Electronic Journal of Probability

Electron. J. Probab. **24** (2019), no. 25, 1–2.
ISSN: 1083-6489 https://doi.org/10.1214/19-EJP291

Second Errata to "Processes on Unimodular Random Networks"*

David Aldous[†] Russell Lyons[‡]

Abstract

We correct a few more minor errors in our paper, *Electron. J. Probab.* **12**, Paper 54 (2007), 1454–1508.

Keywords: amenability; equivalence relations; infinite graphs; percolation; quasi-transitive; random walks; transitivity; weak convergence; reversibility; trace; stochastic comparison; spanning forests; sofic groups.
AMS MSC 2010: Primary 60C05, Secondary 60K99; 05C80.
Submitted to EJP on November 2, 2018, final version accepted on March 6, 2019.

Our first set of errata, *Electron. J. Probab.* **22** (2017), paper no. 51, 4 pp., corrected several minor misstatements and several somewhat incorrect proofs. Here we correct a few more.

(i) In Section 2, the definition of canonical representative that was given to prove its existence is incomplete and incorrect. A correct proof of its existence follows.

Write $\prec$ for the total order that was defined on locally finite, connected networks with vertex set $\mathbb{N}$, root 0, and mark space $\mathbb{N}^{\mathbb{N}}$. Given a locally finite, connected, rooted network G and $r \geq 1$, let $\mathcal{H}_r$ be the class of networks on $\mathbb{N}$ with root 0 that are rooted-isomorphic to G and whose vertices within distance r of 0 form an interval, $[0, N_r]$. Let $\mathcal{H}_r^{\min}$ be the subset of $\mathcal{H}_r$ such that the network induced on $[0, N_r]$ is minimal for $\prec$ (there are only finitely many possibilities for the induced network, so there is a unique minimum induced network). Then $\mathcal{H}_r^{\min} \supseteq \mathcal{H}_{r+1}^{\min}$ for all r by the definition of $\prec$. Hence, there is a unique element $H \in \bigcap_{r=1}^{\infty} \mathcal{H}_r^{\min}$: the network of H induced on $[0, N_r]$ is determined by $\mathcal{H}_r^{\min}$. This network H is the desired canonical representative of G.

(ii) At the end of Question 2.5, the assertion that ν is not $\mathrm{Aut}(T)$-invariant is not always correct. Indeed, if the functions f_a, f_b, and f_c are constant, then ν *is* invariant. Nonetheless, ν is not invariant in any other case. To see this, suppose, without loss of generality, that f_a is not constant. Let e_1 and e_2 be two (distinct) edges that have the same Cayley label, a, and that are incident to a common third edge, e_3. Then under ν, precisely one of the following possibilities occurs:

*Main article: https://doi.org/10.1214/EJP.v12-463.

[†]University of Calif., Berkeley. E-mail: aldousdj@berkeley.edu
[‡]Indiana University, Bloomington. E-mail: rdlyons@indiana.edu

- $X(e_1)$ and $Y(e_2)$ are not independent because $I_{e_1} \cap J_{e_2} = \{e_3\}$;
- $Y(e_1)$ and $X(e_2)$ are not independent because $J_{e_1} \cap I_{e_2} = \{e_3\}$; or
- $X(e_1)$ and $Y(e_2)$ are independent and $Y(e_1)$ and $X(e_2)$ are independent.

In each of these three cases, we can determine which edges form the sets I_{e_1}, I_{e_2}, J_{e_1}, and J_{e_2}, and therefore we can orient e_1 and e_2 towards ξ. This orients all edges labeled a, but such an orientation is not invariant under $\mathrm{Aut}(T)$.

(iii) When a map $\psi : \Xi \to \Xi$ is used to define a percolation on a given measure μ on $\mathcal{G}_*$, the notation $\mu \circ \psi^{-1}$ was used for the measure obtained by changing the marks according to ψ. It should have been explained that ψ induces a map on $\mathcal{G}_*$ by applying ψ to all the marks of a network. Denote this induced map still by ψ in order to make the notation used meaningful. This occurs before Definition 6.4, in Definition 8.1, and later.

(iv) For Theorem 8.5, the proof that (ii) implies (iii) has a gap, because the bounded convergence theorem may not apply unless the vertex degrees are uniformly bounded. We do not know whether (ii) is equivalent to the others without such a boundedness assumption, but it can be strengthened to be equivalent: Namely, replace (8.4) by

$$\lim_{n\to\infty} \int \sum_{x\in\mathsf{V}(G)} \sum_{y\sim x} |\lambda_n(G,o,x) - \lambda_n(G,o,y)| \; d\mu(G,o) = 0\,.$$

That is what is proved from (i) and what is used to prove (iii).

(v) In Theorem 8.13, $\iota_{\mathsf{E}}(G)$ was not defined for a graph, G; it means

$$\iota_{\mathsf{E}}(G) := \inf\left\{ \frac{|\{(x,y)\,;\; x\in K,\, y\notin K,\, (x,y)\in \mathsf{E}\}|}{|K|}\,;\; K \subset \mathsf{V} \text{ is finite} \right\}.$$

Also, in (iii), μ should be assumed extremal.

(vi) In Example 9.6, $\widehat{Z}$ should be defined as $1 + (1/2)\overline{\deg}(\mu) + Z$.