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CORRECTION

STRONG APPROXIMATION OF THE EMPIRICAL PROCESS OF GARCH SEQUENCES

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In Theorem 1.1, condition (1.7) is unnecessary and can be replaced by $\gamma < 0$ [which is also necessary for the existence of a stationary GARCH(p, q) sequence]. To see this, observe that by (1.4) and the definition of γ there is an integer $m \geq 1$ such that $E \log \|A_0 A_{-1} \cdots A_{-(m-1)}\| < 0$. For any $k \geq 1$ we have

$$\left\| \prod_{0 \leq j \leq k} A_{-j} \right\| = \left\| \left\{ \prod_{0 \leq \ell \leq [k/m]-1} \prod_{\ell m \leq i < (\ell+1)m} A_{-i} \right\} \prod_{m[k/m] \leq r \leq k} A_{-r} \right\|$$

$$\leq \left\{ \prod_{0 \leq \ell \leq [k/m]-1} \left\| \prod_{\ell m \leq i < (\ell+1)m} A_{-i} \right\| \right\} \prod_{m[k/m] \leq r \leq k} \|A_{-r}\|,$$

where an empty product is meant to be 1. Applying the Rosenthal inequality for the sum

$$\sum_{0 \leq \ell \leq [k/m]-1} \log \left\| \prod_{\ell m \leq i < (\ell+1)m} A_{-i} \right\| + \log \prod_{m[k/m] \leq r \leq k} \|A_{-r}\|$$

(instead of the sum of the individual $\log A_{-i}\|$'s, as in the original proof) yields that Lemma 2.1 remains valid with $|\gamma_1|$ in (2.3) replaced by a suitable positive constant. For the same reason, Lemma 2.2 also remains valid under $\gamma < 0$ and (2.2). Since only (2.3) and (2.4) are used in the rest of the proof of Theorem 1.1, our claim is proved.

We note, incidentally, that since the second row of the matrix A_0 is a $(p + q - 1)$-dimensional unit vector, we have $\|A_0\| \geq 1$ and thus condition (1.7) of Theorem 1.1 cannot hold.

Finally, in line 6 of page 790, R^{q-1} should be replaced by R^{q-2}.

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