Corrigendum to: The sum of powers of subtree sizes for conditioned Galton–Watson trees[*]

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Abstract

A serious typographical error in [1] is corrected.

Keywords: conditioned Galton–Watson tree; simply generated random tree; additive functional; tree recurrence; subtree sizes; Brownian excursion; random analytic function; generating function; singularity analysis; Hadamard product of sequences; method of moments; polylogarithm.
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1   Corrigendum

There is a typographical error in [1, Theorem D.1]; the variance given in (D.2) there is incorrect and should be

$$\mathbb{E}\,|\zeta|^2 = \frac{1}{2\sqrt{\pi}}\,\mathrm{Re}\,\frac{\Gamma(\mathrm{i}t - \frac{1}{2})}{\Gamma(\mathrm{i}t)}. \tag{1.1}$$

The formula (D.2) in [1] has, incorrectly, $\Gamma(\mathrm{i}t - 1)$ in the denominator, which comes from (D.5) which has the same error. Formula (D.8) in the proof is correct, with denominator $\Gamma(\mathrm{i}t)$, and yields (D.5) and (D.2) with the same denominator, i.e., (1.1).

Theorem D.1 in [1] also claims that $\mathbb{E}\,|\zeta|^2 > 0$. The proof is based on the incorrect formula given there, but luckily the same proof applies also to the correct formula. In (D.14) we obtain $\Gamma(1 - \mathrm{i}t)$ instead of $\Gamma(2 - \mathrm{i}t)$ (and an immaterial change of sign); hence we have to show that $\Gamma(1 - \mathrm{i}t)/\Gamma(\frac{3}{2} - \mathrm{i}t)$ is not real for $t \neq 0$. Thus, in (D.15), we should have $-\,\mathrm{Im}\int_1^{3/2}\psi(s - \mathrm{i}t)\,\mathrm{d}s$. We use (D.18) as before, and now see that if $t < 0$, then $0 > \arg\big(\Gamma(1 - \mathrm{i}t)/\Gamma(\frac{3}{2} - \mathrm{i}t)\big) > -\pi/4$, which completes the proof that the variance in (1.1) is nonzero.

1.1 Other typographical errors

We take the opportunity to point out a few other typographical errors in [1]:

- (12.75): the lower summation limit should be $m = 0$.
- Three lines after (12.75): "every i" should be "every "$i \geqslant 1$".
- Appendix C, two lines before the statement of Theorem C.1: $\alpha^{-1}Y(\alpha)$ there should be $\alpha^{-1/2}Y(\alpha)$.

References

[1] James Allen Fill & Svante Janson. The sum of powers of subtree sizes for conditioned Galton-Watson trees. *Electron. J. Probab.* **27** (2022), Paper No. 114, 77 pp. MR4475879