Comment: Illusions, Then and Now

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Abstract. The situation of mathematical statistics today resembles the situation of Laplacean probability in the mid-19th century. Strong claims are made about how probability theory should be used. Yet it is widely misused and increasingly not used in the analysis of data. Perhaps we need a philosophy of probability that makes more modest and realistic claims.

Key words and phrases: Laplace, Cournot, cognitive illusions, norms, foundations of probability.

Miller and Gelman make a convincing case that Laplace's chapter on illusions in the later editions of his *Essai philosophique* anticipated much of the 20th and 21st century work on heuristics and biases. Their analysis is fascinating, containing much to applaud.

There are two issues that I would like to have seen addressed more fully: the degree of originality in Laplace's treatment of cognitive illusions, and the distinction between getting the probabilities right and deviating from "norms of rationality." Here are some thoughts that could lead to a fuller discussion of these issues.

Originality. The most thorough assessment of the sources of Laplace's thinking on this topic is given by Bru and Bru in the section "Illusions" in *Les jeux de l'infini et du hasard* [3], Vol. 2, pp. 517–520 and 525–533. As they document, Laplace was keenly interested in psychology for an extended period of time and interacted with many other scientists who shared this interest. Condorcet, who had a somewhat troubled relationship with Laplace, also merits mention; he discussed the gambler's fallacy in print in 1785 [5], p. 144. On the general theme that the probability calculus can correct errors to which humans are prone, we might also cite the textbook published by Lacroix in 1816 [8].

Norms. According to the modern subjectivist view of probability, an event may not have an objectively correct probability, but there are norms of rationality that prescribe how an individual's probabilities for different events at different times should cohere. Laplace, on the other hand, did not talk about "norms"; he contended that for an individual with given information there is a correct probability for a given event, a probability that correctly measures the reason the individual has to believe that the event will occur. Miller and Gelman blur

this distinction, but it is relevant to the late 20th century literature on heuristics and biases, because much of that literature, including the work by Kahneman and Tversky in the 1970s, took as its starting point Laplace's picture, not the modern subjectivist dogma.

But a much more important question is raised by the similarity of Laplace's situation in the 1820s to our situation as we step into the 2020s. As the King James Bible reminds us, pride goeth before destruction, and a haughty spirit before a fall. If the stance of mathematical statistics in 2020 is comparable to Laplace's in 1820, should we not now expect a comeuppance similar to that suffered by Laplace's legacy?

As Miller and Gelman suggest, we have come nearly full circle. Starting from the work of the English biometric school around the beginning of the 20th century, we have arrived at about the same place as Laplace arrived starting from the work of Bernoulli and De Moivre around the beginning of the 18th century. Today, as in the 1820s, we know how to build probability models, and we believe that these models can correct errors to which humans are prone. A further feature common to Laplace's situation and ours, not mentioned by Miller and Gelman, can be seen in Laplace's synthesis of the direct (i.e., frequentist or Bernoullian) and inverse (i.e., Bayesian) modes of argument. When he discovered the large-sample approximation at the core of both the central limit theorem and the (very misleadingly named) Bernstein–von Mises theorem, Laplace saw that the direct and inverse arguments give the same answer in large samples, and he mostly shifted to the simpler direct argument. This reconciliation of the two approaches is echoed by accommodations and compromises that we see in mathematical statistics today.

The first blow to Laplace's picture was dealt by his own spectacular illusions. In the last decade of his life, he wrote to his fellow mathematicians around Europe to publicize the power of his large-sample methods, and one of his favorite examples in this letter-writing campaign was

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his estimation of Jupiter's mass relative to the Sun. Combining all the relevant measurements that had been made by that time, he announced bounds on this ratio, bounds on which one could bet a million to one. Five years after his death it was discovered that a crucial part of the data on which he relied was seriously flawed, and the true ratio lay outside his million to one bounds. The assumptions that Laplace had made—independence and absence of systematic error—were illusions.

This was only the beginning of the discredit of Laplace's methods in the 19th century. His and Condorcet's speculations about using probability to fix the sizes of juries were justly ridiculed. By 1843, Cournot was already deploring the p-hacking of census results using Laplace's asymptotics [4], and Bienaymé, the most capable mathematical statistician of the mid-19th century, spent most of his energies combating the application of probability to statistics [2]. The 19th century saw an unprecedented flood of data, and most users of this data concluded that it could speak for itself; probability was not needed [6]. Though a champion of probability, Cournot ridiculed the conceits of Bayes, Condorcet, and Laplace, emphasizing that for many questions it is only possible to justify nonnumerical "philosophical probabilities" [4]. By the middle of the century, geodesy, a field dominated by the French before and during Laplace's heyday, had abandoned Laplace's methods, turning instead to the methods developed by Gauss's followers, which paid much more attention to systematic errors and relied on efficient computation rather than asymptotics [7]. By the end of the century, the most prominent mathematician in France, Joseph Bertrand, would ridicule Laplace's entire undertaking as a delusion [1].

Is it inevitable that all probabilistic methods will again be conflated with extravagant claims about rationality, earning ridicule from 21st century data scientists just as Laplace's methods earned ridicule from 19th century practitioners? To avoid this path, we need less prideful foundations for probability.

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