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Erratum: Concentration bounds for stochastic approximations

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Abstract

We correct an error in [2] pointed out to us by Bernard Bercu.

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1 Introduction

We correct an error of the previous manuscript [2] gently pointed out to us by Bernard Bercu. We will freely use the notations of [2] and suppose the reader is familiar with them.

The error is the following: at the end of the proof of Proposition 5.2 we omitted to report a factor $\sqrt{k}$ coming from the contribution $\mathbf{E}[|S_k|]$. The claim of the proposition should therefore be modified in the following way:

$$\delta_n \leq \exp\left(-\underline{\lambda}\Gamma_n\right)|\theta_0-\theta^*|$$
$$+[H]_1\sigma_Y\left(\gamma_n n^{1/2}+\sum_{k=1}^{n-1}e^{-\underline{\lambda}(\Gamma_n-\Gamma_{k+1})}\sqrt{k}(\gamma_k-\gamma_{k+1})+\sum_{k=1}^{n-1}e^{-\underline{\lambda}(\Gamma_n-\Gamma_{k+1})}\sqrt{k}\gamma_k\gamma_{k+1}\overline{\lambda}\right),$$

where we recall $\Gamma_n := \sum_{k=1}^n \gamma_k$, $\sigma_Y := \mathbf{E}\left[F^2(Y)\right]^{1/2} < +\infty$, with $F : y \mapsto \mathbf{E}\left[|y-Y|\right]$.

This does not affect the bound concerning the bias δ_N in Theorem 2.2 when the step sequence $(\gamma_n)_{n\geq 1}$ is s.t. $\gamma_n = \frac{c}{n}$. However, when $\gamma_n = \frac{c}{n^\rho}$, $\rho \in (1/2, 1)$ the above inequality does not provide a satisfactory control. In particular it gives an explosive bound for $\rho \in (1/2, 3/4]$. For $\rho \in (3/4, 1)$ it gives the convergence of δ_n to 0 at a sub-optimal rate.

2 Correct control of the bias in Proposition 5.2

This problem can be fixed by modifying Proposition 5.2. The new control writes:

$$\forall n \geq 1,\ \delta_n \leq \exp\left(-\underline{\lambda}\Gamma_n\right)|\theta_0-\theta^*| + [H]_1\sigma_Y\left(\sum_{k=0}^{n-1}e^{-2\underline{\lambda}(\Gamma_n-\Gamma_{k+1})}\gamma_{k+1}^2\right)^{\frac{1}{2}}. \tag{2.1}$$

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To prove (2.1), we start from the identity $z_{n+1} = (I - \gamma_{n+1} J_n) z_n + \gamma_{n+1} \Delta M_{n+1}$ obtained in the proof of Proposition 5.2.
Take now the square of the L^2-norm in the previous equality (instead of the L^1-norm as in [2]). Recalling that ΔM_{n+1} is a martingale increment, we derive:

$$\mathbf{E}[|z_{n+1}|^2] = \mathbf{E}[|(I - \gamma_{n+1} J_n) z_n|^2] + \gamma_{n+1}^2 \mathbf{E}[|\Delta M_{n+1}|^2] \leq \exp(-2\underline{\lambda}\gamma_{n+1})\mathbf{E}[|z_n|^2] + \gamma_{n+1}^2 [H]_1^2 \sigma_Y^2.$$

For the last inequality we used (exploiting assumption **(HUA)**) $||I - \gamma_{n+1} J_n|| \leq \exp(-\underline{\lambda}\gamma_{n+1})$, $\|.\|$ standing for the matrix norm on $\mathbf{R}^d \otimes \mathbf{R}^d$, and the inequality $\mathbf{E}[|\Delta M_{n+1}|^2] \leq [H]_1^2 \sigma_Y^2$ which follows from **(HL)**.

A direct induction yields for all $n \geq 1$:

$$\mathbf{E}[|z_n|^2] \leq \exp\left(-2\underline{\lambda}\Gamma_n\right) |z_0|^2 + [H]_1^2 \sigma_Y^2 \left(\sum_{k=0}^{n-1} e^{-2\underline{\lambda}(\Gamma_n - \Gamma_{k+1})} \gamma_{k+1}^2 \right).$$

Eventually, equation (2.1) should be reported in Theorem 2.2. As already indicated, the associated bound for the bias when $\gamma_n = \frac{c}{n}$ writes

$$\delta_N \leq \frac{|\theta_0 - \theta^*|}{N^{\underline{\lambda} c}} + K[H]_1 \sigma_Y \frac{1}{N^{\underline{\lambda} c \wedge \frac{1}{2}}},\ K := K(c),$$

which remains the same as previously, whereas for $\gamma_n = \frac{c}{n^\rho},\ \rho \in (1/2, 1)$ equation (2.1) yields

$$\delta_N \leq \exp\left(-\frac{\underline{\lambda} c}{1-\rho} N^{1-\rho}\right) |\theta_0 - \theta^*| + [H]_1 \sigma_Y \frac{K}{N^{\frac{\rho}{2} - \epsilon}}, K := K(c),\ \forall \epsilon > 0,$$

which also improves the formerly indicated bound. Let us also indicate that in the current framework, i.e under **(HUA)**, **(HL)**, computations similar to those leading to (2.1) can already be found in Duflo [1], see e.g. p. 56.

References

[1] Duflo, Marie. *Algorithmes stochastiques. (French) [Stochastic algorithms]*, Mathématiques & Applications (Berlin) [Mathematics & Applications], 23. Springer-Verlag, Berlin, 1996. xiv+319 pp. ISBN: 3-540-60699-8 MR-1612815

[2] N. Frikha and S. Menozzi, *Concentration bounds for stochastic approximations*, Electronic Communications in Probability 17 (2012), 1–15 (DOI: 10.1214/ECP.v17-1952)