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Antoine Gombaud, Chevalier de Méré

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Abstract. In 1654, Antoine Gombaud, Chevalier de Méré, approached Blaise Pascal with a question about the throw of dice and drew his attention to the problem of points, which had been around for 250 years or more in the Italian "abbaco" literature. A correspondence ensued between Pascal and Pierre de Fermat which is widely regarded as marking the birth of probability calculus. While historians of probability have rightfully focused on Pascal and Fermat, they have generally ignored Gombaud's part, portraying him only as a gambler, sometimes an avid one. Through a careful examination of Gombaud's life and philosophy, it is argued here that his role was more important than has usually been attributed to him. In addition, a review of the historical background to the problem of points shows that gambling was not as central to the early development of probability theory as has often been assumed.

Key words and phrases: Antoine Gombaud, Blaise Pascal, Chevalier de Méré, Pierre de Fermat, problem of points.

1. INTRODUCTION

In 1654, the French polymaths Blaise Pascal and Pierre de Fermat exchanged letters in which they solved some dicing puzzles and, more significantly, the long-standing problem of points. It is generally agreed that this correspondence was an important milestone in the development of modern probability concepts.

The problems had been proposed to Pascal by Antoine Gombaud, Chevalier de Méré. In the history of probability, Gombaud has generally been ignored or portrayed negatively as a mere gambler, sometimes as an inveterate one. Gouraud [31] even views his problems as "frivolous." When Gombaud's life and deeds are examined, however, a very different picture emerges.

The purpose of this paper is to shed light on Antoine Gombaud and his relation to the development of probability theory. A sketch of his life is first presented in Section 2, including the circumstances in which he met Pascal. The rapport of Gombaud to gambling is then examined in Section 3. In Section 4, the problem of points which Gombaud brought to Pascal's attention is described, along with its historical roots. Gombaud's dice puzzles are then discussed in Section 5, and his general relation to mathematics is considered in Section 6. Concluding comments can be found in Section 7.

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2. ANTOINE GOMBAUD

This section presents a short biography of Antoine Gombaud, Chevalier de Méré, as it pertains to the development of probability theory. Extensive information about his life is provided, for example, by Chamaillard [10], Revillout [53] or Taillé and Foisseau [61]; see also [30] for a complete collection of his writings, and a 50-page introduction in Vol. 1 by Charles-Henri Boudhors, which recounts the life of Gombaud and his family.

Born around 1607 in the county of Angoulême, France, Antoine was the third son of Benoît Gombaud, seigneur de Beaussais et Méré. Benoît embarrassed himself financially, and part of his estate was sold to cover debts. Despite financial setbacks, he promoted his sons' interests. Thus, Antoine Gombaud came to inherit the family estate (see Figure 1) on the deaths of his father and then his brothers; however, he was never a very wealthy man.

Antoine joined the lay religious Catholic Order of Malta at the age of 16, after which he was known as the Chevalier de Méré. At the time, this Order was also a relatively powerful military organization. Antoine Gombaud took up military service in France and was active in some conflicts, particularly naval battles, that occurred between the 1630s and the mid-1640s. He appeared at the court of Louis XIII when Armand Jean du Plessis, known as the Cardinal de Richelieu, was the king's chief minister.

FIG. 1. *The Château de Beaussais, property of the Gombaud family, circa* 1900. *Postcard from Rémy Foisseau's personal collection.*

2.1 Gaming in 17th Century France

Louis XIII enjoyed hunting but not gaming. Public gambling houses called "académies des jeux" were suppressed during his reign. There were 47 such academies operating in Paris alone at that time; see [60], pages 85–87. These closures affected mostly the common people; the nobility continued to gamble in secret.

Gombaud was seen as a good sport ("un beau joueur") who played in at least one of the academies once they reopened. On the rakish side of his personality, he indulged in several love affairs and was involved in duels which left him with lifelong effects from an injury.

On the accession of Louis XIV to the throne in 1643, things changed. In the transition years, the king's chief minister was Jules Mazarin, an Italian cardinal, diplomat, and politician. Mazarin introduced gaming to Louis XIV's court in 1648 and the "académies des jeux" reopened. As a young man, Louis XIV himself enjoyed gaming.

The types of games that were played in the "académies des jeux" from the mid-17th to the early 18th century can be gleaned from books that were written to provide the rules for the games played in these establishments. Sanchez [55] has examined several of these books published over more than a century, tracking the percentage of pages devoted to card games, table games such as billiards, dice games, physical games such as a forerunner of tennis, and mind games ("jeux d'esprit").

By the early 18th century, dice games and those exercising the mind disappeared from these books. During this period, card games grew substantially in popularity. The first of these rule books, entitled *La Maison academique*, appeared in 1654, that is, the year during which the Pascal–Fermat correspondence took place.

While living in Paris, Gombaud became involved in literary circles. This usually took place in salons organized and hosted by prominent women, often literary figures themselves. Gombaud is known to have attended the salon of French writer Madeleine de Scudéry [11]. This was a weekly gathering where new literary work was critiqued and philosophical discussions took place around literary and scientific ideas such as the mechanist theory of matter espoused by mathematician, physicist, and philosopher René Descartes.

2.2 "L'honnête homme"

It was probably within this salon culture that Gombaud developed and refined his philosophy of "honnêteté" (honesty) and "honnête homme" (honest man). The philosophy was honed in response to "La Fronde," a series of rebellions that took place in France between 1648 and 1653. Gombaud's ideas are found in his letters published in 1682 [28] and in essays such as *De la vraïe honnêteté* and *Suite de la vraïe honnêteté* published posthumously; see [29], pages 1–95.

This philosophy found its roots in the highly popular and influential book by Baldassare Castiglione entitled *Il libro del cortegiano* [*The Book of the Courtier*], originally published in 1528 and first translated into French in 1537. Written in dialogue form between courtiers in the Duchy of Urbino, *The Courtier* describes the ideal conduct and behavior of a gentleman attached to a Renaissance court. Castiglione's book had its roots in classical Roman writings such as Cicero's *De officiis* [*On Obligations*].

In *The Courtier*, Castiglione was "concerned with social advancement as much as with polite behavior"; see [23], pages 389–390. While akin to Castiglione's treatise regarding social behavior, Gombaud's ideal of "honnêteté" was to behave so as to achieve happiness for those around the "honnête homme"; see [57], page 25. The approach was both moral and aristocratic.

It is difficult to define the philosophy of "honnêteté" precisely but it could be described as an areligious morality reconciling the search for happiness with reason. In particular, happiness might be achieved by following the codes of propriety and social decorum, through eloquence of speech and proper use of language, through wisdom that leads to self-control and a control over one's destiny, by learning to discriminate between the excellent and the mediocre through accurately judging one's surroundings, and by displaying seemingly intuitive elegance. Above all, the "honnête homme" establishes his superiority over the group by bringing happiness to its members.

Incidentally, there was also a notion of "honnête femme" (honest woman) which, as a 17th-century ideal, involved an acquiescence with a high degree of dependence on men. As described by Turner ([63], p. 167), her qualities include "her sense of obedience or duty to father and husband and her sense of her moral reputation or gloire."

Gombaud's mastery of this approach eventually led him to tutor Françoise d'Aubigné, later known as Madame de Maintenon, in the art of courtly courtesy and "honnêteté." She then became governess to Louis XIV's children and eventually his mistress. They married sometime after the death of Queen Maria Theresa of Spain (Marie-Thérèse d'Autriche, in French).

2.3 Meeting with Pascal

One of Gombaud's acquaintances was Artus Gouffier, Duc de Roannez. They were both from the province of Poitou, while Blaise Pascal was from Auvergne. Roannez acquired the office of governor of Poitou in 1651; Gombaud lent him 10,000 livres to this end; see [43], page 253.

The duke had an interest in mathematics and was acquainted with Pascal's family for many years, but he only started being more closely associated with Blaise himself in 1651, when he followed him on the path of Jansenism.

According to Michaut ([45], pp. 68–69), it is the duke who introduced Pascal to Gombaud and other members of higher society. One of them was French writer Damien Mitton who, along with Gombaud, was a theorist of the ideal of the "honnête homme" [46].

Mitton had met and befriended Gombaud at the "académies des jeux" but unlike him, he was a bourgeois, not an aristocrat. Described by the poet and writer Tallemant des Réaux as a "grand joueur" ([32], pp. 32–34), he eventually became Pascal's model of a "libertin."

Around 1652, Gombaud, Mitton, Pascal and the Duke of Roannez traveled together to Poitou. Gombaud's description of the trip is revealing about both Pascal and Gombaud. Here is a translation in modern English of the key passage of the original French text:

> "I [Gombaud] traveled with the D. D. R. [Duc de Roannez], a fair and thoughtful man whom I find eminently sympathetic. M. M. [Mr. Mitton], whom you know and who is universally appreciated at the court, came along; and as it was a short trip rather than a long journey, we only thought of enjoying ourselves, and discussed a wide variety of topics. The D. D. R. has a mathematical mind, and in order to be entertained along the way, he had invited a middle-aged man [Pascal], who was not much known back then, but who has since made a name for himself. This man excelled at mathematics, but he knew nothing else. This science does not teach you social behavior, and this man, who had no taste or manners, was constantly mingling in our conversation with remarks that were almost always surprising and that often made us laugh. He admired the thoughts and eloquence of M. du Vair [a French author who passed away in 1631], and was relating funny lines from the Marquis d'O [a long-deceased "mignon" of Henry III]. We did our best to disabuse him, while always addressing him in good faith. After two or three days, he grew conscious of his deficiencies and limited himself henceforth to listening and questioning in order to be better informed about whatever topic was being discussed, and from time to time he would draw out tablets on which he jotted down a remark. We were impressed that by the time we reached P... [Poitiers], almost everything he said made good sense, and in line with what we could have said ourselves, and to be truthful, he had improved considerably."[1]

For a different translation, see Duclaux [18], page 97.

As an "honnête homme," it was Gombaud's ambition to act as the master that would transform Pascal into a man of the world. With time, however, Gombaud's philosophy of the "honnête homme" gradually went out of fashion, and along with it Gombaud himself. In a letter to her daughter dated November 24, 1679, Madame de Sévigné, famous for her correspondence, referred to Gombaud's writing style as wretched; see [42], page 745.

Gombaud passed away on December 29, 1684. When the news reached Versailles, the Marquis de Dangeau ([41], p. 111) wrote in his diary, on January 23, 1685:

> "I learned of the death of the Chevalier de Méré; he was a man of great wit who had written books that did him no great credit."[2]

Never mighty, Gombaud had still fallen.

3. THE CHEVALIER AND GAMBLING

As mentioned earlier, Gombaud's philosophy of "honnêteté" and the "honnête homme" had its roots in Castiglione's *The Courtier*. In one of his dialogues, Castiglione commented on gambling. It is in an interchange between two Italian courtiers, Gaspare Pallavicino and Federico Fregóso, who eventually became a cardinal.

[1] "Je fis un voyage avec le D. D. R. qui parle d'un sens juste & profond, & que je trouve de fort bon commerce. M. M. que vous connoissez, & qui plaît à toute la Cour, étoit de la partie; & parce que c'étoit plûtôt une promenade qu'un voyage: nous ne songions qu'à nous réjoüir, & nous discourions de tout. L. D. D. R. a l'esprit mathematique, & pour ne se pas ennuyer sur le chemin, il avoit fait provision d'un homme d'entre d'eux âges, qui n'étoit alors que fort peu connu, mais qui depuis a bien fait parler de lui. C'étoit un grand Mathematicien, qui ne savoit que cela. Ces sciences ne donnent pas les agrémens du monde, & cét homme qui n'avoit ny goût, ni sentiment, ne laissoit pas de se mêler en tout ce que nous disions, mais il nous surprenoit presque toûjours, & nous faisoit souvent rire. Il admiroit l'esprit, & l'éloquence de M. du Vair, & nous rapportoit les bons mots du Lieutenant Criminel d'O; nous ne pensions à rien moins qu'à le desabuser: cependant nous lui parlions de bonne foi. Deux ou trois jours s'étant écoulez de la sorte, il eut quelque défiance de ses sentimens, & ne faisant plus qu'écouter, ou qu'interroger, pour s'éclaircir sur les sujets qui se presentoient, il avoit des tablettes qu'il tiroit de tems en tems, où il mettoit quelque observation. Cela fut bien remarquable, qu'avant que nous fussions arrivez à P... il ne disoit presque rien qui ne fut bon, & que nous n'eussions voulu dire, & sans mentir c'étoit étre revenu de bien loin."

[2] "J'appris la mort du Chevalier de Méré; c'étoit un homme de beaucoup d'esprit qui avoit fait des livres qui ne lui faisoient pas beaucoup d'honneur."

FIG. 2. *Georges de La Tour, Les Joueurs de dés* (1650–51). *©Preston Hall Museum, Stockton-on-Tees*.

As in other parts of *The Courtier*, Federico is the more prudent man. Here is what Castiglione had to say about gambling ([9], p. 140):

> 'And which games?' asked signor Gaspare.
> Federico answered with a laugh: 'For this let us go for advice to Fra Serafino, who invents new ones every day.'
> 'Joking apart,' answered signor Gaspare, 'does it seem to you that it is wrong for the courtier to play at cards and dice?'
> 'To me, no,' said Federico, 'unless he does so too assiduously, and in consequence neglects things of greater importance, or indeed for no other reason than to win money and cheat his partner, and then, when he loses, is so dismayed and angry as to prove his avarice.'

3.1 Gombaud's View on Gambling

Gambling was fashionable in the early 1650s (see Figure 2), and there is no doubt that Gombaud gambled, too. In fact, he was sometimes present, with Pascal as his playing partner, at Mitton's home and at those of some others including that of the satirical poet, Guillaume Bautru; see [10], page 86. However, Gombaud's general opinion on gambling, parallel to what appears in *The Courtier*, is reflected in two of his letters to Mitton; see [28], pages 196–198, 298–300.

In the first letter, Gombaud writes about the pleasures of being in the countryside and of being away from Parisian high society. In the second letter, he reiterates his love of the countryside and then tells Mitton:

> "Yet you feel sorry for me as soon as I leave Paris, and you think that honest people who are anywhere else are to be pitied. I have to say, though, that I also pity you for being confined in games, for worrying about nothing but your luck [or fortune], and for laying your eyes on nothing else but this artificial world, as is the case for nearly all courtiers to whom the greatest wonders of nature are unknown." [3]

Like Castiglione, Gombaud advocated moderation in playing games of chance and the gambling associated with it. It was part of his persona as an "honnête homme." Based on the quote alone, Mitton appears to be the gamester, not Gombaud. Between 1653 and 1660, Mitton's home was a meeting place for several gamblers, though his interest in gambling decreased with age.

Gombaud was obviously far from being an inveterate gambler. Rather, he was an experienced player who won more often than he lost, but never to excess. He knew the rules of various games, probably a number of card games, very well. He also lived his philosophy of the "honnête homme" while at play, which implied a strict adherence to rules. At the court of Louis XIV, he was sometimes called on to referee disputes that arose at various tables, especially those where military men played. After listening to both sides, Gombaud's decision in the dispute was the final word; see [10], pages 26–27.

In harmony with Castiglione's comments on gambling in *The Courtier*, Gombaud's philosophy of gaming was to play both correctly and graciously; see [10], "Lettres et fragments choisis," pages 94–97. He went one step further, which many today would find patronizing. As part of gracious play, he advocated letting others, especially women, win sometimes. This, he thought, could lead to a greater benefit than in actually winning the game. In general, Gombaud did not think highly of players who took their gambling so seriously as to neglect others.

3.2 Gombaud's Book on Gambling

Late in his life, Gombaud wrote a book on the Spanish card game *Hombre* [27]. The book has no probability calculations in it. Rather, it describes the rules of the game and how to play it. It is written in the form of a letter to a woman who wants to know about the game, as it was just coming into fashion.

In his book, Gombaud mentions how the game was played in Paris, Versailles, and Saint-Germain-en-Laye, the latter two places being the residences of Louis XIV at the time that the book was written. Also mentioned is that Queen Maria Theresa of Spain played the game. The book, which was reproduced in its entirety in the 1697 edition of *La Maison academique*, has every appearance

[3] "Cependant vous me plaignez si tôt que je m'éloigne de Paris, & vous pensez que par tout ailleurs les honnestes gens sont à faire pitié. Mais je vous avouë aussi que je vous plains à mon tour d'estre confiné dans le jeu, de ne soupirer qu'aprés la fortune, & de n'avoir des yeux que pour le monde artificiel, comme presque tous les Courtisans à qui les plus grandes beautez de la nature sont inconnuës."

of an "honnête homme" giving sound advice to a woman who wants to do well at court by participating in the typical court activity of playing cards.

3.3 Leibniz's View on Gombaud

In the mid to late 1690s, the German mathematician and philosopher Gottfried Wilhelm Leibniz was in correspondence with his friend and colleague in Paris, encyclopedist Gilles Filleau des Billettes, about the mathematical developments of 1654 and about the man who had suggested these probability problems.

Leibniz, who knew of Gombaud but never met him, described him as a "gentilhomme joueur" with extraordinary insight. Leibniz had been in Paris about 20 years earlier and had heard about the events of 1654 from the Duc de Roannez. In response to Leibniz's question about the unknown man, Billettes, who had met Gombaud, described him as a "grand joueur"; see [43], page 370.

Today, "grand joueur" usually has the pejorative translation of inveterate gambler and ties in with modern beliefs about Gombaud. Without further qualification, as in "joueur de tennis," the word "joueur" generally translates to player but circa 1700, it meant either player or gambler [7]. It is very reasonable to translate "gentilhomme joueur" circa 1700 as a gentleman player, who, for example, could referee card game disputes at court. Likewise, "grand joueur" could be rendered as an expert player, who, for example, was knowledgeable enough to write in detail about the latest fashionable card game.

4. THE PROBLEM OF POINTS

The earliest known correspondence between Pascal and Fermat is about a dicing problem. A player agrees to play for an amount staked by rolling a die eight times in a row; see [16], pages 288–289. If a 6 shows on any throw, the player wins the pot. After three throws, 6 has not shown. If the player decides to forego his fourth throw, what fraction of the pot should this player be given?

In the undated letter which first presents an intentional setup with false reasoning that looks back to the three previous throws, Pascal correctly reasons that the answer to the question should be 1/6 of the pot. The insight, which was not explicitly stated, is that probability calculations are statements about the future, not the past. This insight is the key to solving the problem of points.

The first dated correspondence is from Pascal to Fermat on 29 July 1654 ([16], p. 290):

> "J'admire bien davantage la méthode des parties que celle des dés; j'avois vu plusieurs personnes trouver celle des dés, comme M. le chevalier de Méré, qui est celui qui m'a proposé ces questions, et aussi M. de Roberval: mais M. de Méré n'avoit jamais pu trouver la juste valeur des parties ni de biais pour y arriver, de sorte que je me trouvois seul qui eûsse connu cette proportion."

David's translation ([13], p. 84) begins after the semicolon.

> "I have seen several people obtain that for dice, like M. le Chevalier de Méré, who first posed these problems to me and also M. de Roberval. But M. de Méré has never found the true value for the division of stakes nor the method of deriving it so I find myself alone in discovering this."

This ties in directly with what the French mathematician Montmort ([17], p. xxi) wrote in his *Essay d'analyse sur les jeux de hazard*, provided here in English translation of the original French:

> "The Chevalier de Méré had proposed to him [Pascal] this problem, he had also proposed to him a few others on dice: for example, determine in how many throws you can obtain a certain pair, & some others of this type that are relatively easy. This knight, who was more of a fine wit than a geometer, solved the dicing problems, but neither he nor M. de Roberval could solve the problem of points. M. Pascal proposed it to M. Fermat who was writing to him in friendship and as a mathematician, & who in this field was only inferior to M. Descartes."[4]

Gilles de Roberval was a prominent French mathematician who held positions initially at Collège Gervais in Paris and who was then Chair of Mathematics, a position once held by Peter Ramus, at the Collège royal, also in Paris. Like the Italian mathematician Niccolò Tartaglia before him, Roberval thought that a solution to the problem could not be obtained; see [43], page 371.

Montmort probably saw the Pascal–Fermat correspondence in Fermat's *Varia opera mathematica* published in 1679. Gombaud could solve some of the "easier" problems in probability but not the crucial one, the problem of points. Roberval, who was also a good mathematician, could not solve the problem of points either.

Gombaud was a writer, but not a first-rate one. He enjoyed the literary discussions in the salons. Judging from

[4]"Le Chevalier de Méré lui avoit proposé ce Problême, il lui en avoit aussi proposé quelques autres sur les dez: Par exemple, déterminer en combien de coups on peut amener une certaine rafle, & quelques autres de cette sorte assez faciles. Ce Chevalier, qui étoit plus bel esprit que Geometre, resolut les Problêmes sur les dez, mais ni lui ni M. de Roberval ne purent resoudre celui des partis. M. Pascal le proposa à M. Fermat avec qui il étoit en commerce d'amitié & de Geometrie, & qui en cette Science n'étoit inférieur qu'à M. Descartes."

his letters [28], he honed his ideas about the "honnête homme" through discussions in these salons. Likewise, Gombaud had some mathematical abilities but was not a first-rate mathematician. His mathematical interests had a similar flavor to the salon culture. He threw out and discussed mathematical problems with people who exhibited a range of mathematical abilities.

The question that was thrown to this group of mathematicians, and which could only be solved by Pascal at first, was the problem of points. Here is how Pascal described the problem and his solution to Fermat in a letter dated July 29, 1654 ([16], pp. 290–291):

> "Voici à peu près comme je fais pour savoir la valeur de chacune des parties, quand deux joueurs jouent, par exemple, en trois parties, et chacun a mis 32 pistoles au jeu:[5]
>
> Posons que le premier en ait deux et l'autre une; ils jouent maintenant une partie, dont le sort est tel que, si le premier la gagne, il gagne tout l'argent (qui est au jeu, savoir 64 pistoles); si l'autre la gagne, ils sont deux parties à deux parties, et par conséquent, s'ils veulent se séparer, il faut qu'ils retirent chacun leur mise, savoir chacun 32 pistoles.
>
> Considérez donc, Monsieur, que, si le premier gagne, il lui appartient 64; s'il perd, il lui appartient 32. Donc, s'ils veulent ne point hasarder cette partie et se séparer sans la jouer, le premier doit dire: 'Je suis sûr d'avoir 32 pistoles, car la perte même me les donne; mais pour les 32 autres, peut-être je les aurai, peut-être vous les aurez, le hasard est égal. Partageons donc ces 32 pistoles par la moitié et me donnez, outre cela, mes 32 qui me sont sûres.' Il aura donc 48 pistoles et l'autre 16."

David's translation ([13], p. 85) begins at the second paragraph.

> "Suppose that the first player has gained 2 points and the second player 1 point. They now have to play for a point on this one condition, that if the first player wins he takes all the money which is at stake, namely 64 pistoles, and if the second player wins each has 2 points, so they are on terms of equality, and if they leave off playing each ought to take 32 pistoles.
>
> Thus, if the first player wins, 64 pistoles belong to him, and if he loses, 32 pistoles belong to him. If then the players do not wish to play this game, but to separate without playing it, the first player would say to the second: 'I am certain of 32 pistoles even if I lose this game and as for the other 32 pistoles perhaps I shall have them and perhaps you will have them; the chances are equal. Let us divide these 32 pistoles equally and give me also the 32 pistoles of which I am certain.' Thus, the first player will have 48 pistoles and the second 16."

4.1 Origins of the Problem

The problem of points was not new. By the time of the Pascal–Fermat correspondence, it had been around for at least 250 years. Although set in a gambling context, it was not originally a gambling problem. When first posed, it was set as an exercise for commercial mathematics students in Renaissance Italy.

Essentially the same problem considered by Pascal shows up in an Italian manuscript circa 1400 [54, 56]. In the manuscript, two players are playing a "schacchi" (chess) tournament. They have each staked one ducat to make up the prize. As in the Pascal–Fermat version of the problem, the two ducats are given to the first player to win three games. However, the opponents are forced to stop playing after one of them has won two games. In contrast to the Pascal–Fermat version, though, the other chess player has not won any games.

How should the two ducats be divided between the players? Using the logic that Pascal applied in his letter to Fermat, the second chess player must win the next two games in order to come even with the first. There are four possibilities for the player who is behind two games: win/win, win/lose, lose/win, and lose/lose. In only one of these four (equally likely) situations does this player draw even (win/win). Then the first player has $1\,1/2$ ducats for certain and an equal chance at the remaining $1/2$ ducat. If the game were to terminate at that point, the player who is ahead two games should receive $1\,3/4$ ducats and the player who is behind should receive $1/4$ ducat.

The poser of the problem had a correct procedure different from the above, but found the wrong answer due to an algebraic blunder at the end of their calculations. With one exception, incorrect answers to similar problems would continue to be offered for 250 years or so.

As expressed in the Pascal–Fermat correspondence or the chess game problem, the name given to these puzzles is "the problem of parts" or the "division of stakes," based on who gets what part of the prize. If only the number of games played is counted, one speaks of the "problem of points," one point being assigned to winning a game.

4.2 Roots in Commercial Mathematics

To understand the nature of the problem of points and how, at heart, it is not a gambling problem, context is important. The chess problem in the manuscript from circa

[5] According to various sources, 1 pistole = 10 livres was roughly 1/10th of the annual salary of a lackey or a coachman. A small house (1 chimney, two doors, two windows) would have costed around 200 livres. By that standard, the stakes were relatively high.

1400 was set for students in a course of study in elementary mathematics related to business practice in late 14th or early 15th century Italy. In the manuscript describing the problem, there is no mention of probability, chance or randomizing device. The audience was not comprised of university students. Rather, teaching of this material was done in so-called "abbaco" schools that taught business arithmetic [64]. The problem was probably conceived as an exercise in the relatively new mathematics called algebra that had been imported from Arabic scholars about 200 years before.

The development of commercial mathematics and "abbaco" schools is directly tied to the rise of trade in the Mediterranean region, especially with the Muslim world [37]. Early Italian trade began with Venetian visits to Constantinople in the 8th century. Formal trading relations between Venice and Constantinople were established in the late 10th century. The Venetian example was followed by Pisa and Genoa in the early 12th century.

Italian trade at Muslim ports in North Africa began as early as the 9th century and picked up substantially in the 11th and 12th centuries following increasing shipping volume related to the Crusades. Genoa and Pisa were the main traders into North Africa. Trade with the latter peaked in the 12th and 13th centuries, and then changed to other parts of the Mediterranean [20, 39]. Part of what encouraged trade was the suppression of piracy and brigandage [64].

With the rise in trade, "abbaco" books, or method books in commercial arithmetic, began to appear along with "abbaco" schools to teach these commercial arithmetic methods. Van Egmond [64] has provided a catalog of Italian "abbaco" manuscripts and printed books written prior to 1600. He has also given a brief analysis of these books, noting that they were typically reference manuals for merchants or "abbaco" teachers. They were also written in a characteristic style with standard contents including definitions of the basic arithmetic operations, business problems, and recreational mathematics problems. These were often, but not always, accompanied by a discussion of elementary geometry and algebra as well as miscellaneous material such as calendars and astrology. The geometry in these books is mostly arithmetical, dealing with lengths, areas, and volumes [50].

A prime example of a mathematician working in this milieu is Leonardo Bonacci, or Fibonacci. In the prologue of his *Liber abaci*, Fibonacci [52] gives a sketch of his mathematical education. He initially studied in Bougie, or what is now Bejaia, a Mediterranean port in northeastern Algeria. His father had been posted there by the Pisan government. Later he traveled extensively in the Mediterranean area studying mathematics all the while, and then returned to Pisa in about 1200 CE at the age of 30.

Fibonacci wrote the first version of his arithmetical treatise *Liber abaci* in 1202; a second edition appeared in 1228. What became typical of "abbaco" books like his is that Arabic, rather than Roman, numerals were used.

The problem of points is not treated in Fibonacci's *Liber abaci*. A related problem that became a stumbling block to the solution of the problem of points is covered instead. This is the division of profits upon completion of a business venture; see [52], pages 213–226. The profits are divided in proportion to the amounts invested by those participating in the venture. Some used this approach to try to solve the problem of points.

In the context of the problem of points, the division of profits looks to the past, that is, the games that have been played. As pointed out by Pascal, however, probability looks to the future, that is, the games that are left to play.

4.3 "Abbaco" Books Listing the Problem of Points

Here is a list of "abbaco" books and manuscripts known to contain the problem of points. In each case, the problem is given as stated in the book or manuscript.

1. *Vatican Library Urb. Lat. 291*: The problem was stated in manuscript form circa 1400 and was studied by Franci [24]. According to Van Egmond ([64], p. 216), the main body of the manuscript is based on the work on algebra by Gherardo da Cremona, a mathematician based in Toledo, Spain.

The question is more difficult than the usual problem of points as it involves three players rather than two. Suppose A, B, C play a series of games, the nature of which is unspecified, with stakes of two soldi (silver coins). The first player to win three games gets the two soldi. After A has won two games, B one game, and C none, how should the two soldi be divided among the three players? The author of the manuscript provides a correct solution, a feat not achieved for another 250 years.

2. *National Library of Florence Magl. Cl. XI. 120*: Also written circa 1400 ([64], p. 119), this manuscript is entitled *Regole de l'arzibra* or rules of algebra. It was carefully studied by Rigatelli [54]. Two problems are considered.

The first problem is the chess game mentioned earlier. Two contestants enter a best-of-five contest for which the reward is one ducat. They stop after one player has won two games and the other none. The correct method to obtain the solution is described, but an algebraic blunder is made in the last step.

The second problem is similar to the first and its solution is left incomplete. In this case, the two players engage in a best-of-seven series which is stopped after one of the contestants has won three games and the other none.

3. *Library of Siena L. VI. 45*: This manuscript was written in 1495 by the mathematician Filippo Calandri; see Van Egmond [64], pages 192–193. It was also studied by Rigatelli [54]. The production of the manuscript was

30 years after the first books were printed in Italy using movable type. The first books were typically religious in nature or were editions of ancient Roman authors such as Cicero. The printer carried all the financial risk in bringing books to press and so printed only those books which could sell well. In the manuscript, Calandri considered a number of "abbaco" and geometrical problems, two of them related to the problem of points. Calandri could not solve successfully either of the two problems of points.

In the first problem, two players play a version of the Florentine football game called "palla grossa" with each player putting up 3 lira for a total prize of 6 lira. The winner is the first to catch 5 balls. However, after one player has caught 4 balls and the other 3, the ball is punctured and play cannot be continued. How should the prize be divided between the two players?

In the second problem, three persons are shooting crossbows for a prize of 3 denari. The first to make three hits wins the prize. When one of the contestants has made two hits, the second a single hit, and the third none, a crossbow breaks and the game has to end. How should the prize be divided? This is the same problem as stated in item 1 above, but with crossbows added.

4. *Pacioli [49]*, fols. 197r and 198v: *Summa de arithmetica*, by Luca Pacioli, is a compendium of material on arithmetic, algebra, and geometry related to business problems and often based on the work of others. It contains the first printed treatment of double entry bookkeeping. The setup of Pacioli's two versions of the problem of points is very similar to Calandri's, differing only in the numbers and some terminology.

As in Calandri's first problem, two companies play a ball game with a "palla," but not a "palla grossa." The score is counted in 10's. The first to score 60 wins 10 ducats. When one side has 50 points and the other has 20, the game is discontinued due to some incident. How should the prize be divided?

Pacioli looks at the solution in three different ways and comes up with the same answer: the one who is ahead should take 5/7 of the 10 ducats and the one who is behind should take 2/7 of the amount. The thinking is directly related to the division of profits in a business enterprise. The profits shared are in proportion to the amounts brought to the table.

As in Calandri's second problem, there is a crossbow contest with three participants. This time the first person to make 6 hits wins 10 ducats. After the first contestant has made 4 hits, the second 3 hits, and the third 2 hits, they no longer wish to continue. How should the 10 ducats be divided among the three players?

5. Cardano [8]: The book *Practica arithmetica*, by Gerolamo Cardano, marks a distinct change from earlier treatments of the problem of points. It was written in Latin rather than in the vernacular. Smith [58] described the book as "one of the most pretentious arithmetics of the 16th century," but added that "it did much to influence the advanced teaching of the subject." As Cardano taught in a university, the book was probably meant for students and professors. In addition to the change in language, the problem of points was described generically.

Two persons play a series of games of an unspecified type that should end when one of them has won 10 games. After 16 games, one player has won 7 and the other has 9. What share should each player have if the series is stopped early after the 16 games? Cardano uses this and other examples to demonstrate that Pacioli's solution is incorrect. However, the solution he offers is also incorrect.

6. Tartaglia [62]: Niccolò Tartaglia considered the problem of points in his *General trattato di numeri et misure* published in 1556. Tartaglia took Pacioli's problem with a slight change in numbers (one player now has 30 rather than 20 points) and demonstrated why Pacioli's solution could not be correct. He doubted that a solution was possible but gave one anyway, also incorrect.

Tartaglia's book was very popular in Italy for several decades. Guillaume Gosselin translated the book into French under the title *L'arithmetique de Nicolas Tartaglia*. It was published in 1578 and 1613. The problem of points does not seem to appear in this book, but it is possibly from a similar source that Gombaud first came across the issue.

7. Peverone [51]: Giovanni Francesco Peverone was an "abbaco" teacher. Peverone's solution to the problem of points appears in his *Due brevi e facili trattati* published in 1558. His treatment and solution of the problem is a translation into Italian of Cardano's problem originally written in Latin.

8. Forestani [22]: Lorenzo Forestani was a Roman Catholic priest and a member of the Franciscan Order. His *Practica d'arithmetica, e geometria* contains a well-stated, but incorrectly solved, version of the problem of points. The problem was put in terms of a specific kind of game.

An old gentleman in his villa enjoyed ball games (*giuco di palla*). He called to two young men who were laborers to play the game in his presence. He offered 4 ducats to the first to win 8 games. The ball was lost after one player had won 6 games and the other had won 3. The gentleman gave the 4 ducats to the young men and told them to divide it between them. How should the 4 ducats be divided?

4.4 Additional Comments

It is remarkable that except for Cardano, the versions of the problem of points reviewed above were not cast in a gambling context. Rather, they were often expressed as athletic contests with a prize for winning. The prize was usually financed by an entrance fee for the contest or by a third party. Cardano's formulation might stem from the fact that early in his career, he was an avid gambler.

It is also worth keeping in mind that the problem of points may very well predate 1400. In view of the Italian trading connections across the Mediterranean, and the fact that much of the Italian early commercial arithmetic may have been imported from the Muslim world, the problem of points may have Arabic origins [44]. While such connections may exist, to date no one has found them nor anything that predates 1400 in Italy.

In this section, the focus has been on the nature of the problem of points as posed. Readers interested in the technical details of the solutions in the printed books can refer, for example, to Montucla [47], Lubbock and Drinkwater Bethune [38], Kendall [36], David [13], Edwards [19], Schneider [56], Hald [33], and Franklin [25]. Franci [24] and Rigatelli [54] have provided technical details for the manuscripts.

As an interesting aside, Edwards [19] has argued that Pascal was the one responsible for the solution to the problem of points and that in so doing, he predated Huygens [34] in the use of the notion of expectation.

5. GOMBAUD'S PROBLEM

The Chevalier de Méré's problem, as it is generally referred to, appears near the end of the first surviving letter in the correspondence between Pascal and Fermat, dated July 29, 1654; see [16], pages 295–196. Pascal having described to Fermat his solution to the problem of points, he then commented on Gombaud's dicing question. Here is an English translation of the relevant excerpt from [13], page 88, in which the quotation is divided into two parts as it mentions two different issues.

> "I haven't time to send you a solution of a difficulty which has puzzled M. de Méré. He has good intelligences but he isn't a geometer and this, as you realize, is a bad fault. He does not understand even that a mathematical line is infinitely divisible and holds very strongly that it is composed of a finite number of points and never have I been able to dissuade him of this. If you are able to solve the difficulty it would be perfect." [6]

In an undated letter to Pascal, Gombaud responds to Pascal's criticism of his view of the infinite divisibility of a line as expressed in the quotation. This is addressed in Section 6. The second part of the quotation is:

> "He [Méré] said to me that he has found falsehood in the theory of numbers for the following reason. If I undertake to throw a six with one die, there is an advantage in undertaking to do it in 4 throws, as 671 to 625. If I undertake to throw the "Sonnez" with two dice there is a disadvantage in undertaking to do it in 24. And moreover 24 is to 36 (which is the number of pairings of the faces of two dice) as 4 is to 6 (which is the number of faces of one die). This is his "grande [sic] scandale" which makes him say loftily that the propositions are not constant and that Arithmetic is self-contradictory. But you will see it very easily by the principles you have." [7]

More than 40 years later, Leibniz was in correspondence with his friend and colleague Billettes about this very subject, as mentioned in Section 3. In a letter dated October 27, 1697, Billettes commented to Leibniz that Gombaud had named this area of research "géométrie mobile"; see [43], pages 370–371. This expression can be translated as "moving mathematics" or perhaps "fuzzy mathematics." This might be taken to mean that Gombaud thought that the underlying principles of probability, compared to the axioms of Euclidean geometry or the rules of arithmetic, were malleable or vague.

A typical modern solution to the Chevalier de Méré's problem is to calculate $1 - (5/6)^4$ for four throws of a single die and $1 - (35/36)^{24}$ for 24 throws of two dice. These probabilities are approximately equal to 0.5177 and 0.4914, respectively.

In her writings, the English statistician F.N. David ([13], p. 89) enhanced the belief that the Chevalier de Méré was a gamester. She concluded that the difference between the two events was determined by Gombaud's own gambling experience. Her conclusion is in a footnote to the above quotation, namely:

> "The Chevalier de Méré was obviously such an assiduous gambler that he could distinguish empirically between a probability of 0.4914 and 0.5, that is, difference of 0.0086, comparable to that (0.0108) of the gambler who asked advice of Galileo."

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[6] "Je n'ai pas le temps de vous envoyer la démonstration d'une difficulté qui étonnait fort M. de Méré: car il a très bon esprit, mais il n'est pas géomètre; c'est, comme vous savez, un grand défaut et même il ne comprend pas qu'une ligne mathématique soit divisible à l'infini, et croit fort bien entendre qu'elle est composée de points en nombre fini, et jamais je n'ai pu l'en tirer. Si vous pouviez le faire, on le rendrait parfait."

[7] "Il me disait donc qu'il avait trouvé fausseté dans les nombres par cette raison. Si l'on entreprend de faire six avec un dé, il y a de l'avantage à l'entreprendre en quatre coups, comme de 671 à 625. Si l'on entreprend de faire sonnez avec deux dés, il y a désavantage à l'entreprendre en 24 coups. Néanmoins 24 est à 36, nombre de faces de deux dés, comme 4 est à 6, nombre des faces d'un dé. Voilà quel était son grand scandale qui lui faisait dire hautement, que les propositions n'étaient pas constantes et que l'Arithmétique se démentait: mais vous en verrez bien aisément la raison par les principes où vous êtes."

Regrettably, David has put everyone on the wrong track by her interpretation of this quotation from Pascal.

To begin, Spiegelhalter ([59], pp. 205–207) shows in a simulation study that it only becomes apparent after about 400 throws that the event with 4 throws of a single die is better, in terms of higher probability, than the event with 24 throws with two dice.

To make the matter a little more extreme, Pascal wrote about a slightly different issue. As can be seen from the translation of Pascal's letter to Fermat, the 4-throw event is said to be advantageous (probability greater than $1/2$ in modern parlance), while the 24-throw event is disadvantageous (probability less than $1/2$). From Spiegelhalter's graph, the disadvantage in the 24-throw event is only apparent after more than 5000 throws of the two dice. In either situation, that is a lot of gambling and keeping track of the outcomes. And this is all before the concept of long-run frequency was applied to the interpretation of probability and before the word "probability" itself was used to describe the numerical likelihood of chance events.

5.1 Different Approach

Here is a different approach from the one used by Spiegelhalter, but which arrives at the same conclusion. Since Gombaud gave the correct odds for the throw of four dice and said only that it was disadvantageous to bet on seeing two sixes in 24 throws of two dice, standard sample size calculations can be used to illustrate the large number of throws necessary to discover that the throws result in a chance that is less than $1/2$.

Figure 3 shows the risk of error at various sample sizes. What is plotted on the horizontal axis is, for a given number of throws, the probability that the observed proportion of throws is greater than $1/2$ when the probability of a throw is $1 - (35/36)^{24}$. The vertical axis shows the number of throws ranging from about 900 to 32,500. If one allows for a 30% risk of error, it transpires from Figure 3 that only about 900 throws are needed to conclude that the proportion of throws is less than $1/2$. At the extreme end, tightening the risk of error to 0.1% requires in excess of 32,000 throws of the dice. For a moderate risk of error, say 5%, the required number of throws is over 9000.

Using either Spiegelhalter's simulation study or Figure 3, it seems highly unlikely that gambling experience or a private empirical study was the basis of Gombaud's thought that the 24-throw event is disadvantageous.

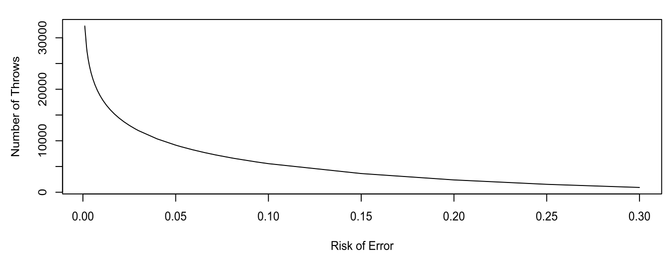

FIG. 3. *The number of throws required from two dice to allow for a set risk of error.*

5.2 Another Take on the Problem

We would like to offer a different take on the chevalier's problem. First, we need to put ourselves in the mindset of probability calculators of the 17th century.

The word "probability" was not used to describe chance calculations until the early 18th century. The Swiss mathematician Jakob Bernoulli was the first to define probability in this context somewhere between 1690 and his death in 1705. Indeed, probability is not used as a word by Bernoulli [3] regarding chance calculations on dice, but is defined in his 1713 posthumous work [4]. It first appears in print as a numerical measure related to chance in Nicolaus Bernoulli's doctoral thesis [6].

Nicolaus Bernoulli had access to his uncle's manuscript which was eventually published in 1713. The probability calculations of 0.5177 and 0.4914 related to the chevalier's problem mentioned earlier would have been foreign to mathematicians in the 17th century. Instead, they would have calculated the number of chances in favor of an outcome. Any comparisons would have been done through the odds—the number of favorable outcomes to the number of unfavorable outcomes.

Gombaud used odds in the one calculation that can be attributed to him. He was able to state correctly that the odds in the four-throw event are 671 to 625. We conjecture that he solved the problem in the following way:

(a) The number of ways of throwing four dice is $6^4 = 1296$. The number of ways of throwing the four dice so that no sixes show is $5^4 = 625$. Hence, the number of ways of throwing at least one six (expressed as throwing a 6) is $6^4 - 5^4 = 671$.

(b) Doing the intermediate calculations, Gombaud would have found that the odds of obtaining a six in one, two, and three throws of a single die are 1 to 5, 11 to 25, and 91 to 125, respectively, with the odds changing in favor of throwing a six for four throws of the die.

For the throws of two dice, we conjecture that Gombaud thought that the transition from an unfavorable event to a favorable one occurred at 24 throws of two dice. The transition would have been based on applying the Rule of Three in arithmetic, a long-standing rule that Gombaud would have known.

Four throws of a single die with six faces results in a favorable event. By the Rule of Three, the required number of throws to get a favorable event should be obtained from the equation $4/6 = x/36$ or 24 throws. This falls in line with the way in which Pascal gives Gombaud's reason: 24 is to 36 as 4 is to 6. When Gombaud did the calculation, he found that this is not the case and concluded that arithmetic is self-contradictory.

How Gombaud did the calculation, rather than carrying out an empirical study or gambling for hours on end at one particular game that no one else played at court or in the "académies des jeux," might be guessed at from Huygens' solution published just three years later [34].

5.3 The Solutions by Huygens and Bernoulli

The Dutch mathematician and astronomer Christiaan Huygens had heard of Pascal and Fermat's work but had not seen the details. Huygens' solution to the chevalier's problem is contained in Propositions X and XI of his *De ratiociniis in ludo aleæ*. Proposition X deals with the throw of one die and Proposition XI deals with two dice. Both are set in terms of finding the transition from the number of throws where the event is disadvantageous to where it is advantageous.

In Proposition XI, Huygens calculates that, in four throws of two dice, the odds are 178,991 to 1,500,625 for seeing two sixes. He calculates the 24 throws of two dice in three steps. To get the previous odds, Huygens essentially calculates $36^4 - 35^4$ to 35^4 (rather than calculating the number of chances, he computes expected values). He proceeds to eight dice by multiplying 35^4 by 35^4 and 36^4 by 36^4. Then he makes calculations on 16 dice by multiplying 35^8 by 35^8 and 36^8 by 36^8. The final calculations are 35^{16} by 35^8 and 36^{16} by 36^8.

All these calculations involve very large numbers. His explanation of how he handles the large numbers is given in English translation by the Scottish physician, writer and mathematician, John Arbuthnot. The following is excerpted from Arbuthnot ([1], pages 38–39):

> "In this operation, because that which is principally sought is the number of throws, which makes the hazard equal on both sides, viz. to him who undertakes and him who offers, you may, without any sensible error, from the numbers (which else would grow very great) cut off some of the last figures. And so I find, that he who undertakes to throw 12 with two dice, at 24 times, has some loss, and he who undertakes it at 25 times, has some advantage."

Note that Huygens or Artbuthnot, like Gombaud, does not provide any values for the odds in this case. His calculations are based on the same principle that is used in calculations with large numbers on a modern computer.

Doing some quick calculations in R, in the case of 24 dice, the odds of $36^{24} - 35^{24}$ to 35^{24} are 1.103×10^{37} to 1.142×10^{37}, while the odds for 25 dice are 4.086×10^{38} to 3.997×10^{38}. Ignoring the exponents, the leading numbers are enough to know that there is a disadvantage at 24 dice and an advantage at 25 dice. Gombaud probably did the same kind of calculation but by hand.

Prior to his death in 1705, Jakob Bernoulli examined the Chevalier de Méré's problem; his work was published posthumously in 1713. After reading the Pascal–Fermat correspondence in [15], Bernoulli ([5], p. 157) commented on Gombaud's perplexity:

> "But those who are initiated [in mathematics] are hardly delayed by this sort of ἐναντιοφανέι αζ [apparent contradiction], knowing that there can rightly be given countless problems which with the application of calculation are discovered to come out otherwise than appeared at the beginning. And therefore they are zealously careful, according to what I have warned more than once, lest they yield rashly to analogies."

Bernoulli ([5], p. 160) also looked for easier ways to find the number of throws required to move from disadvantage to advantage. For the throw of two dice, one of his solutions gives the transition number, say n, as

$$n = \log_{10}(2)/\{\log_{10}(36) - \log_{10}(35)\} = 24.6,$$

that is, a disadvantage at 24 throws and an advantage at 25.

The Chevalier de Méré's gambling proclivities moved from assiduous, in the initial words of F. N. David, to inveterate, by modern popular writers on mathematics such as Darling [12] and others. As noted earlier, Gombaud did gamble, but in view of the arguments given in Section 3, assiduous and inveterate are very far from the mark that would describe how Gombaud gambled. The "honnête homme" gambled in a very different way.

6. GOMBAUD AND MATHEMATICS

In the preface to his *De ratiociniis in ludo aleæ*, Huygens [34] wrote that the problems he had solved in his short book were previously solved by some outstanding French mathematicians without giving any names. He did not know how they were solved, and so the solutions he gave were his own. Huygens went on to say that it was the custom of these mathematicians to challenge one another with difficult problems and to keep their solutions secret. What Pascal wrote to Fermat on July 29, 1654 is that Gombaud initiated the challenge.

As described earlier, Gombaud was an intellectual who moved in literary circles. The letter that Pascal sent to Fermat on July 29, 1654 also states that Gombaud did not believe in the infinite divisibility of the line, and hence was a poor mathematician. Kavanagh ([35], pp. 51–54) interpreted Pascal's statement about Gombaud in a very negative light. In what follows, we try to put the chevalier's mathematics in a historical perspective, taking into account the fact that many other writers held similar beliefs.

Gombaud was very familiar with arithmetic since he thought that his calculations related to the throws of one and two dice contradicted the rules of arithmetic. He also believed that he had made important contributions to mathematics in 1654. In an undated letter to Pascal ([28], pp. 115–116), he wrote:

> "As you know, I have discovered in mathematics some facts that are so rare that the most knowledgeable of the ancient scholars never said anything about them, and which astounded the best European mathematicians. You have written about my discoveries, as did Mr. Huguens, Mr. de Fermac and so many others who admired them. You must conclude from this that I am not encouraging anyone to despise this science, and to be truthful it can be useful provided that one doesn't take it too seriously; for its objects of inquiry generally seem useless to me; and the time devoted to it could be better used." [8]

On first reading of this passage, and without the context of the whole letter, it appears that Gombaud is incredibly egotistical and is making outrageous claims about his contribution to the development of the probability calculus. This quotation begs two questions:

(a) What were Gombaud's mathematical abilities?
(b) How could he make such a claim when from the Pascal–Fermat correspondence it is obvious that it is they who solved the elusive problem of points?

6.1 The First Question

To try to answer the first question, we looked into Gombaud's education. What is there is sparse and circumstantial; but it does point to a possible basic mathematical education given to sons of the French nobility.

Gombaud lost his father when he was 10 or 11 years of age. His mother, who was of noble lineage, or perhaps his family, was responsible for having Antoine educated. From his letters, we know that Gombaud attended a collège, which one is unknown but may be conjectured; see [61], pages 30–47. The most likely one was Collège Sainte-Marthe de Poitiers run by the Jesuits. There he would have received instruction in rhetoric, Latin grammar, and classical authors such as Horace, Ovid, and Virgil. The Jesuits did not teach mathematics to their students at that time.

There is no information about any instruction in mathematics Gombaud may have received from elsewhere. The fact that he knew of some of the rules of arithmetic, such as the Rule of Three, and that he knew enough about geometry to question the indivisibility of the line, which was a current topic of discussion, points to at least a basic knowledge of mathematics.

One possibility is that Gombaud was privately tutored outside the collège or had a tutor board with him at the collège. This was a relatively common practice among the nobility who wanted their sons to enter military service; see [48], pages 61–62. Knowledge of mathematics, especially fortifications which involved arithmetic and geometry, was necessary for this service.

While Gombaud may have had some education in mathematics, he was not a serious mathematician. There are two pieces of evidence that point to this conclusion.

The first is that he does not appear to have been a member of the circle of mathematicians and physicists that gathered around the French physicist Marin Mersenne. Several mathematicians mentioned previously, including Fermat, Gassendi, Huygens, Pascal, and Roberval, were part of this group. It was the precursor to the "Académie royale des sciences." A list containing many of those associated with the Mersenne circle is given in [14].

The second piece of evidence comes from Gombaud's letter to Pascal, which states his opinions on mathematics, the emerging new physics and his contribution to the development of probability theory. According to Gombaud, mathematics and mathematical reasoning can be useful but have their limitations. They cannot help to promote one's position in high society, the circle in which Gombaud moved. What is necessary for advancement in this society is understanding "the inner workings of things" through reasoning and discussion. This all falls in line with Gombaud's philosophy of the "honnête homme."

6.2 Around the Indivisibility of the Line

As for the mathematical disagreement Gombaud and Pascal had on the indivisibility of the line, it was linked to recent work around physics and Gombaud's understanding of it. In the letter, Gombaud mentions the Greek philosopher Epicurus. This suggests that he was an atomist, a belief that matter was composed of indivisible fundamental components. With this as a worldview, a mathematical model that included infinite divisibility of objects such as lines could not possibly reflect reality.

Gombaud also mentions the immovability of space and so may have adhered to the views of Pierre Gassendi, who was a prominent atomist [21, 26]. As Gassendi worked

[8] "Vous sçavez que j'ay découvert dans les Mathematiques des choses si rares que les plus sçavans des anciens n'en ont jamais rien dit, & desquelles les meilleurs Mathematiciens de l'Europe ont esté surpris; Vous avez écrit sur mes inventions aussi-bien que Monsieur Huguens, Monsieur de Fermac & tant dautres qui les ont admirées. Vous devez juger par-là que je ne conseille à personne de mépriser cette Science, & pour dire le vray elle peut servir pourveu qu'on ne s'y attache pas trop; car d'ordinaire ce qu'on y cherche si curieusement me paroist inutile; & le temps qu'on y donne pourroit estre bien mieux employé."

and taught in Paris during the 1640s, Gombaud may have crossed his path. Gassendi also had disagreements with Descartes, of whom Gombaud speaks disparagingly in the letter to Pascal.

In an article on the Epicurean philosopher Zenon in his *Dictionnaire historique et critique* completed as the end of the 17th century, Pierre Bayle commented on Gombaud's letter to Pascal, particularly about infinite divisibility. He said that some of Gombaud's objections to infinite divisibility were good enough and others were very bad.

6.3 The Second Question

What Gombaud says about his contribution to the development of the probability calculus is a little more problematic. It might be explained by looking at a similar situation. About sixty years after the Pascal–Fermat correspondence, another French nobleman, Pierre Rémond de Montmort claimed priority of solution for another probability problem that was discussed among mathematicians.

In a 1715 letter, Montmort claimed priority of solution to the Problem of Le Her; the relevant section of the letter is quoted by Bellhouse and Fillion; see [2], pages 38–39. The problem was discussed in correspondence among Montmort, Nicolaus Bernoulli and Francis Waldegrave, younger brother of the Henry Waldegrave, 1st Baron Waldegrave who married an illegitimate daughter of King James II of England, and uncle to Henry's son, Jame Waldegrave, 1st Earl Waldegrave. The solution to the problem is usually attributed to Waldegrave. However, in Montmort's mind, he initiated the discussion and encouraged Bernoulli and Waldegrave in their work. He was therefore responsible for the solution to the problem.

The same situation applies to Gombaud and Pascal, with the addition that Gombaud was the master and Pascal the disciple. The problem was Gombaud's originally, and he was responsible for getting the solution found by discussing it with others. Whether or not he actually solved all the problems himself was irrelevant in his view, although he did solve the dicing puzzles on his own.

6.4 Dating and Contents of Gombaud's Letter

There is some disagreement over the dating and content of Gombaud's letter to Pascal. In his biography of Gombaud, Revillout ([53], p. 41) suggests that the passage quoted at the beginning of this section was added later to the letter. His reasoning was that in the first part of his letter to Pascal, Gombaud was writing about the controversy over his denial of the indivisibility of the line. Revillout places this controversy in 1651, before the Pascal–Fermat correspondence. As seen already in Section 5 in the quotation of a letter from Pascal to Fermat, Pascal referred to this controversy in a letter of 1654 concluding that Gombaud was not a good mathematician.

James Franklin, in his book *The Science of Conjecture* ([25], p. 304), translated only the first two sentences of the passage into English. He dated the letter to 1658 or 1659, "though possibly retouched later," probably referring to Gombaud's claim of priority in developing the probability calculus. He went on to discuss how Leibniz looked into Gombaud's contribution to probability.

At the extreme end of opinion regarding the dating of the letter, Mesnard ([43], pp. 256–259) provided a lengthy analysis of the letter. He doubted that the letter from Gombaud to Pascal was ever sent.

Our own view is that the letter was written in 1658 or 1659 and that no changes or additions were made to it. We base the dating of the letter on some internal evidence in the letter and events in the later 1650s external to it.

From Pascal's letter to Fermat on July 29, 1654, we know that Gombaud and Pascal talked about the infinite divisibility of the line at some time prior to this date. Circa 1650, infinite divisibility had been a topic of discussion because of the initial work of Bonaventura Cavalieri published in 1635; see [40], pages 12–20.

As part of this discussion, in about 1658 Pascal wrote a short piece titled *Esprit de géométrie*, which addresses the infinite divisibility of space. It was not published for almost a decade ([40], pp. 28–32), but people knew of it and talked about it. This is also about the time when Pascal rejected the philosophy of the "honnête homme" and delved seriously in the study of Christian theology [65].

With this as background, it is reasonable to conjecture that Pascal wrote to Gombaud about his latest work. This brought about objections from Gombaud as well as concern from Gombaud that Pascal was abandoning the philosophy of the "honnête homme." In Gombaud's view, it used to be that he was the master and Pascal the disciple trying to familiarize himself with the philosophy of the "honnête homme." Consequently, Gombaud chastised Pascal for his belief and implied that holding such beliefs would prevent him from ever rising in society.

7. CONCLUSION

From the investigation described herein, it transpires that Antoine Gombaud was a courtier who developed and espoused the philosophy of "honnêteté" and the "honnête homme." He certainly gambled, as was the norm for a courtier, but there is no evidence to the effect that he was an assiduous, let alone inveterate, gambler as many have thought. It is much more likely that Gombaud applied his philosophy of "honnêteté" to his gambling activities. As such, he would have gambled modestly. He was also experienced enough at card games that as a model of honesty, he would be called on to referee disputes in these games at the court of Louis XIV.

Further, a careful look at the sources indicates that the probability problems that Pascal and Fermat considered were challenge problems given to Pascal by Gombaud in the same fashion that literary questions were posed and discussed in the Paris salons. It is likely out of friendship and consideration for Gombaud's status in literary circles that Pascal agreed to look into the problems. As both of them had been gambling partners, it is not surprising that gambling may have occurred to them as a convenient way to formulate some of these questions.

Nevertheless, there is ample evidence to suggest that gambling per se did not hold a central place in this initiative, and that the problems that Pascal and Fermat solved were not primarily motivated by a need to advise gamblers. Above all, this paper shows that in the history of probability, Antoine Gombaud and the role he played in the advent of modern probability theory has not been given the attention they deserved.

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