040733	.0059789	.0077453	.0093508	.0107871	.0120565+	.0131685-	.0141367	.0149767
	0.3	.0022021	.0043424	.0063674	.0082376	.0099298	.0114357	.0127591	.0139116	.0149094	.0157704
	∞	.0022397	.0044147	.0064694	.0083631	.0100726	.0115905-	.0129216	.0140789	.0150795+	.0159421
a = 7.5	0.0	.0010465+	.0020628	.0030228	.0039076	.0047062	.0054153	.0060370	.0065776	.0070449	.0074477
	0.1	.0016237	.0032049	.0047067	.0061018	.0073737	.0085160	.0095306	.0104246	.0112084	.0118936
	0.2	.0019575-	.0038621	.0056679	.0073408	.0088601	.0102180	.0114168	.0124657	.0133778	.0141682
	0.3	.0020690	.0040796	.0059810	.0077361	.0093229	.0107341	.0119733	.0130518	.0139850+	.0147899
	∞	.0020931	.0041257	.0060457	.0078152	.0094124	.0108305+	.0120741	.0131551	.0140897	.0148954
a = 8.0	0.0	.0009822	.0019359	.0028368	.0036671	.0044164	.0050817	.0056651	.0061722	.0066105+	.0069884
	0.1	.0015525-	.0030643	.0045003	.0058342	.0070502	.0081422	.0091120	.0099664	.0107152	.0113696
	0.2	.0018599	.0036693	.0053841	.0069716	.0084123	.0096987	.0108332	.0118248	.0126862	.0134317
	0.3	.0019492	.0038429	.0056333	.0072851	.0087777	.0101044	.0112687	.0122816	.0131577	.0139130
	∞	.0019643	.0038719	.0056737	.0073341	.0088329	.0101634	.0113301	.0123443	.0132211	.0139768

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