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Erratum: Multivariate Stein factors for a class of strongly log-concave distributions*

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Abstract

The strong continuity argument in [6] did not identify an appropriate Banach space. We do so here. A corrected version has been uploaded to arxiv.org/abs/1512.07392.

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Lem. 3.1 of [6] should read as follows.

Lemma 3.1 (Overdamped Langevin properties)**.** *If* $\log p \in C^2(\mathbb{R}^d)$ *is strongly concave, then the overdamped Langevin diffusion* $(Z_{t,x})_{t\geq 0}$ *with infinitesimal generator* (1.1) *and* $Z_{0,x} = x$ *is well-defined for all times* $t \in [0,\infty)$*, has stationary distribution* P*, and satisfies* strong continuity *on* $L = \{f \in C^0(\mathbb{R}^d) : \frac{|f(x)|}{1+\|x\|_2^2} \to 0$ *as* $\|x\|_2 \to \infty\}$ *with norm* $\|f\|_L \triangleq \sup_{x\in\mathbb{R}^d} \frac{|f(x)|}{1+\|x\|_2^2}$*, that is,* $\|\mathbb{E}[f(Z_{t,\cdot})] - f\|_L \to 0$ *as* $t \to 0^+$ *for all* $f \in L$*.*

Proof. Consider the Lyapunov function $V(x) = \|x\|_2^2 + 1$. The strong log-concavity of p, the Cauchy-Schwarz inequality, and the arithmetic-geometric mean inequality imply that

$$\begin{aligned}(\mathcal{A}V)(x) &= \langle x, \nabla \log p(x)\rangle + d = \langle x, \nabla \log p(x) - \nabla \log p(0)\rangle + \langle x, \nabla \log p(0)\rangle + d \\ &\leq -k\|x\|_2^2 + \|x\|_2 \|\nabla \log p(0)\|_2 + d \leq \left(\frac{1}{2} - k\right)\|x\|_2^2 + \frac{1}{2}\|\nabla \log p(0)\|_2^2 + d \leq k' V(x)\end{aligned}$$

for some constants $k, k' \in \mathbb{R}$. Since $\log p$ is locally Lipschitz, [5, Thm. 3.5] implies that the diffusion $(Z_{t,x})_{t\geq 0}$ is well-defined, and [7, Thm. 2.1] guarantees that P is a stationary distribution. The argument of [4, Prop. 15] with [5, Thm. 3.5] substituted for [5, Thm. 3.4] and [3, Sec. 5, Cor. 1.2] now yields strong continuity. □

In addition the final component of the proof of Thm. 1.1 of [6] should read as follows.

Solving the Stein equation Finally, we show that u_h solves the Stein equation (1.2). Introduce the notation $(P_t h)(x) \triangleq \mathbb{E}[h(Z_{t,x})]$. Since $(P_t)_{t\geq 0}$ is strongly continuous on the Banach space L of Lemma 3.1 and $h \in L$, the generator $\mathcal{A}$, defined in (1.1), satisfies

$$h - P_t h = \mathcal{A}\int_0^t \mathbb{E}_P[h(Z)] - P_s h \, ds \quad \text{for all} \quad t \geq 0$$

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by [2, Prop. 1.5]. The left-hand side limits in L to $h - \mathbb{E}_P[h(Z)]$ as $t \to \infty$, as

$$|h(x) - \mathbb{E}_P[h(Z)] - (h(x) - (P_t h)(x))| = \left| \int_{\mathbb{R}^d} \mathbb{E}[h(Z_{t,y})] - \mathbb{E}[h(Z_{t,x})]\ p(y)dy \right|$$
$$\leq M_1(h) \int_{\mathbb{R}^d} \mathbb{E}\big[\|Z_{t,y} - Z_{t,x}\|_2\big]\ p(y)dy \leq M_1(h)\, \mathbb{E}_P[\|Z - x\|_2] e^{-kt/2}$$

for each $x \in \mathbb{R}^d$ and $t \geq 0$. Here we have used the stationarity of P, the Lipschitz relation (3.1), the first-order coupling inequality (3.7) of Lemma 3.3, and the integrability of Z [1, Lem. 1] in turn. Meanwhile, the right-hand side limits to $\mathcal{A}u_h$, since $\mathcal{A}$ is closed [2, Cor. 1.6]. Therefore, u_h solves the Stein equation (1.2).

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