The Annals of Mathematical Statistics
1969, Vol. 40, No. 1, 329–332

ABSTRACTS OF PAPERS

(Abstracts of papers to be presented at the Central Regional meeting, Iowa City, Iowa, April 23–25, 1969. Additional abstracts will appear in future issues.)

2. An asymptotically optimum nonparametric test for grouped data. Malay Ghosh, University of North Carolina.

Sen [*Ann. Math. Statist.* **38**(1967) 1229–1239] obtained an asymptotically most powerful non-parametric test for the regression parameter for grouped data under simple linear regression model. The present paper considers a similar problem under multiple regression model and proposes an asymptotically optimum non-parametric test. The asymptotic distribution of the proposed test statistic has been derived by using techniques essentially those of Hájek [*Ann. Math. Statist.* **33**(1962) 1124–1147] and Sen, while asymptotic optimality properties in the sense of Wald [*Trans. Amer. Math. Soc.* (1943) 426–483] have been studied by showing the asymptotic equivalence of the proposed test statistic to the likelihood ratio test statistic for the same testing problem. The results have been applied to some particular cases and asymptotic relative efficiency results have also been studied. (Received 4 November 1968.)

3. Some bounds in packing problem for t = 4 (preliminary report). Bodh Raj Gulati, Eastern Connecticut State College.

Let $m_t(r + 1, s)$ denote the maximum number of points in finite projective geometry $PG(r,s)$ of r dimensions based on Galois field $GF(s)$, where s is a prime or power of a prime, so that no t of these points are linearly dependent. Bose has shown (*Sankhyā*, **8**) that $m_t(r + 1, s)$ also symbolises the maximum number of factors that can be accommodated in a symmetrical factorial design in which each factor is at $s = p^n$ levels, blocks are of size s^{r+1}, so that no t-factor or lower order interaction is confounded. These bounds which have been lately useful in error correcting codes and information theory were introduced by Bose, Barlotti, Seiden, Segre, Qvist, Tallini and many others for $t = 3$, but no general results are available for $t = 4$. For this case, the "packing problem" reduces to the investigation of maximum number of points in $PG(r, s)$ so that no four of these lie on the same plane. In this paper, the following results have been established: (i) $m_4(4, s) \leqq s + 2$ for $s = p^n$ where p is an odd prime; (ii) $m_4(4, s) \leqq s + 3$ for $s = 2^n$; (iii) $m_4(5, s) \leqq s^2 + 2$ for $s = 2$ and $s \geqq 3$ and odd; (iv) $m_4(r + 1, s) = s^{r-2} + 2$ for $r \geqq 5$ and $s \geqq 3$ and odd. For some other values of r and s, bounds are being investigated. (Received 13 November 1968.)

4. Estimation in the presence of outliers I: estimation of the mean of a normal distribution. J. S. Mehta and R. Srinivasan, Temple University.

We consider a sample of n observations all of which are hoped to be from $N(\mu, \sigma^2)$ but where it is feared that one or more spurious observations, from $N(\mu + a\sigma, \sigma^2)$ or $N(\mu, k\sigma^2)$, may be present in the sample. Independent estimators of μ which make use of the information from the possibly spurious observations are proposed for the cases when σ^2 is known or unknown. The behavior of these estimators is compared with that of the ones studied by G. C. Tiao and Irwin Guttman [Analysis of outliers with adjusted residuals, *Technometrics*, **9**(1967), 541–559], and Irwin Guttman and Dennis Smith [Investigation of rejection rules for outliers in small samples from the normal distribution, Technical Reports No.

90–93, Department of Statistics, University of Wisconsin; to appear in *Technometrics*]. Our estimators are shown to be better in general. (Received 4 November 1968.)

5. Estimation in the presence of outliers II: estimation in the variance of a normal distribution. J. S. MEHTA AND R. SRINIVASAN, Temple University.

We consider a sample of n observations all of which are hoped to be from $N(\mu, \sigma^2)$ but where it is feared that one or more spurious observations, from $N(\mu, k\sigma^2)$, may be present in the sample. Independent estimators of σ^2 which make use of the information from the possibly spurious observations are proposed. The behavior of these is compared with that of the ones studied by Irwin Guttman and Dennis Smith [Investigation of the rejection rules for outliers in small samples from the normal distribution, Technical Reports No. 90–93, Department of Statistics, University of Wisconsin; to appear in *Technometrics*]. Our estimators are shown to be better in general. (Received 4 November 1968.)

(An abstract of a paper to be presented at the Annual meeting, New York, New York, August 19–22, 1969. Additional abstracts will appear in future issues.)

1. Minimum bias approximation of models by polynomials of low order (preliminary report). ROBERT J. HADER, ALLISON R. MANSON, ROBERT G. PARISH, North Carolina State University. (By title)

We assume that the true response is either of the form $y_u = \alpha + \beta e^{\gamma x_u} + \epsilon_u$ or $y_u = \sum_{i=0}^{d+k-1} \beta_i x_u^i + \epsilon_u$ $(u = 1, \cdots, n)$, with the random variables ϵ_u distributed independently with mean zero and common variance σ^2. In addition we have made the assumption that there is some prior knowledge as to the approximate value of γ, and allow γ to vary over some grid of values about this prior value. To the above models, simple polynomials of the form $\hat{y}_u = \sum_{i=0}^{d-1} b_i x_u^i$, are fitted, and minimum bias is obtained for the exponential model alone, and simultaneously for the two models together. This protection is achieved by the estimation procedure, called "Minimum Bias Estimation." This estimation procedure minimizes the squared bias, integrated over a specific region (R) of interest of the x variate. The bias arises because $\hat{y}_u$ fails to represent precisely either of the assumed true models. Subject to achieving this minimum bias, the estimation procedure minimizes the Var $\hat{y}_u$ integrated over R for any given design. Sufficient flexibility remains to satisfy additional design criteria, e.g. minimize the integrated Var $\hat{y}_u$ over designs. Assuming that the true model is the exponential alone, with γ varying over a grid of values, symmetric designs in the x_u variate have been found which minimize the integrated variance of $\hat{y}_u$ over designs, for particular values of d. These designs depend on the particular grid of γ's. (Received 1 November 1968.)

(Abstracts of papers not connected with any meeting of the Institute.)

1. Conditional estimation of the standard deviation of the logistic distribution by the use of selected order statistics. JON N. BEYER, Space Systems Division, Los Angeles Air Force Station, ALBERT H. MOORE, Air Force Institute of Technology, Wright-Patterson AFB, and H. LEON HARTER, Aerospace Research Laboratories, Wright-Patterson AFB.

A technique is developed to obtain linear, minimum-variance, unbiased estimators for the standard deviation of a logistic population with known mean. Coefficients for multiplying ordered observations are developed for complete and censored samples for $n = 2(1)20$. Each sample of size n is censored from above and all m-order-statistic estimators are obtained where $m \leqq n$. Then the smallest subset of l order statistics from the set of m available order statistics is found which yields a 99% efficiency relative to the m-order-statistic estimator. If $l < m$, the coefficients for the l-order-statistic estimator are tabled. The variances of the m-order-statistic linear unbiased estimators and of the maximum-likelihood m-order-statistic estimators, as given by the asymptotic formula, are presented for comparative purposes. (Received 9 November 1968.)

2. Bayesian estimation of means of a complete two-way classification and a three component hierarchical design with random effect model. N. C. GIRI, University of Montreal.

The problems of estimating the means in the two-way random effect model $y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_{ij} + 1_{ijk}$ and in the three component hierarchieal random effect model $y_{ijk} = \mu + \alpha_i + \beta_{ij} + 1_{ijk}$ has been considered here from a Bayesian view point. In the case of complete two-way random effect model the posterior distributions of α_i, β_j, γ_{ij} are obtained under the assumption that α_i, β_j, γ_{ij} are independently normally distributed random variables with means 0 and variances σ_4^2, σ_3^2 and σ_2^2 respectively. In the case of three component hierarchical random effect design model the posterior distributions of α_i, β_{ij} are obtained from the usual assumption that α_i, β_{ij} are independently distributed normal random variables with means 0 and variances σ_3^2 and σ_2^2 respectively. In both cases it is assumed that the independent random errors are normally distributed with means 0 and variances σ_1^2. It has been shown that in both cases the distributions of these estimates are clustered more closely together than are the corresponding distributions with fixed effect model. (Received 15 October 1968.)

3. Resolution IV fractional factorial designs. BARRY H. MARGOLIN, Yale University.

Fractional factorial designs of resolution IV permit estimation of all the main effects with no aliasing by two-factor interactions. This paper produces a lower bound for the number of observations required for a general fractional factorial design to be of resolution IV. This lower bound agrees with a lower bound obtained by Rao for orthogonal arrays of strength 3. In addition, it is proved that this lower bound is attainable for the $t \cdot 2^n$ factorial design series for t even and $n \equiv 3 \pmod 4$ in plans which permit orthogonal estimation of the main effects. Finally, Webb's conjecture that there exist no resolution IV 2^n factoral designs with $2n$ runs except for those constructed by the fold-over technique is proved to be valid. (Received 7 October 1968.)

4. Nonparametric tests of multivariate sphericity. PAUL J. SMITH, University of Michigan.

The hypothesis of p-dimensional symmetry is the hypothesis that the distribution of a random vector $\mathbf{X}$ is the same as the distribution of $\mathbf{CX}$, where $\mathbf{C}$ is any orthogonal transformation of $\mathbf{R}^p$. This means that the density of a sample of size N is invariant under the action of the group $\mathcal{S} = \mathcal{O}_p \sim \mathcal{S}_N$, the wreath product of the p-dimensional orthogonal group and the symmetric group on N objects. Using this fact, one can characterize the class of all distribution-free tests of sphericity, following the development of Lehmann and Stein (1949) and Bell and Doksum (1967). It is possible to give the most powerful distribution-free test against a simple alternative, but it is seen that this test is unsuitable for practical applications. Therefore, a group of transformations is defined and invariant tests are constructed. This group generates the null hypothesis class, so the tests based on the maximal invariant are distribution-free. This maximal invariant is the ranks of the norms of the sample vectors and their directions, thought of as p-dimensional unit vectors. Under H_0 these unit vectors are distributed uniformly on the unit p-sphere and independently of the norms. These facts are used to form invariant tests. The first family of tests are formed by dividing $\mathbf{R}^p$ into congruent radial regions. This partition divides the sample into subsamples, and a multisample rank test is applied to the ranks of the norms in these subsample. The second method involves a partition of the unit p-sphere into randomized blocks. The partitioning process ranks the direction vectors, so that one can perform a goodness-of-fit test and a rank test of independence simultaneously. In both cases one eventually derives tests based on existing procedures, so that no new tables are required. (Received 1 November 1968.)

5. Conditional linear estimation of the scale parameter of the first asymptotic distribution of extreme values. WILLIAM SHELNUTT, Aerospace Research Laboratories, Wright-Patterson AFB, ALBERT H. MOORE, Air Force Institute of Technology, Wright-Patterson AFB, and H. LEON HARTER, Aerospace Research Laboratories, Wright-Patterson AFB.

A technique is developed to obtain linear, minimum-variance, unbiased estimators for the scale parameters of the first asymptotic extreme-value distributions of smallest and largest values with known mean. Coefficients for multiplying ordered observations are developed for complete and censored samples for $n = 1(1)20$. Each sample of size n is censored from above and all m-order-statistic estimators are obtained where $m \leq n$. Then the smallest subset of l order statistics from the set of m available order statistics is found which yields a 99% efficiency relative to the m-order-statistic estimator. The Cramér-Rao lower bound for the variances of the estimators for complete samples is derived and tabled for $n = 1(1)20$. For censored samples the asymptotic variances of the maximum-likelihood m-order-statistic estimators are presented for comparative purposes. (Received 9 November 1968.)