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THE ABSTRACT RIEMANNIAN PATH SPACE

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Abstract On the Wiener space Ω, we introduce an abstract Ricci process A_t and a pseudo-gradient $F \to F^\sharp$ which are compatible through an integration by parts formula. They give rise to a $\sharp$-Sobolev space on Ω, logarithmic Sobolev inequalities, and capacities, which are tight on Hölder compact sets of Ω. These are then applied to the path space over a Riemannian manifold.

Keywords Wiener space, Sobolev spaces, Bismut-Driver formula, Logarithmic Sobolev inequality, Capacities, Riemannian manifold path space.

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I. Introduction

Since the construction of the Riemannian Brownian motion by Itô and the introduction of the Malliavin calculus in the Wiener space Ω, a great amount of recent work has been devoted to the extension of stochastic calculus to the path space Σ of a Riemannian manifold [1,2,3,4,7,8,9,10,11,12,13,14,15,25,27,30]. The stochastic development map I introduced by Itô takes the Wiener measure μ onto an isomorphic measure $\nu = I(\mu)$ on Σ.

One is guided by the flat case of the Wiener space of $\mathbb{R}^m$ where the Girsanov integration by parts formula plays a dominant part. In the case of the path space of a Riemannian manifold, this formula is replaced by the Bismut-Driver formula which introduces the Ricci curvature of M.

These considerations encounter two kinds of difficulties, the first one due to the use of stochastic calculus, the second due to differential geometry.

Our first goal to show that these two problems can be dealt with separately. We begin by describing a stochastic framework without involving differential geometry.

On the Wiener space Ω we introduce a pseudo-differential operator $F \to F^\sharp$ depending on a so-called "Ricci process" A which is set *a priori*.

This pseudo-differential operator $F^\sharp(\omega, \varpi)$ on $\Omega \times \Omega$ is linear in the second variable, as $F'(\omega, \varpi)$ which has been introduced for the classical flat case [18,19]. It should be noticed that in the flat case we have $A = 0$ and $F^\sharp = F'$.

With the help of a *damped* derivation, that is a modified derivation $F \to F^\flat$, we easily obtain a pseudo-Clark formula (which is equivalent to an integration by parts formula), a closed Dirichlet form, a spectral gap inequality and even a logarithmic Sobolev inequality (by using the Maurey-Ledoux method of the classical Gaussian case).

The closable Dirichlet form gives rise to $\sharp$-Sobolev spaces $W^{1,2,\sharp}$ and, with natural supplementary hypotheses, to $\sharp$-Sobolev spaces $W^{1,p,\sharp}$.

In fact, it turns out that the i.b.p. (integration by parts formula) for the damped derivation and the pseudo-Clark formula do not depend on the Ricci process A_t. This last process is only involved for the link between $\sharp$ and $\flat$-derivations.

Therefore, in Section IV, we deal with the problem of constructing some "concrete" $\flat$-derivation. For this purpose , we generalize an idea of [16], by using some kind of rotation of $\mu \otimes \mu$ instead of translation as we do usually for the Girsanov formula. It is to be noticed that the problem to generalize the Cameron-Martin space is avoided by this construction.

In Section V, we apply these results to the path space Σ of a compact Riemannian manifold M (endowed with a Driver connection): first we choose a $\flat$-derivation in the sense of Section IV, and after that, we choose the convenient A_t to obtain the true i.b.p. Bismut-Driver formula.

It should be noticed that for simplicity, M is embedded in a finite dimensional space E, but the Riemannian structure of M is not necessarily induced by an euclidean structure on E. The link between M and E is explicited by a Weingarten type tensor field on M.

In Section VI, we return to the general situation (without Riemannian manifold), and define the natural capacity $C_{1,p,\sharp}$ associated on the Wiener space Ω, to the $\sharp$-Sobolev spaces. We show that under a natural hypothesis, these capacities are tight on Hölder compact sets of Ω.

In the case of a Riemannian manifold, the Itô map transforms the $\sharp$-Sobolev space $W^{1,p,\sharp}(\Omega,\mu)$ onto a Sobolev space $W^{1,p,\sharp}(\Sigma,\nu)$ for $\nu = I(\mu)$. The associated capacity is tight on the Hölder compact sets of Σ (and this improves the results of [10]). We can then specify the Itô map as a quasi-isomorphism from Ω onto Σ.

In conclusion, we can say that the abstract setting presented here must be considered as an attempt to simplify rather than to generalize the stochastic Riemannian path theory. Nevertheless, this setting allows us to see that there are many different ways to do a reasonable differential calculus on the Wiener space, or on the Riemannian path space.

II. Preliminaries and notations

Let μ be the Wiener measure on $\Omega = \mathcal{C}_0([0,1],\mathbb{R}^m)$ with its natural filtration $\mathcal{F}_t$. We denote by $W_t(\omega)$ the canonical Brownian motion. If F is an elementary functional (cylindrical functional) $F = f(W_{t_1},\ldots,W_{t_n})$, the differential $F'(\omega,\varpi)$ is defined on $\Omega\times\Omega$ by the formula

$$F'(\omega,\varpi) = \sum_i \partial_i f(W_{t_1}(\omega),\ldots,W_{t_n}(\omega))W_{t_i}(\varpi).$$

The norm of the Gaussian Sobolev space $W^{1,2}$ is defined by

$$\|F\|_{1,2}^2 = \mathbb{E}(F^2) + \mathbb{E}(F'^2),$$

where the second expectation is taken with respect to $\mu\otimes\mu$.

For $F \in W^{1,2}$ one has

$$F'(\omega,\varpi) = \int_0^1 D_tF(\omega)dW_t(\varpi),$$

where D_tF is the square integrable rough Borel process with values in $\mathbb{R}^m$ which is worth

$$D_tF(\omega) = \frac{d}{dt}\overline{\mathbb{E}}(F'(\omega,\varpi)W_t(\varpi)),$$

where $\overline{\mathbb{E}}$ stands for the partial expectation w.r. to ϖ.

Note that we have two independent Brownian motions $W_t(\omega)$ and $W_t(\varpi)$, in short we shall denote them respectively W_t and $\overline{W}_t$. We can easily get the Clark–Ocone–Haussmann formula

$$F - \mathbb{E}(F) = \int_0^1 \mathcal{F}_tD_tF(\omega)dW_t(\omega),$$

where $\mathcal{F}_t$ is the conditional expectation operator w.r. to $\mathcal{F}_t$. It is enough to check the formula for $F(\omega) = \exp f(\omega)$ where f is a continuous linear functional on Ω (*cf.* [20]). Let

$$G(\omega) = \int_0^1 g_t(\omega)dW_t(\omega),$$

a zero mean value random variable, where g_t is a predictable process. Define

$$\overline{G}(\omega,\varpi) = \int_0^1 g_t(\omega)dW_t(\varpi),$$

and the Girsanov derivative of F in the direction of G

$$D_G F(\omega) = \overline{\mathbb{E}}(F'\overline{G}) = \overline{\mathbb{E}}\left(F'(\omega,\varpi)\int_0^1 g_t(\omega)dW_t(\varpi)\right).$$

From the Clark formula we can deduce the Cameron-Martin-Girsanov integration by parts formula

$$\mathbb{E}(FG) = \mathbb{E}(F'\overline{G}) = \mathbb{E}(D_G F) = \int_0^1 \mathbb{E}(g_t D_t F)dt.$$

III. Generalizations

Let us now be given an arbitrary predictable process A_t with values in the linear operators of $\mathbb{R}^m$. For some reasons which will be clarified later on (Section V), it will be called *the Ricci process.*

With every $g \in L^p(\mu \otimes dt, \mathbb{R}^m)$ is associated $\widetilde{g}$ which is the solution of the ordinary differential equation

$$\widetilde{g}_t = g_t - \frac{A_t}{2}\int_0^t \widetilde{g}_s ds.$$

One has $\displaystyle\int_0^t \widetilde{g}_s ds = \int_0^t C_t C_s^{-1} g_s ds$ and $\displaystyle\widetilde{g}_t = g_t - \frac{A_t C_t}{2}\int_0^t C_s^{-1} g_s ds,$

where C_t is the resolvant, that is the solution of

$$C_t = I - \frac{1}{2}\int_0^t A_s C_s ds.$$

1 Lemma: *Let N_p be the norm in the space $L^p(\mu \otimes dt, \mathbb{R}^m)$, it holds*

$$N_p(g) \leq \mathrm{e}^{K/2} N_p(\widetilde{g}) \qquad \text{and} \qquad N_p(\widetilde{g}) \leq \mathrm{e}^{K/2} N_p(g),$$

where K is the uniform norm of the process A_t. Then $g \to \widetilde{g}$ is an automorphism of $L^p(\mu \otimes dt, \mathbb{R}^m)$, which preserves the predictable subspace.

Proof: It is easy to get

$$N_p(g) \leq [1 + K/2]N_p(\widetilde{g}) \leq \mathrm{e}^{K/2} N_p(\widetilde{g}).$$

On the other hand

$$|\widetilde{g}_t| \leq |g_t| + \frac{K}{2}\int_0^t |\widetilde{g}_s| ds,$$

and next by Gronwall lemma

$$|\widetilde{g}_t| \leq |g_t| + \frac{K}{2}\int_0^t \mathrm{e}^{K(t-s)/2}\,|g_s| ds.$$

The right hand-side is a convolution, so that

$$N_p(\widetilde{g}) \leq N_p(g)\left[1 + \frac{K}{2}\int_0^1 \mathrm{e}^{Ks/2}\,ds\right] = \mathrm{e}^{K/2} N_p(g). \qquad \square$$

Now, let L^2_0 be the zero mean value subspace of L^2, and let $G \in L^2_0$

$$G = \int_0^1 g_t(\omega) dW_t(\omega)$$

we associate it with

$$\widetilde{G}(\omega, \varpi) = \int_0^1 \widetilde{g}_t(\omega) dW_t(\varpi)$$

2 Corollary: *$G \to \widetilde{G}$ is an isomorphism of $L^2_0(\mu)$ onto the closed subspace of $L^2(\mu \otimes \mu)$ which consists in all the Wiener functionals on $\Omega \times \Omega$ which are linear in the second variable.*

3 Definition: *The* pseudo-gradient *or* pseudo-differential *is any operator $F \to F^\sharp(\omega, \varpi) \in L^2(\mu \otimes \mu)$ which satisfies the following conditions:*

- *a) The domain $\mathcal{D}$ is dense in $L^2(\mu)$,*
- *b) $F^\sharp(\omega, \varpi)$ is linear in the second variable ϖ,*
- *c) The integration by parts formula holds: $\mathbb{E}(FG) = \mathbb{E}(F^\sharp \widetilde{G})$.*
- *d) $F \to F^\sharp$ is a derivation, that is $\gamma(F)^\sharp = \gamma'(F) F^\sharp$, for every $F \in \mathcal{D}$ and every $\mathcal{C}^1$-Lipschitz function γ.*

As above, we define the rough process $D^\sharp_t F \in \mathbb{R}^m$ such that

$$F^\sharp(\omega, \varpi) = \int_0^1 D^\sharp_t F(\omega) dW_t(\varpi),$$

and the Girsanov pseudo-gradient of F in the direction of G by the formula

$$D^\sharp_G F(\omega) = \overline{\mathbb{E}}[F^\sharp(\omega, \varpi) \widetilde{G}(\omega, \varpi)].$$

4 Definition: *Let F be in the domain of the pseudo-gradient, we define the damped pseudo-gradient $F^\flat$ by the formulae*

$$F^\flat(\omega, \varpi) = \int_0^1 D^\flat_t F(\omega) dW_t(\varpi),$$

where

$$D^\flat_t F = D^\sharp_t F - \frac{1}{2} \int_t^1 C_t^{-1*} C_s^* A_s^* D^\sharp_s F \, ds.$$

It turns out that it is the solution of the ODE

$$D^\flat_t F = D^\sharp_t F - \frac{1}{2} \int_t^1 A_s^* D^\flat_s F \, ds,$$

where A_s^* is the adjoint operator of A_s. Indeed, consider the adjoint J^* of the operator J of $L^2(\mu \otimes dt)$ defined by $J(g) = \widetilde{g}$. It is easily seen that $D^\flat_. F = J^*(D^\sharp_. F)$. Moreover we get the estimates

$$N_2(F^\flat) \leq \mathrm{e}^{K/2} N_2(F^\sharp) \qquad \text{and} \qquad N_2(F^\sharp) \leq \mathrm{e}^{K/2} N_2(F^\flat).$$

Now the integration by parts formula writes

$$\mathbb{E}(FG) = \mathbb{E}(F^\sharp \widetilde{G}) = \mathbb{E}(F^\flat \overline{G}) = \mathbb{E}(D^\flat_G F),$$

where $D^\flat_G F$ is the damped pseudo-gradient of F in the direction of G. Finally, $F^\sharp$ and $F^\flat$ have equivalent norms and are defined on the same domain.

Note that the damped pseudo-gradient $F^\flat$ is easily seen to be also a derivation.

5 Theorem: *(Pseudo-Clark's formula) We have*

$$F(\omega) - \mathbb{E}F = \int_0^1 \mathcal{F}_t D^\flat_t F(\omega) dW_t(\omega).$$

Proof: It is straightforward, thanks to the integration by parts formula for $F^\flat$. □

6 Corollary: *(spectral gap) One has*

$$\mathbb{E}(F^2) - \mathbb{E}(F)^2 = \int_0^1 \mathbb{E}[(\mathcal{F}_t D^\flat_t F)^2] dt \leq \mathbb{E}(F^{\flat 2}) \leq \mathrm{e}^K \, \mathbb{E}(F^{\sharp 2}).$$

Proof: Obvious. □

7 Theorem: *$F^\sharp$ and $F^\flat$ are closable in L^2.*

Proof: Assume that F_n and $F^\flat_n$ respectively converge to 0 and H. One gets

$$0 = \mathbb{E}(H\overline{G}),$$

for every $G \in L^2_0$. Let γ be a $\mathcal{C}^1$ bounded Lipschitz function vanishing at 0 and such that $\gamma'(0) = 1$. Replace F_n with $\Phi\gamma(F_n)$ where Φ is a bounded element of the domain. The damped pseudo-gradient $F^\flat_n = \Phi^\flat \gamma(F_n) + \Phi\gamma'(F_n) F^\flat_n$ converges to ΦH in L^2. So we get $\mathbb{E}(H\Phi\overline{G}) = 0$, for every G. Take G as the Wiener integral

$$G(\omega) = \int_0^1 g_t dW_t(\omega),$$

where g_t do not depend on ω. Now H writes

$$H(\omega, \varpi) = \int_0^1 h_t(\omega) dW_t(\varpi),$$

so that we get

$$0 = \mathbb{E}(H\Phi\overline{G}) = \int_0^1 g_s \mathbb{E}(\Phi h_s) ds = \iint \Phi(\omega) g_s h_s(\omega) ds d\mu(\omega).$$

As $\Phi \otimes g$ runs through a total set in $L^2(\mu \otimes dt)$, we get $H = 0$. □

8 Corollary: *The two Dirichlet forms*

$$\mathcal{E}^\sharp(F,F) = \mathbb{E}(F^{\sharp\,2}) \qquad \text{and} \qquad \mathcal{E}^\flat(F,F) = \mathbb{E}(F^{\flat\,2})$$

are local. More precisely, we have

$$|F|^\sharp(\omega,\varpi) = [1_{\{F>0\}}(\omega) - 1_{\{F\leq 0\}}(\omega)]F^\sharp(\omega,\varpi),$$
$$|F|^\flat(\omega,\varpi) = [1_{\{F>0\}}(\omega) - 1_{\{F\leq 0\}}(\omega)]F^\flat(\omega,\varpi).$$

The proof is the same as the one in the classical case ([5]).

9 Theorem: *(Logarithmic Sobolev inequality) We have*

$$\mathbb{E}(F^2 \operatorname{Log} F^2) - \mathbb{E}(F^2)\operatorname{Log}\mathbb{E}(F^2) \leq 2\mathbb{E}(F^{\flat\,2}) \leq 2\,\mathrm{e}^K\,\mathbb{E}(F^{\sharp\,2}).$$

Proof: We follow the idea of [3]. There is a simplification thanks to the use of the damped pseudo-gradient $F^\flat$. It is sufficient to consider the case $F \geq \varepsilon > 0$, denote M_t the martingale $\mathcal{F}_t F$, then $dM_t = \mathcal{F}_t D_t^\flat F\,dW_t$ according to the pseudo-Clark formula. The Itô formula gives

$$\mathbb{E}(M_1 \operatorname{Log} M_1) - \mathbb{E}(M_0 \operatorname{Log} M_0) = \frac{1}{2}\mathbb{E}\int_0^1 \frac{1}{M_t}[\mathcal{F}_t D_t^\flat F]^2\,dt.$$

Replace F by F^2, so that $D_t^\flat F$ is replaced by $2FD_t^\flat F$. Applying the Cauchy-Schwarz inequality to the conditional expectation, we get

$$[\mathcal{F}_t D_t^\flat F^2]^2 = 4[\mathcal{F}_t(FD_t^\flat F)]^2 \leq 4\mathcal{F}_t(F^2)\mathcal{F}_t(D_t^\flat F)^2.$$

Hence,

$$\mathbb{E}(F^2 \operatorname{Log} F^2) - \mathbb{E}(F^2)\operatorname{Log}\mathbb{E}(F^2) \leq 2\mathbb{E}\left(\int_0^1 [D_t^\flat F]^2\,dt\right). \qquad \square$$

Extension to L^p

10 Proposition: *Assume that $W_t^\sharp$ belongs to L^p for every $t \in [0,1]$. If F is an elementary function $F = f(W_{t_1},\ldots,W_{t_n})$, put for $(\omega,\varpi) \in \Omega\times\Omega$*

$$F^\sharp(\omega,\varpi) = \sum_i \partial_i f(W_{t_1}(\omega),\ldots,W_{t_n}(\omega))W_{t_i}^\sharp(\omega,\varpi).$$

So the pseudo-gradient extends to L^p by this formula. The same property holds for the damped pseudo-gradient $F^\flat$.

Proof: The L^p-domain contains elementary functions, so it is dense in L^p. Closability is obtained, via Burkholder's inequality, in the same way as in the L^2 case. The L^p norm equivalence for the two pseudo-gradients comes from the fact that J is also an automorphism of $L^p(\mu\otimes dt)$. $\square$

11 Definition: *The $\sharp$-Sobolev space $W^{1,p,\sharp}$ with respect to the $\sharp$ derivative is the completion of the elementary functions under the norm*

$$\|F\|_{1,p,\sharp}^p = \mathbb{E}(|F|^p) + \mathbb{E}(|F^\sharp|^p).$$

Notice that $W^{1,p,\sharp}$ is a subspace of L^p for the norm is closable. The damped norm $\|F\|_{1,p,\flat}$ which is equivalent to the previous one is defined in the same way, so the $\flat$-Sobolev space is the the same as the $\sharp$ one.

12 Remarks:

1) It follows that it may happen many different Ricci processes generating the same $\sharp$-Sobolev space, since the $\sharp$-Sobolev space only depends on the $\flat$-derivation.

2) It follows from a result of [8] an example of a concrete Ricci process (defined by a Riemannian manifold,*cf.* Section V) generating a $\sharp$-Sobolev space different from the Gaussian Sobolev space. Nevertheless the difference $D_tF - D_t^\flat F$ is singular to the predictable σ-algebra when F belongs to the two domains.

3) The choice of the J operator may seem to be quite arbitrary (consider for example the same formula but with the Ricci process under the integration sign). The same properties as above would hold. In fact the formula that we have taken was motivated by the example of the path space of a Riemannian manifold.

IV. Some concrete derivations

Choosing $F^\flat$

Let $\beta_t(\omega, \varpi)$ and $\gamma_t(\omega, \varpi)$ two predictable processes which are square integrable on $\Omega \times \Omega \times [0, 1]$, with values in skew-symmetric operators of $\mathbb{R}^m$. We assume that β is linear in the second variable (first Wiener chaos in the second variable). Generalizing an idea of [16], we put for $\varepsilon \in \mathbb{R}$,

$$\begin{cases} \omega^\varepsilon(t) = W_t^\varepsilon(\omega, \varpi) = \displaystyle\int_0^t \mathrm{e}^{\varepsilon\beta_s(\omega,\varpi)}\, dW_s(\omega\cos\varepsilon + \varpi\sin\varepsilon), \\ \varpi^\varepsilon(t) = \overline{W}_t^\varepsilon(\omega, \varpi) = \displaystyle\int_0^t \mathrm{e}^{\varepsilon\gamma_s(\omega,\varpi)}\, dW_s(-\omega\sin\varepsilon + \varpi\cos\varepsilon). \end{cases}$$

It is easily seen that the couple $(W_t^\varepsilon, \overline{W_t}^\varepsilon)$ is an $\mathbb{R}^m \times \mathbb{R}^m$-Brownian motion under $\mu \otimes \mu$, so that its distribution does not depend on ε. For a regular $F(\omega)$, define

$$\dot{F}(\omega, \varpi) = \frac{d}{d\varepsilon} F(\omega^\varepsilon)\bigg|_{\varepsilon=0}.$$

Note that properties a), b), d) of definition 2 are satisfied. It remains to prove an integration by parts formula to see that in fact $F \to \dot{F}$ is a damped pseudo-gradient, that is

$$\mathbb{E}(FG) = \mathbb{E}(\dot{F}\overline{G}),$$

for every $G(\omega) = \int_0^1 g_s(\omega) dW_s(\omega)$.

Put

$$H(\omega, \varpi) = F(\omega)\overline{G}(\omega, \varpi) = F(\omega) \int_0^1 g_s(\omega) dW_s(\varpi).$$

We have,

$$H^\varepsilon(\omega, \varpi) = F(\omega^\varepsilon) \int_0^1 g_s(\omega^\varepsilon) dW_s(\varpi^\varepsilon),$$

and for every ε

$$\mathbb{E}(H^\varepsilon) = \mathbb{E}(H),$$

so that for a bounded regular g

$$\mathbb{E}\left[\frac{dH^\varepsilon}{d\varepsilon}\right] = 0,$$

$$\frac{dH^\varepsilon}{d\varepsilon}\Big|_{\varepsilon=0} = \dot{F}(\omega, \varpi) \int_0^1 g_s(\omega) dW_s(\varpi) - F(\omega) \int_0^1 g_s(\omega) dW_s(\omega) + \cdots$$

$$\cdots + F(\omega) \int_0^1 \dot{g}_s(\omega, \varpi) dW_s(\varpi) + F(\omega) \int_0^1 g_s(\omega)\gamma_s(\omega, \varpi) dW_s(\varpi).$$

The last two terms have zero expectation since they are ends of martingales w.r. to ϖ. Hence, by density of regular g, we are done.

13 Remarks:

1) In fact, $\dot{F}$ depends on β but not on γ as it is seen below, so that we can take $\gamma = 0$.

2) In the case $\beta = 0$, one has $\dot{F} = F'$, so that we exactly get the Cameron-Martin-Girsanov integration by parts formula.

3) If $F = f(W_{t_1}, \ldots, W_{t_n})$ is an elementary function, we have

$$\dot{F} = F' + \sum_i \partial_i f(W_{t_1}, \ldots, W_{t_n}) \int_0^{t_i} \beta_s(\omega, \varpi) dW_s(\omega).$$

Hence, for every G,

$$\sum_i \mathbb{E}\left[\partial_i f(W_{t_1}, \ldots, W_{t_n}) \int_0^{t_i} \beta_s^G(\omega) dW_s(\omega)\right] = \mathbb{E}(\dot{F}\overline{G} - F'\overline{G}) = 0,$$

where β^G is defined by

$$\beta_t^G(\omega) = \overline{\mathbb{E}}\left[\beta_t(\omega, \varpi) \int_0^1 g_s(\omega) dW_s(\varpi)\right].$$

Choosing $F^\sharp$

Now, take an arbitrary Ricci process A_t in order to define $F^\sharp$ in such a way that $F^\flat = \dot{F}$. So, put

$$\dot{F}(\omega, \varpi) = \int_0^1 D_t^\flat F(\omega) dW_t(\varpi),$$

and

$$D_t^\sharp F(\omega) = D_t^\flat F(\omega) + \frac{1}{2}\int_t^1 A_s^* D_s^\flat F(\omega) ds.$$

It is easy to verify that $F^\sharp$ is a pseudo-gradient in the sense of Definition 2, and that $F^\flat = \dot{F}$.

V. The Riemannian manifold paths

Let M be an m-dimensional compact submanifold of a finite dimensional vector space E. First we assume that M is endowed with a Riemannian structure $\mathcal{G}$. Second we assume that we are given a Driver connection ∇, that is [7] a) ∇ is $\mathcal{G}$-compatible, *i.e.* $\nabla\mathcal{G} = 0$

b) For every tangent vectors ξ and η we have

$$\langle T(\xi, \eta), \eta\rangle = 0$$

where T is the torsion tensor of ∇. It is known [7,9] that this implies $\nabla = \overline{\nabla} + \frac{1}{2}T$ where $\overline{\nabla}$ is the Levi-Civita connection.

Consider the natural connection D on the vector space E. We define a Weingarten type tensor by writing

$$V(\xi, \eta) = D_\xi \eta - \nabla_\xi \eta$$

This is an E-valued tensor field which is not symmetric as we have $V(\xi, \eta) - V(\eta, \xi) = -T(\xi, \eta)$. Let Σ be the space of continuous paths starting at a point $o \in M$, and let W_t be the canonical Brownian motion of $\mathbb{R}^m$. According to Itô and Driver, we get an M-Brownian motion X_t starting at o by solving the Itô–Stratonovich SDE

$$\mathbf{I}(0) \qquad \begin{cases} dX_t = H_t \circ dW_t \\ \\ dH_t = V(\circ dX_t, H_t) \end{cases},$$

where $X_0 = o$, and H_0 a fixed isometry of $\mathbb{R}^m$ onto $T_o(M)$. These initial conditions are in force all over the section. It is known that X_t is M-valued and is an M-Brownian motion. Moreover H_t is an isometry of $\mathbb{R}^m$ onto $T_{X_t}(M)$. The second equation means that H_t is a stochastic parallel field over X_t.

This system has a unique solution (X_t, H_t) for $t \in [0, 1]$, and X_t is an M-Brownian motion.

The Bismut-Driver formula

14 Theorem: *There exists a process* β *in the sense of Section IV such that*

$$X_t^\sharp = H_t \overline{W}_t,$$

for the Ricci process $A_t = H_t^{-1}[\mathrm{Ric} + \Theta]H_t$, *where* Ric *is the Ricci tensor field of* ∇, *and* $\Theta = \mathrm{Trace}\,(\nabla T) = \sum_i \nabla_i T(e_i, \cdot)$.

Proof: First we search for the damped pseudo-gradient (*i.e.* $F^\flat$) in the form of Section IV. So, take β_t and γ_t as in Section IV. We have $X_t^\varepsilon(\omega, \varpi) = X_t(\omega^\varepsilon)$, and we get the new Itô–Stratonovich system

$$\mathbf{I}(\varepsilon) \qquad \begin{cases} dX_t^\varepsilon = H_t^\varepsilon \circ dW_t^\varepsilon \\ \\ dH_t^\varepsilon = V(\circ dX_t^\varepsilon, H_t^\varepsilon) \end{cases},$$

with the same initial conditions as $\mathbf{I}(0)$. Taking the derivative with respect to ε at 0, we get

$$\dot{\mathbf{I}}(0) \qquad \begin{cases} d\dot{X}_t(\omega,\varpi) = \dot{H}_t(\omega,\varpi) \circ dW_t(\omega) + H_t(\omega) \circ dW_t(\varpi) + H_t \circ (\beta_t dW_t) \\ \\ d\dot{H}_t = V(\circ d\dot{X}_t, H_t) + V(\circ dX_t, \dot{H}_t) + V'(\dot{X}_t, \circ dX_t, H_t) \end{cases},$$

where V' is a suitable tensor field. Obviously the vector $\dot{X}_t$ belongs to $T_{X_t}(M)$, so that it writes

$$\dot{X}_t(\omega,\varpi) = H_t(\omega)\xi_t(\omega,\varpi).$$

Hence,

$$d\dot{X}_t(\omega,\varpi) = dH_t(\omega) \circ \xi_t(\omega,\varpi) + H_t(\omega) \circ d\xi_t(\omega,\varpi) = V(\circ dX_t, H_t\xi_t) + H_t \circ d\xi_t.$$

In the same way, we have

$$\dot{H}_t(\omega,\varpi) = H_t(\omega)\alpha_t(\omega,\varpi) + V(\dot{X}_t(\omega,\varpi), H_t(\omega)),$$

where α_t is a skew-symmetric operator of $\mathbb{R}^m$ since H_t is an isometry. By identification with the first line of $\dot{\mathbf{I}}(0)$, we get

$$d\xi_t = \alpha_t \circ dW_t + d\overline{W}_t + \tau_t(\circ dW_t, \xi_t) + \beta_t dW_t,$$

where $\tau_t = H_t^{-1} T H_t$ is the stochastic parallel transport of T, and where $\overline{W}_t$ stands for $W_t(\varpi)$. On the other hand, $H_t\alpha_t$ is the stochastic covariant derivative of H_t w.r. to ε, so that we have

$$d(H_t\alpha_t) = \mathrm{Riem}(\circ dX_t, \dot{X}_t)H_t + V(\circ dX_t, H_t\alpha_t),$$

where Riem is the curvature tensor of the connection ∇ at X_t. By comparison with the second line of $\mathbf{I}(\mathbf{0})$ we get

$$d\alpha_t = r_t(\circ dW_t, \xi_t),$$

where $r_t = H_t^{-1} \operatorname{Riem} H_t$ is the stochastic parallel transport of Riem. Then we obtain the Itô–Stratonovich system

$$\mathbf{II} \qquad \begin{cases} d\xi_t = \alpha_t \circ dW_t + d\overline{W}_t + \tau_t(\circ dW_t, \xi_t) + \beta_t dW_t \\ \\ d\alpha_t = r_t(\circ dW_t, \xi_t) \end{cases} .$$

By stochastic contraction, we get

$$d\alpha_t dW_t = -\mathrm{ric}_t \xi_t \, dt,$$

where $\mathrm{ric}_t = H_t^{-1} \operatorname{Ric} H_t$ is the stochastic parallel transport of the Ricci tensor at X_t. In the same way we get

$$\tau_t(\circ dW_t, \xi_t) = \tau_t(dW_t, \xi_t) + \frac{1}{2} d\tau_t(dW_t, \xi_t) + \frac{1}{2}\tau_t(dW_t, d\xi_t).$$

So we obtain the new Itô–Stratonovich system

$$\begin{cases} d\xi_t = (\alpha_t + \beta_t) dW_t + d\overline{W}_t + \tau_t(dW_t, \xi_t) - \frac{1}{2}\mathrm{ric}_t \xi_t \, dt - \frac{1}{2}\theta_t \xi_t \, dt + \frac{1}{2}\tau_t(dW_t, d\xi_t) \\ \\ d\alpha_t = r_t(\circ dW_t, \xi_t) \end{cases} ,$$

where $\theta_t = d\tau_t(dW_t, \cdot) = H_t^{-1} \Theta H_t$ is the stochastic parallel transport of $\Theta = \operatorname{Trace}(\nabla T)$. Observe that the coefficient $\tau_t(\cdot, \xi_t)$ of dW_t is skew-symmetric valued, thanks to the Driver condition; and that $\xi(\omega, \varpi)$ is linear in ϖ.

At this time, we already obtained a $\flat$-derivation and even a family of $\flat$-derivation (one for each process β), with the good i.b.p. formula. In addition, if we take an arbitrary bounded Ricci process, we get also a $\sharp$-derivation with a good i.b.p.

Nevertheless, we want to have $X_t^\sharp = H_t \overline{W}_t$. In order to obtain such a $\sharp$-derivation, put $A_t = \mathrm{ric}_t + \theta_t$, we get

$$d\xi_t = [\widetilde{\alpha}_t + \beta_t] dW_t + d\overline{W}_t - \frac{1}{2} A_t \xi_t \, dt + \frac{1}{2}\tau_t(dW_t, d\xi_t),$$

where $\widetilde{\alpha}_t = \alpha_t + \tau_t(\cdot, \xi_t)$ and where $A_t = \mathrm{ric}_t + \theta_t$.

Introduce *a priori* the solution η_t of

$$d\eta_t = d\overline{W}_t - \frac{1}{2} A_t \eta_t \, dt, \qquad \text{which is} \qquad \eta_t(\omega, \varpi) = \int_0^t C_t C_s^{-1} d\overline{W}_s.$$

Observe that η_t is linear in ϖ, and now choose the particular β process by putting

$$\beta_t(\omega, \varpi) = -\int_0^t r_s(\circ dW_s, \eta_s) - \tau_t(\cdot, \eta_t),$$

which is skew-symmetric valued (∇ is a Driver connection), and which is also linear in ϖ. It turns out that the couple $(\xi_t, \alpha_t) = (\eta_t, \int_0^t r_s(\circ dW_s, \eta_s))$ is the solution of the last system. The proof of Theorem 14 is completed. □

15 Corollary: *If $F = f(X_{t_1}, \dots, X_{t_n})$ is an M-elementary function, we have*

$$\mathbb{E}(FG) = \mathbb{E}\left[\sum_i \partial_i f(X_{t_1}, \dots, X_{t_n})\, H_{t_i} \int_0^{t_i} \widetilde{g}_s ds\right].$$

Proof: We have

$$\dot{F}(\omega, \varpi) = \textstyle\sum_i \partial_i f(X_{t_1}, \dots, X_{t_n}) \dot{X}_{t_i} = \sum_i \partial_i f(X_{t_1}, \dots, X_{t_n}) H_{t_i} \eta_{t_i},$$

$$\mathbb{E}(FG) = \mathbb{E}(\dot{F}\overline{G}) = \mathbb{E}\left[\textstyle\sum_i \partial_i f(X_{t_1}, \dots, X_{t_n})\, H_{t_i} \displaystyle\int_0^{t_i} \widetilde{g}_s ds\right] = \mathbb{E}(F^\sharp \widetilde{G}),$$

as it can be easily seen from the obvious relation

$$\overline{\mathbb{E}}\left[\eta_t \int_0^1 g_s d\overline{W}_s\right] = \int_0^t \widetilde{g}_s ds = \overline{\mathbb{E}}\left[\overline{W}_t \int_0^1 \widetilde{g}_s d\overline{W}_s\right].$$

16 Remarks:

a) As we have $X_t^\sharp = H_t \overline{W}_t$ and $X_t^\flat = H_t \eta_t$, we can verify that with the notations of definition 4, we have for $\tau \in [0, 1]$ two lines of vectors

$$D_t^\sharp X_\tau = 1_{\{t<\tau\}} H_\tau^*, \qquad D_t^\flat X_\tau = 1_{\{t<\tau\}} C_t^{-1*} C_\tau^* H_\tau^*,$$

or in terms of columns of *covectors* $\in (\mathbb{R}^m)^*$, which is better

$$\widetilde{D_t^\sharp X_\tau} = 1_{\{t<\tau\}} H_\tau, \qquad \widetilde{D_t^\flat X_\tau} = 1_{\{t<\tau\}} H_\tau C_\tau C_t^{-1}.$$

b) Notice that the solution ξ_t of the above system is an affine function of β. For the choice of Bismut we get the i.b.p. of Bismut-Driver (modulo the good Ricci process), for $\beta = 0$ we get $F^\flat = F'$ that is the ordinary flat derivation on the Wiener space. For an arbitrary β, we get

$$D_G^\beta F = D_G^\sharp F + H_t \zeta_t,$$

where ζ is the solution of

$$\zeta_t = \int_0^t \overline{\beta}_s(\omega) dW_s(\omega) - \frac{1}{2}\int_0^t A_s \zeta_s ds,$$

with a suitable skew-symmetric valued predictable process $\overline{\beta}$ depending on G. Hence we have again the i.b.p. and

$$\sum_i \mathbb{E}(\partial_i f(X_{t_1}, \dots, X_{t_n}) H_{t_i} \zeta_{t_i}) = 0.$$

c) More generally, if ζ satisfies the preceding SDE with an arbitrary $\overline{\beta}$, one can prove in the same way that this last expectation vanishes.

VI. Capacities

In the first part of this section we return to the general case (without manifold). We assume that every W_t belongs to the space $W^{1,p,\sharp} = W^{1,p,\flat}$. It is equivalent to say that every $W_t^\sharp$ or $W_t^\flat$ belongs to $L^p(\mu \otimes \mu)$.

A functional capacity is defined on the Wiener space. Put

$$C_{1,p}^\sharp(g) = \mathrm{Inf}\{\ \|f\|_{1,p,\sharp}\ /\ \ f \geq g \text{ almost everywhere }\},$$

for every l.s.c. function $g \geq 0$ on Ω; and put for every numerical function h,

$$C_{1,p}^\sharp(h) = \mathrm{Inf}\{\ C_{1,p}^\sharp(g)\ /\ \ g \text{ l.s.c. },\ g \geq |h|\}.$$

In the same way we define the functional capacity $C_{1,p}^\flat$. Clearly these two capacities are equivalent.

17 Theorem: *Suppose that the process W_t satisfies the inequality*

$$N_p(W_t^\flat - W_s^\flat) \leq k.|t-s|^\alpha,$$

with $p > 2$, $1/p < \alpha < 1/2$ for a given constant k. Then for $0 < \gamma < \alpha - 1/p$, the capacities $C_{1,p}^\sharp$ and $C_{1,p}^\flat$ are tight on γ-Hölder compact sets of Ω.

Proof: The hypothesis means that $t \to W_t$ is a $W^{1,p,\flat}$ valued α-Hölder function. Let β such that $\gamma < \beta < \alpha - 1/p$, consider the Hölder norm

$$q(\omega) = \operatorname*{Sup}_{s \neq t} \frac{|W_t - W_s|}{|t-s|^\beta}.$$

Denote $\mathcal{H}_\alpha$ the space of α-Hölder continuous functions with its natural norm. We have the inclusions $\mathcal{H}_\alpha(L^p) \subset L^p(\mathcal{H}_\beta)$ ([22], proof of Theorem 5), hence the function q belongs to L^p.

Now the space $W^{1,p,\flat}$ is of local type, so that we have the estimate

$$|q^\flat(\omega, \varpi)| \leq \operatorname*{Sup}_{s \neq t} \frac{|W_t^\flat - W_s^\flat|}{|t-s|^\beta},$$

which belongs to L^p for the same reason. Finally q belongs to $W_{1,p}^\flat$.

The q-balls $\{q \leq \lambda\}$ are compact into $\mathcal{H}_\gamma$ and then into Ω, so that the complementary sets U_λ are open, and their capacities are worth

$$C_{1,p}^\flat(U_\lambda) \leq \frac{1}{\lambda}\|q\|_{1,p,\flat},$$

which vanish as λ tends to infinity.

Now suppose that $W_t^\flat$ is given by a concrete derivation as in Section IV. We have

18 Proposition: *Let $p > 4$. If β satisfies the inequality*

$$\int_0^1 \mathbb{E}(|\beta_s|^p)\, ds < +\infty,$$

then $W_t^\flat$ satisfies the hypotheses of Theorem 17 for $1/2 - 1/p > \alpha > 1/p$.

Proof: Put $M_t = \int_0^t \beta_s dW_s$. By Burkholder's inequality we have

$$\mathbb{E}(|M_t - M_s|^p) \leq K_p(t-s)^{\frac{p-2}{2}} \int_s^t \mathbb{E}|\beta_u|^p du \leq k|t-s|^{p\alpha},$$

for $1/2 - 1/p > \alpha > 1/p$, and the result since $W_t^\flat = \overline{W}_t + M_t$. □

Application of capacities to the Riemannian case

First observe that X and H are solutions of the system $\mathbf{I}(0)$, so by [21] they belong to $\mathcal{H}_\alpha(L^p)$ for $1/2 > \alpha > 1/p$. Now we have

$$\beta_t(\omega, \varpi) = -\int_0^t r_s(\circ dW_s, \eta_s) - \tau_t(\cdot, \eta_t),$$

and

$$\eta_t = \int_0^t C_t C_s^{-1} d\overline{W}_s.$$

So for various constants K

$$\mathbb{E}|\eta_t|^p \leq K, \qquad \mathbb{E}|\beta_t|^p \leq K.$$

19 Corollary: *$C_{1,p,\sharp}$ and $C_{1,p,\flat}$ are tight on Hölder compact sets of Ω.*

The Itô map $\omega \to \sigma$ defined by $\sigma(t) = X_t(\omega)$ exchanges the measurable function classes on Ω (resp. Σ). It also exchanges the $\sharp$-Sobolev spaces $W^{1,p,\sharp}(\Omega, \mu)$ constructed on Ω with the $\sharp$-Sobolev spaces $W^{1,p,\sharp}(\Sigma, \nu)$ constructed on Σ. More precisely, we have

20 Theorem: *Let $p > 1$. We can refine the Itô map into a $C_{1,p}^\sharp$-quasi-continuous map with values in a separable subset of $\Sigma \cap \mathcal{H}_\alpha$ for $1/2 > \alpha > 0$. The image capacity $\Gamma_{1,p}^\sharp$ is associated with $W^{1,p,\sharp}(\Sigma, \nu)$ and is tight on Hölder compact sets of Σ and then the Itô map is a quasi-isomorphism.*

Proof: As both capacities $C_{1,p}^\sharp$ and $\Gamma_{1,p}^\sharp$ are increasing with p, we can suppose p as great as we want, and take $\alpha > 1/p$. The Itô map I is an isomorphism of the Wiener measure μ onto its image ν which is carried by Σ, so that $f \to f \circ I$ is an isomorphism of $L^p(\nu)$ on $L^p(\mu)$, and also of $W^{1,p,\sharp}(\Sigma, \nu)$ onto $W^{1,p,\sharp}(\Omega, \mu)$. Let us show first that $\Gamma_{1,p}^\sharp$ is tight on compact sets of Σ. Consider, as above, for $\alpha - 1/p > \beta$,

$$Q(\sigma) = \operatorname*{Sup}_{s \neq t} \frac{|\sigma(t) - \sigma(s)|}{|t-s|^\beta}.$$

Then,

$$Q \circ I(\omega) = \operatorname*{Sup}_{s \neq t} \frac{|X_t - X_s|}{|t-s|^\beta},$$

and

$$|(Q\circ I)^\sharp(\omega,\varpi)| \leq \operatorname*{Sup}_{s\neq t}\frac{|X_t^\sharp - X_s^\sharp|}{|t-s|^\beta} = \operatorname*{Sup}_{s\neq t}\frac{|H_t(\omega)W_t(\varpi) - H_s(\omega)W_s(\varpi)|}{|t-s|^\beta}.$$

By the previous lemma, $t \to H_t$ is Hölder continuous with values in L^p. As $W_t(\varpi)$ shares the same property, we get from the Kolmogorov lemma [21,22] that $(Q\circ I)^\sharp$ is majorized by an element of $L^p(\mu)$.

It follows as above that $Q\circ I$ belongs to $W^{1,p,\sharp}(\Omega,\mu)$ hence Q belongs to $W^{1,p,\sharp}(\Sigma,\nu)$.

The same argument as in Theorem 17 applies and shows that $\Gamma^\sharp_{1,p}$ is tight on compact sets of $\mathcal{H}_\gamma(M)\subset\Sigma$ for $\alpha - 1/p > \beta > \gamma$. Note that β can take any arbitrary value between 0 and 1/2.

It results from [19] that, as for the flat Gaussian Sobolev space $W^{1,p}(\Omega,\mu)$, every linear increasing functional on $W^{1,p,\sharp}(\Omega,\mu)$ (resp. $W^{1,p,\sharp}(\Sigma,\nu)$) is representable by a non-negative measure on Ω (resp. Σ), vanishing on sets which are $C^\sharp_{1,p}$-polar (resp. $\Gamma^\sharp_{1,p}$-polar).

If φ is an elementary function on Σ, $\varphi\circ I$ belongs to $\mathcal{L}^1(C^\sharp_{1,p})$, so that $\varphi\to\varphi\circ I$ is a quasi-isomorphism for the two quasi-topologies. One knows that μ is carried by a separable subspace $\Omega_\alpha\subset\Omega\cap\mathcal{H}_\alpha(T_0(M))$. In the same way ν is carried by a separable subspace $\Sigma_\alpha\subset\Sigma\cap\mathcal{H}_\alpha(M)$. Both are polish spaces, so that they have metrizable compactifications with polish boundaries of null capacity since both capacities are tight on compact sets. We can then apply Theorem 14 of [17]: there exists a quasi-continuous representative $\underset{\sim}{I}:\Omega\to\Sigma$, and there exists $\rho:\Sigma\to\Omega$ quasi-continuous, unique up to polar sets, and such that $\widetilde{f} = f\circ\rho$ where $\widetilde{f}$ is the image of f by the isomorphism $L^1(C^\sharp_{1,p}) \;\to\; L^1(\Gamma^\sharp_{1,p})$. It easily follows that ρ is a quasi-continuous representative of I^{-1}, that is $\rho\circ I = \mathrm{Id}_\Omega$ quasi-everywhere on Ω, and $I\circ\rho = \mathrm{Id}_\Sigma$ quasi-everywhere on Σ.□

21 Corollary: *For any $\varepsilon > 0$, there exist compact sets $K_1\subset\Omega$, $K_2\subset\Sigma$, whose complementary sets are of capacities $\leq\varepsilon$, and such that $\rho = I^{-1}$ is a homeomorphism of K_1 onto K_2.*

22 Remark: It is to be noticed that in all of these results, the compact sets of Ω (resp. of Σ) which are involved can always be taken in a given space $\mathcal{H}_{\alpha-1/p}$ for any $1/2 > \alpha > 1/p > 0$.

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