

A TWO-STEP APPROACH TO MODEL PRECIPITATION EXTREMES IN CALIFORNIA BASED ON MAX-STABLE AND MARGINAL POINT PROCESSES¹

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In modeling spatial extremes, the dependence structure is classically inferred by assuming that block maxima derive from max-stable processes. Weather stations provide daily records rather than just block maxima. The point process approach for univariate extreme value analysis, which uses more historical data and is preferred by some practitioners, does not adapt easily to the spatial setting. We propose a two-step approach with a composite likelihood that utilizes site-wise daily records in addition to block maxima. The procedure separates the estimation of marginal parameters and dependence parameters into two steps. The first step estimates the marginal parameters with an independence likelihood from the point process approach using daily records. Given the marginal parameter estimates, the second step estimates the dependence parameters with a pairwise likelihood using block maxima. In a simulation study, the two-step approach was found to be more efficient than the pairwise likelihood approach using only block maxima. The method was applied to study the effect of El Niño-Southern Oscillation on extreme precipitation in California with maximum daily winter precipitation from 35 sites over 55 years. Using site-specific generalized extreme value models, the two-step approach led to more sites detected with the El Niño effect, narrower confidence intervals for return levels and tighter confidence regions for risk measures of jointly defined events.

1. Introduction. Environmental extreme data are often spatial in nature, as data are recorded at a network of monitoring stations over time. Extreme weather and climate events may also exhibit spatial dependence because their occurrences are influenced by atmospheric circulation of a very large spatial scale. The large-scale modes of climate variability, such as El Niño-Southern Oscillation (ENSO) and the Pacific decadal oscillation (PDO), have profound impacts on the precipitation regime over North America, especially during the winter [e.g., [Ropelewski and Halpert \(1986, 1996\)](#), [Gershunov and Barnett \(1998\)](#)]. As an example, El Niño usually lasts for at least one season and brings substantially increased extreme precipitation over a vast region of North America [[Shang, Yan and Zhang \(2011\)](#),

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Zhang et al. (2010)]. Rare events that occur at multiple locations within a very short time interval can cause more damage, consume more resources and demand more delicate emergency rescue. For strategic emergency management and loss mitigation, understanding and predicting extreme events in a spatial context is needed. Although univariate extreme value modeling has been well developed [e.g., Coles (2001)], spatial extreme modeling has not gained a sharpened focus until recently [e.g., Buishand, de Haan and Zhou (2008), Davison and Gholamrezaee (2012), de Haan and Pereira (2006), Padoan, Ribatet and Sisson (2010)]. Two recent reviews are Davison, Padoan and Ribatet (2012) and Bacro and Gaetan (2012); the latter focuses on spatial max-stable processes, while the former covers additionally latent variable approaches and copula approaches.

A max-stable process extends the multivariate extreme value distribution to an infinite dimension; every multidimensional marginal distribution of it is a multivariate extreme value distribution [de Haan (1984)]. For only a few parametric models, statistical inference is practically viable: the Smith model [Smith (1990)], the Schlather model [Schlather (2002)], the Brown–Resnick model [Kabluchko, Schlather and de Haan (2009)], the geometric Gaussian model [Davison, Padoan and Ribatet (2012)] and the extremal- t model [Opitz (2013)]. Wadsworth and Tawn (2012) proposed to superimpose two max-stable processes to obtain a new model, which can produce more realistic event realizations than, for example, the Smith model by itself. Inferences for max-stable process models are challenging because the joint density for multiple sites is only available for bivariate or trivariate marginal distributions. In fact, the trivariate marginal density has been derived only recently for the Smith model [Genton, Ma and Sang (2011)] and the Brown–Resnick model [Huser and Davison (2013)]. The pairwise likelihood approach based on the bivariate marginal distributions of block maxima has been used in applications [Davison and Gholamrezaee (2012), Padoan, Ribatet and Sisson (2010)].

For univariate extreme value analysis based on generalized extreme value (GEV) distributions, daily records which contain more information than annual maxima can be exploited. Two well-known threshold approaches are the peaks over threshold (POT) approach [Balkema and de Haan (1974), Pickands (1975)] and the point process approach [Leadbetter, Lindgren and Rootzén (1983), Pickands (1971)]. Ferreira and de Haan (2015) recently showed that, for the probability weighted moment estimator [Hosking, Wallis and Wood (1985)], the block maximum method is asymptotically more efficient in mean squared error than the POT method under certain conditions. Nonetheless, for shorter records, the threshold methods may be more efficient than the block maximum method when the number of exceedances is larger than the number of blocks on average or when the shape parameter is positive [e.g., Katz, Parlange and Naveau (2002), Tanaka and Takara (2002)]. For multivariate extremes, as Falk and Michel (2009) pointed out, an extension of the univariate threshold approach needs to solve which distributions describe the exceedances and how exceedances are defined in a multivariate setting. These problems are being actively investigated [e.g., Falk and Guillou

(2008), Falk, Hüsler and Reiss (2010), Ferreira and de Haan (2014), Rootzén and Tajvidi (2006), Thibaud and Opitz (2013)]. Bivariate threshold-based inferences have been applied to max-stable process models through the composite likelihood approach. Bacro and Gaetan (2014) considered two bivariate exceedance distributions, one from the tail approximation for bivariate distribution in Ledford and Tawn (1996) and the other from the bivariate extension of a generalized Pareto distribution (GPD) in Rootzén and Tajvidi (2006). No clear winner of the two approaches was found in a simulation study, and their performance depends on the spatial dependence level. Similar approaches have been adopted by Wadsworth and Tawn (2014) for spatial extremes and Huser and Davison (2014) in a space–time setting.

Without resorting to a spatial version of threshold-based approaches, we propose a two-step approach that utilizes daily records in addition to block maxima from each site for max-stable process models. The first step estimates the marginal parameters using an independence likelihood constructed from the univariate threshold-based point process approach with daily records. Given the marginal parameter estimates, the second step estimates the dependence parameters from a pairwise likelihood with block maxima. The two-step approach has been studied recently for multivariate models to overcome the computational difficulty in maximum likelihood estimation [Joe (2005), Zhao and Joe (2005)]. Our two-step approach is different, however, in that we use different data in the two steps: the first step uses daily records, while the second step uses block maxima. Compared to the bivariate threshold-based approaches, the marginal parameter estimator from the two-step approach is robust to misspecification of the spatial dependence. The more efficient marginal estimator helps improve the efficiency of the dependence parameter estimator compared to the composite likelihood estimator based on only block maxima.

The rest of the article is organized as follows. Our motivating application, annual maximum winter daily precipitation in California, is presented in Section 2. The spatial max-stable process model defined by all univariate marginal distributions and a spatial dependence structure, and the dependence measure extremal coefficient, are introduced in Section 3. In Section 4 we present details of the two-step approach, the asymptotic properties of the estimator, and describe how to estimate the limiting variance. A simulation study is reported in Section 5. The proposed method is applied to the precipitation data from 36 sites in California over 55 years in Section 6, providing more compact confidence regions for joint return levels. Section 7 concludes with some discussion.

2. Extreme winter precipitation in California. Recent studies suggest that the El Niño/Southern Oscillation (ENSO) has significant impact on extreme precipitation in North America [Zhang et al. (2010)]. Southern Oscillation refers to the variation in the sea surface temperature of the tropical waters in the eastern Pacific Ocean. The “warm” events and the “cool” events are referred to as El Niño

and La Niña, respectively, and their strength is measured by the Southern Oscillation Index (SOI), the normalized sea level pressure difference between Tahiti and Darwin. With SOI as a covariate in sitewise GEV modeling, El Niño was found to be associated with a substantial increase in the likelihood of extreme precipitation over a vast region of southern North America [Zhang et al. (2010)]. Focusing on the California stations, Shang, Yan and Zhang (2011) reported similar findings with spatial dependence incorporated through a Smith model, which enabled inference and predictions of joint extremal events at multiple sites within the same year. Nevertheless, two practical issues were not satisfactorily addressed. First, realizations from the Smith model are of too regular shape. Second, collaborators who are familiar with threshold-based univariate extreme value analysis wondered if the full records of daily precipitation can lead to a more efficient analysis than that based on block maxima alone. These issues motivated our two-step approach and a revisit of the extreme winter precipitation in California.

Daily precipitation records at all monitoring stations in California were extracted from the second version of the Global Historical Climatology Network (GHCN), compiled and quality-controlled at the National Climatic Data Center of the National Oceanic and Atmospheric Administration (available at <http://www.ncdc.noaa.gov/oa/climate/ghcn-daily/>). As precipitation in California occurs predominantly in winter, we restrict our attention to the winter season, which is defined as the period from December 1st to March 31st in the following year [Zhang et al. (2010)]. Due to missing data, the block maxima in a given winter at a given site was considered to be valid only if no more than 10% of the daily records were missing in that winter [Shang, Yan and Zhang (2011)]. For comparison, we used the same time periods and sites as the balanced data in Shang, Yan and Zhang (2011), covering daily winter precipitation from 1948 to 2002 for 36 sites. The 36 sites in California are shown in Figure 1, superimposed with the elevation map of the state. The distance between the two furthest sites is 1188 km. As in Shang, Yan and Zhang (2011), possible covariates to be included in the GEV parameters for each site are longitude, latitude, elevation and SOI. The latitude and longitude are in degrees, and the elevation is in 100 meters. The SOI for each winter is the average of the four monthly SOI values of the winter months, ranging from -3.14 to 1.88 with a sample average -0.15 for the data period.

3. Spatial extreme model with max-stable process. A max-stable process model for spatial extremes consists of two parts: marginal distributions and spatial dependence structure. The marginal distribution at each site is a GEV distribution, which may incorporate temporal nonstationarity through temporally varying covariates such as the SOI. In particular, let $M(s, t)$ be the maximum at site s in block t in a spatial domain $D \subset \mathbb{R}^2$. The distribution of $M(s, t)$ is

$$(3.1) \quad M(s, t) \sim \text{GEV}(\mu(s, t), \sigma(s, t), \xi(s, t)),$$

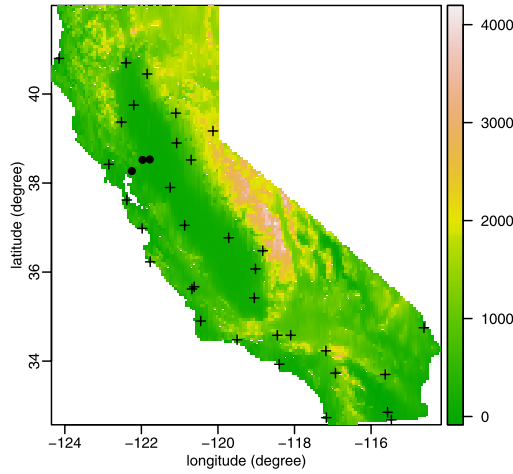


FIG. 1. Locations of the 36 monitoring stations in California superimposed with the elevation map (meter). The three sites in solid circles are Napa, Winters and Davis, near the Sacramento area.

where $\mu(s, t)$, $\sigma(s, t)$ and $\xi(s, t)$ are the location, scale and shape parameters, respectively, of the GEV distribution. Covariate information is incorporated into the parameters through $\mu(s, t) = X_\mu^\top(s, t)\beta_\mu$, $\sigma(s, t) = X_\sigma^\top(s, t)\beta_\sigma$, and $\xi(s, t) = X_\xi^\top(s, t)\beta_\xi$, where $X_\mu(s, t)$, $X_\sigma(s, t)$ and $X_\xi(s, t)$ are the covariate vectors for μ , σ and ξ , respectively, \top denotes transpose, and $\beta = (\beta_\mu^\top, \beta_\sigma^\top, \beta_\xi^\top)^\top$ is the vector containing all marginal parameters.

The spatial dependence structure ensures that every finite-dimensional marginal distribution is a multivariate GEV distribution. The multivariate extreme value property essentially requires that every finite-dimensional marginal copula must be an extreme value copula [Gudendorf and Segers (2010)]. Without loss of generality, let $Z(s, t) = F^{-1}[G_{s,t}\{M(s, t)\}]$, where F is the distribution function of a unit Fréchet variable with inverse function F^{-1} , and $G_{s,t}(\cdot; \beta)$ is the distribution function of $\text{GEV}(\mu(s, t), \sigma(s, t), \xi(s, t))$ with parameter vector β . Consider any p sites $x_i \in D, i = 1, \dots, p$. The copula of $\{M(x_1, t), \dots, M(x_p, t)\}$ is the same as the copula of $\{Z(x_1, t), \dots, Z(x_p, t)\}$. This copula is determined by a max-stable process (MSP) model with dependence parameter θ for process $Z(s, t)$ for any t :

$$(3.2) \quad Z(s, t) \sim \text{MSP}(\theta).$$

The MSP has a marginal unit Fréchet distribution at each s and marginal extreme value copulas for any multidimensional marginal distribution.

The parametric form of an MSP is determined from its spectral representation [de Haan (1984), Schlather (2002)]. Let $\{U_j\}_{j \geq 1}$ be a Poisson process on \mathbb{R}_+ with intensity du/u^2 . Let $W_j(x), x \in D, j \geq 1$, be independent copies of a nonnegative stationary process $W(x)$ with $E\{W(x)\} = 1$ for all $x \in D$. Then,

$$Z(x) = \sup_{j \geq 1} U_j W_j(x), \quad x \in D,$$

is a stationary MSP with unit Fréchet margins. Three practically viable MSP models are obtained by different choices of $W(x)$ with parameter vector θ [e.g., Davison, Padoan and Ribatet (2012)]. The Smith model takes $W_j(x) = g(x - V_j)$, where g is the density of a zero mean bivariate normal random vector with variance matrix Σ , and V_1, V_2, \dots are the points of a homogeneous Poisson process of unit rate in D . Isotropy is obtained when $\Sigma = \tau I_2$, where I_2 is the two-dimensional identity matrix and $\tau > 0$ is a scalar. The Schlather model takes $W_j(x) = \max\{0, \sqrt{2\pi}\varepsilon_j(x)\}$, where $\varepsilon_1, \varepsilon_2, \dots$ are independent copies of a stationary Gaussian process $\{\varepsilon(x) : x \in D\}$ with unit variance and correlation function ρ . A geometric Gaussian model takes $W_j(x) = \exp\{\delta\varepsilon_j(x) - \delta^2/2\}$, where $\delta > 0$ and $\varepsilon_j(x)$'s are independent copies of a stationary Gaussian process with unit variance and correlation function ρ . Geometric anisotropy can be obtained for the Schlather model and the geometric Gaussian model through using a geometric anisotropic correlation function ρ . The spectral representation of another model, the extremal- t process, was only obtained recently by Opitz (2013). The extremal- t process is the extreme value limit of t processes, which are scale mixtures of Gaussian processes. It is characterized by a degree of freedom ν and a dispersion function ρ (the correlation function of the Gaussian process). The extremal- t process covers the Schlather model when $\nu = 1$ and the Brown–Resnick model when $\nu \rightarrow \infty$.

A useful measure for extremal dependence is the extremal coefficient. For an MSP $Z(s)$ with unit Fréchet margins, the extremal coefficient at p sites x_1, \dots, x_p is the number ζ such that

$$\Pr\{Z(x_1) \leq z, \dots, Z(x_p) \leq z\} = \exp(-\zeta/z), \quad z > 0.$$

The range of ζ is $1 \leq \zeta \leq p$, with 1 and p corresponding to full dependence and independence, respectively. The pairwise extremal coefficient as a function of the pairwise distance can be used in exploratory analysis and model checking. For two sites x_1 and x_2 with $h = x_2 - x_1$, the pairwise extremal coefficient $\zeta(h)$ is $2\Phi\{\sqrt{h^\top \Sigma^{-1} h}/2\}$, $1 + \sqrt{[1 - \rho(h)]/2}$, $2\Phi\{\sqrt{\delta^2[1 - \rho(h)]/2}\}$, and $2T_{\nu+1}\{\sqrt{[1 - \rho(h)][\nu + 1]/[1 + \rho(h)]}\}$ for the Smith, Schlather, geometric Gaussian and extremal- t model, respectively, where Φ is the distribution function of a standard normal variate and T_ν is the distribution function of a Student- t variate with ν degrees of freedom. Unlike the other three models which offer the full range of dependence level from complete independence to complete dependence, the Schlather model has $\zeta(h) \leq 1 + \sqrt{1/2} \approx 1.707$, not allowing full independence of two sites regardless of their distance.

Given observed block maxima from S sites over n years, estimation of model parameters $\eta = (\beta^\top, \theta^\top)^\top$ is challenging because the full joint distribution of S sites is unavailable for $S \geq 3$ in general. Inference about max-stable process models has mostly been based on the composite likelihood approach [e.g., Davison and Gholamrezaee (2012), Padoan, Ribatet and Sisson (2010)]. In particular, a pairwise likelihood is constructed from the bivariate marginal distributions of all pairs. The

three aforementioned MSP models are viable because their bivariate marginal distributions have closed forms and the corresponding density can be derived and used to construct pairwise likelihoods. The pairwise likelihood approach is potentially wasteful of data because it only uses the block maxima.

4. The two-step approach. Suppose that we observe the full record of each block with block size m at S sites over n blocks (e.g., years or seasons). For ease of notation, m is assumed to be the same but our approach can also handle the case where m varies from year to year. Let $Y_{s,t,k}$ be the k th observation within block t at site s , $k = 1, \dots, m$, and let $\mathbf{Y}_{s,t} = \{Y_{s,t,1}, \dots, Y_{s,t,m}\}$. Let $M_{s,t} = \max_k Y_{s,t,k}$ be the block maximum. Our first step estimates the marginal parameters β based on daily records $\mathbf{Y} = \{\mathbf{Y}_{s,t} : s = 1, \dots, S; t = 1, \dots, n\}$. Our second step estimates the dependence parameters θ based on block maxima $\mathbf{M} = \{M_{s,t} : s = 1, \dots, S; t = 1, \dots, n\}$.

Step 1. The first step is based on an independence likelihood constructed from the point process approach for univariate extreme value analysis. This step utilizes the daily record in each block but ignores the spatial dependence across sites. Let $u_{s,t}$ be the threshold chosen for site s and block t , $s = 1, \dots, S$, $t = 1, \dots, n$. This choice accommodates nonstationarity across the blocks. The independence loglikelihood has the form

$$(4.1) \quad l_1(\beta; \mathbf{Y}) = \sum_{t=1}^n \sum_{s=1}^S \ell_{1t,s}(\beta; \mathbf{Y}_{s,t}),$$

where

$$\begin{aligned} &\ell_{1t,s}(\beta; \mathbf{Y}_{s,t}) \\ &= -\left[1 + \xi_{s,t} \left(\frac{u_{s,t} - \mu_{s,t}}{\sigma_{s,t}}\right)\right]^{-1/\xi_{s,t}} \\ &\quad + \sum_{k: Y_{s,t,k} > u_{s,t}} \left[-\log \sigma_{s,t} - \left(\frac{1}{\xi_{s,t}} + 1\right) \log \left\{1 + \xi_{s,t} \left(\frac{Y_{s,t,k} - \mu_{s,t}}{\sigma_{s,t}}\right)\right\}\right]. \end{aligned}$$

The contribution to the independence loglikelihood from site s , $\sum_{t=1}^n \ell_{1t,s}$, is simply the loglikelihood of the point process approach in a univariate extreme value analysis [Smith (1989)]. Since we assume independence from block to block, the contribution from block t is $\ell_{1t} = \sum_{s=1}^S \ell_{1t,s}$. The maximizer of (4.1), $\hat{\beta}_n$, is the estimator of β .

The independence loglikelihood (4.1) also allows temporal dependence within the same block, in which case the temporal dependence is ignored similar to the spatial dependence. Recent studies show that this approach not only uses all threshold excesses for more efficient estimation, but also avoids significant biases that may come with declustering [Fawcett and Walshaw (2007, 2012)]. The variance

of $\hat{\beta}_n$ needs to be estimated with sandwich estimators to adjust for the dependence [Smith (1991)].

Step 2. Given $\hat{\beta}_n$, the second step uses block maxima to estimate the dependence parameters θ based on a pairwise likelihood. Let $f_{ijt}(\cdot; \theta, \beta)$ be the bivariate marginal density of the $(M_{i,t}, M_{j,t})$ from the max-stable process model specified by (3.1) and (3.2) with site i and j in block t . Define pairwise loglikelihood

$$(4.2) \quad l_2(\theta; \hat{\beta}_n, \mathbf{M}) = \sum_{t=1}^n \ell_{2t}(\theta; \hat{\beta}_n, M_{s,t} : s = 1, \dots, S),$$

where the contribution from block t is

$$\ell_{2t}(\theta; \beta, M_{s,t} : s = 1, \dots, S) = \sum_{i=1}^{S-1} \sum_{j=i+1}^S \log f_{ijt}((M_{i,t}, M_{j,t}); \theta, \beta).$$

Our estimator for $\theta, \hat{\theta}_n$, is the maximizer of (4.2).

The asymptotic properties of the two-step estimator $\hat{\eta}_n^\top = (\hat{\beta}_n^\top, \hat{\theta}_n^\top)$ can be derived with the theory of estimating functions [Godambe (1991)]. Let $\psi_{1t}(\beta) = \partial \ell_{1t} / \partial \beta$. Let $\psi_{2t}(\beta, \theta) = \partial \ell_{2t} / \partial \theta$. Then $\hat{\eta}_n$ is the solution to the estimating equations $\sum_{t=1}^n \psi_t(\eta) = 0$, where $\psi_t^\top(\eta) = (\psi_{1t}^\top(\beta), \psi_{2t}^\top(\beta, \theta))$. Under mild regularity conditions, as $n \rightarrow \infty$, $\hat{\eta}_n$ is consistent for the true parameter vector η_0 , and $\sqrt{n}(\hat{\eta}_n - \eta_0) \rightarrow N(0, \Omega)$, where $\Omega = A^{-1}B(A^{-1})^\top$ is the inverse of the Godambe information matrix, with $A = \lim_{n \rightarrow \infty} n^{-1} \sum_{t=1}^n \partial \psi_t(\eta) / \partial \eta^\top$ and $B = \lim_{n \rightarrow \infty} n^{-1} \sum_{t=1}^n \psi_t(\eta) \psi_t^\top(\eta)$. With independent replicates at the block level, Ω can be easily estimated with the sample versions of A and B as outlined in the supplementary material [Shang, Yan and Zhang (2015)]. An alternative, computing-intensive method is a bootstrap applied to the blocks (years) with spatial structure preserved. We assess the validity of the sandwich estimator in our simulation study but use the bootstrap estimator in the real data analysis.

Computationally, the optimization in both steps can be challenging, especially when the dimension of the parameter vector is large. Optimizing with respect to all parameters simultaneously often gives poor results at local maxima [Blanchet and Davison (2011)]. We adapt the profile method suggested for pairwise likelihood maximization by Blanchet and Davison (2011) and apply it to both steps. The profile method maximizes with respect to one parameter at a time while holding all other parameters at their current values, and the process goes through all parameters iteratively until convergence. To be safe, we optimize with respect to all parameters simultaneously one more time after the convergence of the profile method.

Model selection for the two-step approach can be done separately for the marginal GEV models and the MSP model in two steps with the composite likelihood information criterion (CLIC) [Varin (2008), Varin and Vidoni (2005)], which is an adaptation of the Takeuchi information criterion (TIC) [Takeuchi (1976)].

Models with lower CLIC are preferred. In step 1, the CLIC selects the best marginal model without specifying the spatial dependence structure. In step 2, the CLIC is a conditional version given the marginal model selected from step 1 and the marginal parameter estimates.

5. Simulation study. To investigate the performance of the two-step approach using daily records in comparison to the pairwise likelihood approach using block maxima only, a simulation study was conducted. The study region was confined to $[-20, 20]^2$. The marginal distribution of the block maxima at each site s is a GEV distribution with location μ_s , scale σ_s and shape ξ_s . Let $X_1(s)$ and $X_2(s)$ denote the latitude and longitude of site s . The GEV parameters were

$$\begin{cases} \mu_s = \beta_{\mu,0} + \beta_{\mu,1}X_1(s) + \beta_{\mu,2}X_2(s), \\ \sigma_s = \beta_{\sigma,0} + \beta_{\sigma,1}X_1(s) + \beta_{\sigma,2}X_2(s), \\ \xi_s = \beta_{\xi,0}, \end{cases}$$

where $\beta_{\mu,0} = 15$, $\beta_{\mu,1} = -0.2$, $\beta_{\mu,2} = 0.25$, $\beta_{\sigma,0} = 4$, $\beta_{\sigma,1} = -0.04$, $\beta_{\sigma,2} = 0.08$, and $\beta_{\xi,0} = 0.2$. The factors of our simulation study are as follows: the max-stable model, the spatial dependence level, the number of sites S and the sample size n . Three one-parameter isotropic max-stable processes were considered: the Smith model, the Schlather model and the geometric Gaussian model. The Smith model has a single parameter $\theta = \tau$. The Schlather model has an exponential correlation function with range parameter $\theta = \alpha$: $\rho(h) = \exp(-\|h\|/\alpha)$. The geometric Gaussian model also has an exponential correlation function with range parameter $\theta = \alpha$ and the parameter $\delta^2 = 8$ is assumed known. The choice of the value 8 is a compromise between two facts: (1) the random number generation from this model in R package `SpatialExtremes` [Ribatet (2013)] works well only for $\delta^2 < 10$; and (2) the pairwise extremal coefficient from $\delta^2 = 8$ has an upper bound 1.96, close to independence. The Brown–Resnick model, which covers the geometric Gaussian model as a special case and offers full range of dependence level, was not considered here because of lack of fast simulation tools. Three dependence levels were considered: weak, moderate and strong, abbreviated as W, M and S, respectively. The parameter τ for the Smith model was chosen to be 20, 200, 2000 for weak, moderate and strong dependence, respectively, as in Padoan, Ribatet and Sisson (2010). The range parameters for the other three models were chosen such that their pairwise extremal coefficient as a function of distance matches as closely as possible with that from the corresponding Smith model. From nonlinear least squares fits with distance in the range of $[0, 40]$, the parameter values of α for the Schlather model were found to be 5.2, 24.3 and 242.9 for dependence level W, M and S, respectively, and the corresponding α values for the geometric Gaussian model were found to be 25.2, 135.2 and 1252.0, respectively. We considered two levels for the number of sites $S \in \{25, 50\}$ and three levels for sample size $n \in \{20, 50, 100\}$. The performance of the sandwich variance estimator for $n = 20$ was not expected to be good, but we kept $n = 20$ in efficiency comparisons.

For each scenario, 1000 data sets of daily records were generated. The S sites were regenerated for each data set from a uniform distribution over the study region $[-20, 20]^2$. To mimic the California data analysis, we set block size $m = 122$. The m daily observations at S sites within each season were generated as realizations from the target MSP model divided by m . The max-stability ensures that the site-wise maxima of the m observations at the S sites is a realization from the MSP model. For each data set, we used the profile method for both approaches with the same starting values—the pairwise likelihood estimate from R package `SpatialExtremes` [Ribatet (2013)]. The threshold $u_{s,t}$ in the two-step approach was chosen to be the 95th sample percentile at site s in block t .

We first assess the estimator from the two-step approach. The results for $n \in \{50, 100\}$ are summarized in tables in the supplementary material [Shang, Yan and Zhang (2015)]. Consider, for example, the geometric Gaussian model. The biases are very small relative to the truth for all parameters. The empirical standard error of the estimates is higher for stronger dependence or smaller sample size, but it is much less sensitive to the number of sites S , which is consistent with the observation in Padoan, Ribatet and Sisson (2010). The average standard errors are generally in close agreement with the empirical standard errors, suggesting good performance of the sandwich variance estimator for sample size as small as 50. Consequently, the empirical coverage percentage of the 95% confidence intervals for most parameters are close to the nominal level. Under-coverage occurred for α and $\beta_{\xi,0}$ when the dependence is weak; the lowest case was 84% for $S = 50$ and $n = 100$. The coverage for $\log \alpha$ is uniformly better than for α . The under-coverage is unfortunate because sandwich variance estimators tend to underestimate the variance for small to moderate sample sizes. Bias correction [Kauermann and Carroll (2001), Mancl and DeRouen (2001)] might lead to better coverage rate of the confidence intervals in this context, but an investigation is beyond our scope here. The results for the Smith model and the Schlather model were similar or better—no empirical coverage was below 90%.

We now compare the efficiency of the pairwise likelihood approach using block maxima only (M_1) with the two-step approach (M_2). Table 1 reports the relative efficiency in mean squared error for the estimators from the two approaches for each parameter, with the M_2 estimator as the reference. Method M_2 has smaller MSE for all marginal parameters; the relative efficiency of M_1 ranges from 23% to 95%. For example, for the shape parameter $\beta_{\xi,0}$ in the geometric Gaussian model, the relative efficiency of M_1 was 45% for $S = 25$ and $n = 100$, which is the case where the coverage of the confidence interval was low in the two-step approach. This is of great interest since the shape parameter ξ governs the tail behavior of the GEV distribution and plays an important role in predicting return levels. The difference between the two methods decreases as the dependence level increases from weak to strong. For the dependence parameters, the relative efficiency of M_1 ranges from 69% to 103%, with the highest relative efficiency occurring in the weak dependence case under the extremal t process. The efficiency gain in M_2 here

TABLE 1
Relative efficiency (%) in mean squared error of model parameter estimates for the pairwise likelihood approach relative to the two-step approach for Smith, Schlather and geometric Gaussian models

Dep	n	S	τ	Smith							Schlather							Geometric Gaussian								
				$\beta_{\mu,0}$	$\beta_{\mu,1}$	$\beta_{\mu,2}$	$\beta_{\sigma,0}$	$\beta_{\sigma,1}$	$\beta_{\sigma,2}$	$\beta_{\xi,0}$	α	$\beta_{\mu,0}$	$\beta_{\mu,1}$	$\beta_{\mu,2}$	$\beta_{\sigma,0}$	$\beta_{\sigma,1}$	$\beta_{\sigma,2}$	$\beta_{\xi,0}$	α	$\beta_{\mu,0}$	$\beta_{\mu,1}$	$\beta_{\mu,2}$	$\beta_{\sigma,0}$	$\beta_{\sigma,1}$	$\beta_{\sigma,2}$	$\beta_{\xi,0}$
W	20	25	81	67	54	54	72	25	24	30	107	66	64	62	87	36	48	32	87	75	56	57	86	25	30	37
		50	81	70	54	59	80	24	30	34	106	64	65	62	92	41	57	34	83	87	57	60	92	29	34	45
	50	25	80	70	55	57	77	23	28	33	98	71	60	61	87	34	49	37	85	82	58	60	89	24	32	43
		50	78	76	55	62	89	23	32	35	100	66	66	65	87	40	54	40	85	85	55	58	95	25	35	46
	100	25	81	73	56	57	80	24	28	34	97	67	61	62	91	34	49	37	85	84	52	60	94	23	34	45
		50	74	75	57	60	85	25	30	37	101	77	64	72	97	40	61	36	87	89	55	57	96	28	34	55
M	20	25	84	62	51	55	77	29	37	40	93	58	66	64	81	44	54	36	85	63	49	49	86	28	39	54
		50	90	63	51	56	78	30	35	40	96	63	66	66	86	42	55	38	77	67	49	55	80	28	43	53
	50	25	86	69	49	57	78	27	35	45	94	59	65	61	80	42	48	44	80	66	52	51	80	29	38	52
		50	86	64	53	52	80	33	38	45	92	65	66	64	84	42	53	41	82	72	52	54	89	29	42	58
	100	25	87	69	51	57	82	29	39	49	95	62	75	64	81	42	49	40	79	70	50	55	84	31	43	56
		50	84	68	50	51	79	30	38	45	93	66	63	69	85	39	58	40	83	70	48	56	89	29	43	60
S	20	25	78	54	58	54	67	47	48	54	79	55	86	67	69	65	67	45	76	55	53	52	66	44	54	78
		50	69	49	56	50	62	39	49	56	85	55	85	67	67	67	62	52	84	57	57	51	69	48	54	85
	50	25	76	59	55	61	69	48	53	56	82	55	81	63	62	60	62	48	75	57	53	57	64	43	54	85
		50	84	52	48	52	63	38	50	61	88	56	77	66	70	66	72	45	78	61	54	60	67	45	53	85
	100	25	86	57	56	53	66	40	47	70	85	57	78	67	63	57	62	44	76	57	51	52	66	42	51	84
		50	82	63	58	58	68	43	54	68	87	63	85	74	66	59	63	47	74	62	55	58	70	45	54	82

is explained by the fact that the marginal parameters are estimated more precisely in the first step.

How does the efficiency gain in M_2 affect risk analysis such as estimation of joint and individual return levels? Let y_{50} be the joint 50-year return level for two sites s_1 and s_2 , such that $\Pr(Y(s_1) > y_{50}, Y(s_2) > y_{50}) = 1/50$. Given the bivariate marginal distribution, y_{50} can be found numerically for any given parameter vector. We considered three sites in the study region, $s_1 = (10, 10)$, $s_2 = (10, 11)$, and $s_3 = (10, 0)$. The joint 50-year return level was estimated for two pairs, (s_1, s_2) and (s_1, s_3) , which represent pairs that are close and distant, respectively. The relative efficiency of the two methods in estimating the individual 50-year return level at the three sites and the joint 50-year return level at the two sites is summarized in a table in the supplementary material [Shang, Yan and Zhang (2015)]. The relative efficiency of M_1 with M_2 as the reference is poor, ranging from 58% to 91% when the dependence is weak. As the dependence level gets stronger, M_1 becomes almost as competitive as M_2 , which is consistent with the relative efficiency for the shape parameter estimator. The sample size n and the number of sites S seems to have little effect on the relative efficiency for all three models.

Up to this point, both the GEV margin model and the max-stable dependence model have been correctly specified in the fitting. The only possible misspecification for the two-step approach is the distribution of the exceedances over the threshold, which depends on the block size m and the threshold u . In practice, however, neither the marginal model nor the dependence model will be correct for any finite m or u , which may introduce bias in estimation. To understand the limitation of the two-step method, we generated data using t processes, which are in the max-domain of attraction of the extremal- t process [Opitz (2013)]. Details about the data generation, the choice of degree of freedom ν and the results for $\nu \in \{1, 2\}$ are in the supplementary material [Shang, Yan and Zhang (2015)]. The two-step method was more efficient than the pairwise likelihood method in all parameter estimation except in a very few parameters, including $\beta_{\xi,0}$ when $\nu = 2$. A close examination revealed that the MSE for the two-step estimator was dominated by its bias in these cases. The pairwise likelihood approach requires the convergence of the marginal block maximum to a ν -Fréchet distribution, with ν being the degrees of freedom of the t process, and the convergence of the dependence structure to extremal- t copula. The two-step approach requires additionally that the distribution of those observations exceeding the threshold converges to a generalized Pareto distribution with appropriately transformed parameters. For $\nu = 1$, the limiting distribution provided good approximation in all aspects, but for $\nu = 2$, the convergence of the marginal block maxima and exceedances needed m to be much greater than 122. Consequently, the two-step method was more efficient than the pairwise likelihood method for $\nu = 1$, but lost its edge for some parameters for $\nu = 2$.

6. Data analysis.

6.1. *First step—marginal GEV models.* Recall that our main interest is to make inferences about the effect of ENSO on extreme precipitation in California. Let $X_1(t)$ be the SOI in year t . We considered site specific GEV models with SOI in the location parameter:

$$(6.1) \quad \begin{aligned} \mu(s, t) &= \beta_{\mu,s,0} + \beta_{\mu,s,1}X_1(t), \\ \sigma(s, t) &= \beta_{\sigma,s,0}, \\ \xi(s, t) &= \beta_{\xi,s,0}. \end{aligned}$$

This model has $4S$ parameters, but it does not assume any smooth surface of the GEV parameters in covariates such as latitude, longitude and elevation, which may be unrealistic given the complex terrain of California. In fact, in our earlier exploratory analysis, including all the covariates in smooth GEV parameter surfaces led to undesired results: the effects of the SOI made little physical sense and the GEV models did not pass goodness-of-fit tests at many sites.

Model (6.1) was fitted with threshold $u(s, t)$ chosen to be the 98th sample percentile of the daily records in block t at site s in the first step of our two-step approach. The standard errors of the parameter estimates were obtained by the bootstrap method with 1000 bootstrap samples. To check the adequacy of the marginal GEV models, a parametric bootstrap based goodness-of-fit test procedure was performed for the annual winter maximum daily precipitation at each site. Out of 36 sites, the p -values of the Kolmogorov–Smirnov test statistics at 35 sites were insignificant at the 1% level. The choice of 1% level was ad hoc and informal, with the consideration of multiple tests and possible adjustment to control false discovery rate [e.g., [Benjamini and Yekutieli \(2001\)](#)]. The only site that did not pass the goodness-of-fit test was removed from the analysis in the sequel.

The pattern of the marginal parameter estimates at 35 sites is presented in [Figure 2](#). It confirms that there is no obvious smooth surface of these parameters to be characterized by simple functions of covariates such as latitude, longitude and elevation. Our interest is the coefficients of SOI in the location parameter. Their estimates were negative at all sites, and 22 out of the 35 estimates were significantly negative at the 5% level. We also investigated the map of the standardized coefficient estimates, estimates divided by their standard errors, in the supplementary material [[Shang, Yan and Zhang \(2015\)](#)]. The standardized coefficient estimates are the z -scores under the null hypotheses that the corresponding coefficients are zero. Again, no obvious smooth spatially varying pattern was present.

6.2. *Second step—spatial dependence model.* Using the fitted marginal GEV models from the first step, we transformed the block maxima to the unit Fréchet scale. An exploratory analysis with the pairwise extremal coefficients of the transformed data using the Capéraà–Fougères–Genest (CFG) estimator [[Capéraà,](#)

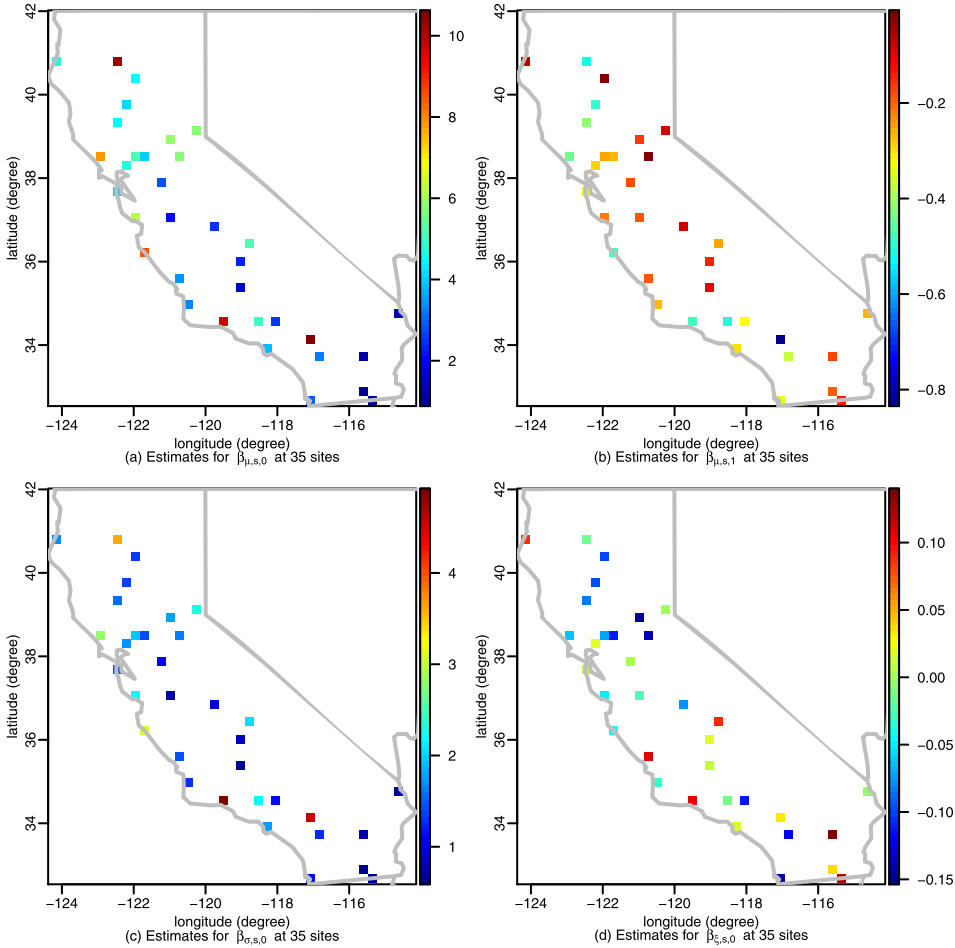


FIG. 2. Marginal parameter estimates at 35 sites. (a) $\beta_{\mu,s,0}$; (b) $\beta_{\mu,s,1}$; (c) $\beta_{\sigma,s,0}$; (d) $\beta_{\xi,s,0}$.

Fougères and Genest (1997), Genest and Segers (2009)] suggested possible anisotropy and elevation effect in the dependence. We considered both the Schlather model and the geometric Gaussian model with a climate space transformation to allow anisotropy and elevation effects [Blanchet and Davison (2011), Cooley, Nychka and Naveau (2007)]. The Smith model was excluded because event realizations from it are too regular to be realistic for practical usage. Let h be the trivariate difference vector of longitude, latitude and elevation between two sites. This vector is transformed into the climate space by Vh with

$$V = \begin{pmatrix} \cos \varphi & \sin \varphi & 0 \\ -\sin \varphi / r & \cos \varphi / r & 0 \\ 0 & 0 & q \end{pmatrix}, \quad r \in (0, 1), \varphi \in [-\pi/2, \pi/2), q \geq 0,$$

where φ is a rotation angle measured counterclockwise from the east direction, r is the ratio of the minor axis to the major axis of the ellipse of the geometric anisotropy, and q gives a weight to elevation in the squared climate distance. The distance in the climate space is $\sqrt{h^\top V^{-1} V h}$, which is then used in the correlation function of the models. For comparison, we also fitted isotropic and geometric anisotropic models in the two-dimensional space without the climate space, that is, $\varphi = 0$, $r = 1$, and $q = 0$. Four correlation functions were considered: exponential, double exponential (also known as Gaussian), Cauchy and Whittle–Matérn [Banerjee, Carlin and Gelfand (2003), Section 2.1]. For the Cauchy and Whittle–Matérn correlation, the shape parameter was fixed at 1 since it is difficult to estimate. In the geometric Gaussian model, the variation parameter δ^2 which controls the upper bound of the extremal coefficient function is not easily identifiable jointly with the range parameter in the correlation function [Davison, Padoan and Ribatet (2012)]. We fixed δ^2 at 9 as a compromise between reliable simulation needed for risk analysis and the near-independence in pairwise extremal coefficient it can provide.

In total, 24 models were fitted and compared with their CLIC value conditioning on the marginal GEV models from the first step. Our final model with the lowest conditional CLIC value (262280.4) is an isotropic geometric Gaussian model which has an exponential correlation function without elevation effect. The range parameter is estimated as 4.95, with standard error 0.73. The spatial dependence decays quickly with distance. The fitted bivariate extremal coefficient for two sites reaches 1.3 and 1.7 when their distance becomes 19.3 and 149.7 kilometers, respectively. For illustration, with downtown San Francisco as the reference point, the extremal coefficients are 1.51, 1.57, 1.89 and 1.92, respectively, at San Jose, Santa Cruz, Santa Barbara and Los Angeles. The spatial dependence is quite weak, giving much room for the two-step approach to improve efficiency compared to the pairwise likelihood approach as shown in the next subsection.

To check the adequacy of the geometric Gaussian model, we first compared the madogram-based pairwise extremal coefficients [Cooley, Naveau and Poncet (2006)] with those predicted from the model. The madogram-based pairwise extremal coefficients are calculated based on the data in the unit Fréchet scale obtained from step 1, instead of ranks, and, hence, it is possible that some of the estimates exceed the theoretical upper limit 2. They are plotted against distance in the supplementary material [Shang, Yan and Zhang (2015)]. The madogram-based estimators with 100 bins are also shown. The fitted extremal coefficient curve crosses the scatters in the middle, suggesting no obvious lack of fit for pairs. To check the fit beyond pairs, we compared the empirical quantiles of the maxima of subsets of sites with the quantiles implied from the model [Blanchet and Davison (2011), Davison and Gholamrezaee (2012)]. For a subset A of all sites, let $Z_A = \max_{d \in A} Z_d$. We have observations of Z_A for n independent years denoted by $z_{A,1}, \dots, z_{A,n}$ with $n = 55$. The distribution of these empirical quantiles can

be approximated from a large number k of simulated realizations from the fitted model, $z_{A,1,k}^*, \dots, z_{A,n,k}^*$, $k = 1, \dots, K$. The empirical quantiles versus the model-based quantiles for four subsets of the sites formed geographically based on their latitudes are plotted in the supplementary material [Shang, Yan and Zhang (2015)]. The pointwise confidence intervals and simultaneous confidence bands were obtained from $K = 5000$ simulated realizations [Davison and Hinkley (1997), Section 4.2.4]. No alarming disagreement between the empirical quantiles and the model quantiles is observed for any of the subset of sites.

6.3. *Risk analysis.* For comparison, we also used the pairwise likelihood approach (M_1) based on block maxima only to fit the same model as selected from the two-step approach (M_2) with daily records. Unlike the two-step approach where the site-specific marginal parameters are estimated separately for each site, the pairwise likelihood approach needs to estimate all the parameters altogether. Our profile method updated the marginal parameter estimates one site at a time first and then updated the dependence parameter; this process was repeated until convergence.

The point estimates from the two approaches are reasonably close. For the marginal GEV models, the standard errors from M_2 are much smaller than those from M_1 for most of the parameter estimates. The box plots of the ratio of the standard errors of the four parameter estimates across 35 sites are presented in the supplementary material [Shang, Yan and Zhang (2015)]. In particular, the three quantiles of the ratio are 0.51, 0.58 and 0.65 for the SOI coefficient $\beta_{\mu,s,1}$, and 0.48, 0.53 and 0.61 for the shape parameter $\beta_{\xi,s,0}$. The reduction in standard errors in estimating $\beta_{\mu,s,1}$ leads to increased power in detecting the SOI effect: significance at 5% was found only at 14 out of 35 sites with M_1 (compared to 22 with M_2). The reduction of standard error in estimating $\beta_{\xi,s,0}$ has important implications on the accuracy of return level estimation given that the shape parameter controls the shape of a GEV distribution. As will be shown next, the reduced standard errors in marginal parameters lead to more efficient inference about marginal risk measures such as return levels at each individual site. For the dependence model, the range parameter was estimated as 6.31 with standard error 0.99 from M_1 , in comparison to 4.95 with standard error 0.73 from M_2 . The reduction in the standard error in the dependence parameter estimate of M_2 might be explained by its more efficient marginal parameter estimates.

In the spatial context, it is of more interest to see how the efficiency gain in both marginal and dependence parameter estimation affects risk measures of jointly defined events. We first look at the joint 50-year return level for two sites, as defined in Section 5. Since SOI is a season-specific covariate, we fix the SOI value at -1 , -0.15 (the sample average) and 1 so that the return levels are interpreted for years with these SOI values separately. For illustration, consider the three stations near the Sacramento area: Napa (122.25°W , 38.27°N), Winters (121.97°W , 38.52°N) and Davis (121.78°W , 38.53°N); see Figure 1. We generated $N = 5000$

TABLE 2

Joint 50-year return levels (cm) for three pairs at three different SOI values based on both pairwise likelihood approach and two-step approach

Pair	Pairwise likelihood (M_1)		Two-step (M_2)	
	95% CI	Width	95% CI	Width
SOI = -1				
Napa & Winters	(10.22, 15.04)	4.82	(10.15, 13.60)	3.46
Napa & Davis	(8.62, 12.35)	3.73	(8.52, 10.37)	1.84
Winters & Davis	(8.33, 11.42)	3.09	(8.17, 9.84)	1.67
SOI = -0.15				
Napa & Winters	(9.92, 14.79)	4.87	(9.85, 13.34)	3.49
Napa & Davis	(8.18, 11.87)	3.68	(8.30, 10.23)	1.93
Winters & Davis	(7.85, 10.83)	2.98	(7.90, 9.59)	1.69
SOI = 1				
Napa & Winters	(9.34, 14.38)	5.04	(9.45, 13.12)	3.66
Napa & Davis	(7.36, 11.14)	3.77	(7.89, 9.92)	2.03
Winters & Davis	(7.05, 10.14)	3.09	(7.48, 9.37)	1.89

realizations of the model parameters from the approximate multivariate normal distribution of the estimator from both M_1 and M_2 . For each realized parameter vector, the joint 50-year return level was obtained numerically for each pair of the three sites. Table 2 shows the 95% confidence intervals of the joint 50-year return levels for the three pairs with the empirical distribution from both M_1 and M_2 at the three SOI values. The decreasing trend of the joint return levels as the SOI value increases is consistent with existing findings [Shang, Yan and Zhang (2011), Zhang et al. (2010)]. Interestingly, the confidence intervals from M_2 are almost inside those from M_1 for all three pairs, with a reduction of 27.3% to 50.7% in length.

To gain further insights about the efficiency gain in assessing bivariate risk measures, we investigated the joint sampling distribution of the site-wise maximum extremal precipitations over every 50 years for all pairs of the three sites. Realizations from the distribution can be drawn for the three sites and then used to assess their joint behavior. The SOI was fixed at the sample average -0.15 for ease of interpretation. For each of the $N = 5000$ parameter vectors drawn from their asymptotic normal distribution, we generated 50 years of data and obtained the sitewise maxima. On the log scale, Figure 3 shows the empirical contours of the 5000 draws from the sampling distribution for the three sites with both M_1 and M_2 . The levels with 50, 75, 90 and 95 percent of coverage are plotted. It is apparent that the joint sampling distribution is much more compact from M_2 than from M_1 . Consequently, much tighter approximate confidence regions are obtained with M_2 than with M_1 . Positive dependence between each pair is clearly visible, with es-

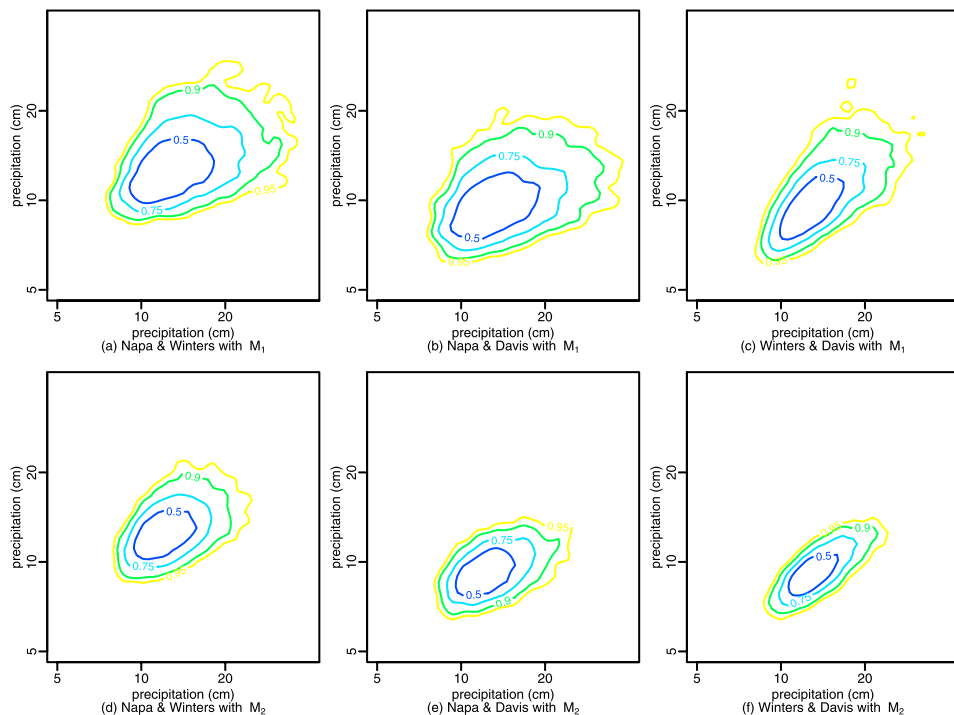


FIG. 3. Contours of the 50-year sample return levels (cm) for three pairs on the log scale. Upper (a), (b), (c): pairwise likelihood approach using block maxima data (M_1); Lower (d), (e), (f): two-step approach (M_2). Left (a), (d): Napa & Winters; Center (b), (e): Napa & Davis; Right (c), (f): Winters & Davis.

pecially stronger dependence between the last pair (Winters and Davis), which is explained by their distance.

7. Discussion. In contrast to the pairwise likelihood approach which utilizes only block maxima, the two-step approach uses more information through daily records and makes more efficient inferences about the parameters. The consistency in marginal GEV parameter estimation is not affected by possible misspecification of the dependence model. Our simulation study showed appreciable efficiency gain of the two-step approach in comparison to the pairwise likelihood approach. The two-step approach is simple to implement with existing software, intuitive for practitioners, and avoids defining multivariate thresholds [Bacro and Gaetan (2014), Huser and Davison (2014), Wadsworth and Tawn (2012)] or multivariate Pareto process modeling [Aulbach and Falk (2012)]. A caveat is that, as with the POT approach or the point process approach, not only the block maxima but also the exceedances over the threshold need to have distributions that are well approximated by the corresponding limiting distribution; not meeting the requirement may lead to bias as illustrated in our simulation study.

In application to maximum daily winter precipitation in California, large scale climate variation ENSO was found to have significant negative impact on the location parameter of the marginal GEV distribution at 22 out of 35 sites with the two-step approach (compared to 14 with the pairwise likelihood approach). Risk analysis with the two-step approach gives much tighter confidence intervals and confidence regions for joint risk measures than the pairwise likelihood approach.

Several methodological aspects merit further investigation. In the first step, we did not address threshold selection, an important and still active problem even for univariate extreme value analysis [e.g., [Guillou and Hall \(2001\)](#), [Thompson et al. \(2009\)](#)]. Recent research has shown a promising approach with quantile regression for nonstationarity with covariate information [[Northrop and Jonathan \(2011\)](#)]. Alternatively, one may use the r largest order statistic to construct the marginal likelihood [e.g., [Coles \(2001\)](#)]. Compared to the bivariate threshold-based approaches, the two-step approach may potentially be less efficient if the distributional approximation over the bivariate threshold is accurate, but its marginal inference is robust to dependence structure misspecification. A study on the robustness-efficiency trade-off would be interesting.

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SUPPLEMENTARY MATERIAL

Additional simulation results and data analysis (DOI: [10.1214/14-AOAS804SUPP](https://doi.org/10.1214/14-AOAS804SUPP); .pdf). We provide a sandwich variance estimator, additional tables summarizing the simulation study and additional figures in analyzing the California precipitation data.

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