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BOOK REVIEW

Correspondence concerning reviews should be addressed to the Book Review Editor, Professor James F. Hannan, Department of Statistics, Michigan State University, East Lansing, Michigan 48823.

NELSON, EDWARD (1967). *Dynamical Theories of Brownian Motion*. Mathematical Notes, Princeton University Press. 142 pp.

Review by J. L. DOOB

University of Illinois

This charming little book, notes for a course on stochastic processes, discusses Brownian motion 'as a natural phenomenon'. The audience had already encountered measure theoretic probability. The natural phenomenon turns out to involve sophisticated mathematics but the stress is frequently on physical significance. The author thinks that stochastic process theory is now in the doldrums, that it can be rescued by the introduction of differential structures, and that a study of dynamical theories of Brownian motion can help in the rescue.

The book starts with an entertaining and enlightening discussion of the early history of Brownian motion analysis, including quotations from authors as far apart as Brown and George Eliot. The first analysis of Browian motion from a modern physical point of view was by Einstein in 1905 and the theory was first put into rigorous mathematics about 20 years later by Wiener. In 1908 Langevin introduced stochastic differential equations into the analysis to get a truly dynamical approach. The Langevin equation was modified by Ornstein and Uhlenbeck in 1930 to derive a new mathematical process which is a second approximation to the physical one.

The author derives the original Brownian motion (Wiener) and the Ornstein-Uhlenbeck processes, allowing external forces, by sophisticated semigroup arguments, leading to the usual differential operators. There is a careful discussion of the kinematics and dynamics of the processes considered, including heavy doses of stochastic integrals and Ito's stochastic differential equations. The concluding chapters discuss the role of probability in quantum theory and 'Brownian motion in the aether' (the latter from a point of view going back to Fényes (1952)) giving on the one hand a quantum mechanical interpretation and on the other a stochastic mechanical interpretation of the Schrödinger equation. The relations between these two interpretations are not yet fully understood.

The unusual combination of sophisticated but intuitive mathematical and physical reasoning makes this book instructive and interesting to readers with the most diverse interests.