The Annals of Mathematical Statistics
1968, Vol. 39, No. 5, 1769

J. KEILSON, *Green's Function Methods in Probability Theory*. Hafner Publishing Co., New York, 1965. viii + 220 pp. $6.50.

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This book is a rather rambling account of certain problems connected with stochastic processes in discrete or continuous time with stationary independent increments, or with such processes modified by boundaries. The problems treated are of classical type—first passage problems, recurrence time distributions, maximum excursions and limiting cases. The emphasis is mostly on distribution theory, little attention being given to sample function behavior; generally speaking, the book's point of view is applied.

In treating the processes modified by boundaries it is convenient to view, e.g., the densities of the unmodified ones as Green's functions for the problems at hand; this is the origin of the term "Green's function methods" in the book's title. Boundary effects may then be at least partially accounted for by "compensation functions," of which the author makes occasional and ingenious use. However it is difficult to accept the apparent claim that a new and unifying method is being set forth, as it is still necessary to draw from that heterogeneous bag-of-tricks stocked with recurrence relations, Laplace transforms, complex variable methods and the like. Probabilistic considerations are often slighted; a particularly horrible example is the messy equation at the top of Section IV.3, which is proved by a contour integration, but which is, in disguised form, the following simple relation for partial sums S_n of iid integrable random variables X_i : $E\{S_n\ ;\ S_N = 0\} = 0$, $N \geqq n$. On the other hand, Chapter VI contains a fascinating two-sided generalization of Spitzer's observation that $\max(0, S_1, S_2, \cdots, S_n)$ and Z_n (obtained recursively from $Z_0 = 0$, $Z_n = (Z_{n-1} + X_n)^+$) have the same distribution, for each n.

The chapter headings are, I Introduction and summary, II Homogeneous Markov processes, III Conjugate transformations and asymptotic behavior, IV The passage problem for homogeneous skip-free processes, V Bounded processes and ergodic Green's functions, VI Ergodic distributions, ruin problems, and extremes, VII Processes with representations of finite dimension. The book concludes with a 70-item bibliography, a dozen Notes, and an index.