# Discussion

## A. H. Welsh

This collection of papers gathering and promoting highly successful applications of Statistics is a good antidote for anyone feeling somewhat defensive about Statistics. The focus on the successful use of Bayesian methods has produced a powerful and stimulating set of stories; the Editors and Authors are all to be congratulated on their successful efforts to bring out the stories behind these analyses. The papers are relatively short (as was required by the Editors) and a good measure of their success is that they both stand alone and motivate the reader to follow up and read the original papers.

The article on the search for the wreckage of flight AF 447 (Stone et al.) is fascinating. The description of the careful and detailed thinking about what might have happened, the evaluation and inclusion of relevant empirical evidence to quantify the possible scenarios and the final success of the analysis in assigning substantial posterior probability to where the wreckage was ultimately found are all inspiring. Like many inspiring articles, it challenges us to think about both the difficult issues of the particular problem considered and general issues about the overall approach. I think a Bayesian analysis is highly appropriate for this problem, but it is not so easy to explain why and it is clear that, as always, the analysis itself has to be done extremely well.

One motivation for doing a Bayesian analysis for this problem (and one that is commonly articulated) is that the event in question is unique so it is not meaningful to think about replications. This is not really convincing because hypothetical replications are hypothetical whether they are conceived of for an event that is extremely rare (and in the extreme happens once) or for events that occur frequently. Moreover, it turns out later that nine past crashes were deemed similar enough to be used to provide information for constructing the prior, making it difficult to argue that the event really is unique.

Another widely used motivation for Bayesian analysis is that it propagates the uncertainty correctly. This

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is true and important, but it is also true that it propagates only the uncertainties that we decide to include in the model. We make choices over what uncertainties to include and we also make relatively arbitrary choices which we subsequently treat as fixed. For example, were the uncertainties in the weights for the different scenarios or the chosen $\alpha$ propagated through to the conclusion? As a practical matter, I do not believe we can or should try to propagate all uncertainty, simply that we should not get too carried away and forget about aspects we have treated as certain. This highlights the fact that the Bayesian approach is a tool that is extremely useful for combining the quantitative information we choose to use and are able to express in terms of distributions but which, like any tool, needs to be used well to be effective; the tool on its own does not solve the problem but needs to be applied by highly skilled people.

The four unsuccessful searches that preceded the final, successful search highlight some of the issues. They too used assumptions and information to select the search location. Presumably they did not use a Bayesian analysis? If they did not (and it is not really possible with the benefit of hindsight to go back and redo this fairly), differences between the particular techniques used may be outweighed by differences in the information and beliefs that fed into the analysis. For example, the fourth search based on possible drift concentrated in a small rectangle relatively far from the actual crash site. Would a Bayesian analysis based on the information used to come up with that search rectangle have produced different results? It is difficult to be sure from the maps but it looks like a passive acoustic search actually covered the crash site but that the wreckage was not discovered. We can interpret this as measurement error or as using an incorrect prior. The searchers tried to find the sonar beacons, not realizing that these had failed and were not operating. The successful search both allowed for this possibility (at least by not ruling out that area as having been previously searched) and, because so much time had elapsed that the beacons could not have been expected to still work, adopted different technology in the search. Had they adopted the belief that the area had been searched so the wreckage could not be there and built this into the

prior, it would not have been found. Thus, it was crucial to adopt the correct beliefs to end up with the right result. The point is that the tool had to be used well and the credit is due to the users rather than simply the tool.

In their very interesting paper on managing Baltic salmon, Kuikka et al. make the point that Bayesian methods make it possible to combine "relevant data from many sources." The paper explicitly acknowledges the role of politics in salmon management and the need to combine empirical data with "data" that is too difficult or expensive to ever be collected. The word relevant is critical here since irrelevant data may at best just increase uncertainty and at worst lead to seriously wrong answers. The choice of what is relevant or not depends ultimately on the user and is not an automatic property of the approach. Kuikka et al. also make the point that biologically realistic models for salmon involve too many parameters to fit without using informative priors. This is mentioned again in Carroll's intriguing paper on dietary consumption; Bayesian computations can be used to fit models that frequentist methods cannot fit. Running a Bayesian computation will produce numbers but, as in any computation, we need to convince ourselves that the numbers are meaningful before we use and interpret them. In particular, it is important to understand clearly whether the model is identifiable or not, whether the model is incorrect in some important way (so the computational issues reflect lack of fit) and the extent to which the prior is driving the analysis. The fact that these questions are not easy to answer with complicated models and high-dimensional parameter spaces does not lessen the importance of trying. Identifiability is important because it is resolved by using informative priors which regularize the likelihood and enable the model to be fitted; even vague priors can be informative in this context. There is no problem with using informative priors but we need to know when the priors are informative, particularly when they are so informative that the posterior is essentially the prior. Conceptually, this may not be so different from the frequentist approach of imposing nonestimable constraints on the parameters. A different kind of identifiability issue arises in Bayesian history matching (Vernon et al.) because it is possible that different scenarios or models can lead to the same observable data, particularly when this is a single slice in time. Here, finding matching simulations seems only part of the really difficult scientific problem being considered.

Another reason a model may be difficult to fit is that it does not describe the data. Forcing it to "fit," for example, by switching to a Bayesian analysis, may not be the best response. It is difficult to check complicated models, particularly hierarchical models with latent variables, measurement error, missing data, etc., but using an incorrect model may be a concern when the model proves difficult to fit.

A challenging issue acknowledged in Carroll is the issue of using survey weights in a Bayesian analysis. We can think about this as a way of estimating the likelihood by the pseudo-likelihood and then using this estimated likelihood in a regular Bayesian analysis. This does involve a combination of design-based and model-based approaches which require different conditioning but, somewhat like approximate Bayesian computation (ABC), it might be viewed as a pragmatic approach to solving difficult problems. It is not clear what the Bayesian costs and benefits are; in frequentist analysis, Chambers et al. (2012) show that pseudo-likelihood estimation is less efficient than maximum likelihood estimation so there is some loss of information. Constructing the likelihood requires including all the design variables in the model. Aside from the fact that, in contrast to the survey weights, the design variables are not usually available to secondary analysts, the study from which the data are taken (NHANES) uses a complicated design (with several nested levels of cluster sampling) which it would not be straightforward to incorporate into the model. Moreover, making the model more complicated may increase the computational difficulties of fitting the model. The use of pseudo-likelihood in Bayesian analysis definitely needs research into its meaning and consequences before we can consider it with equanimity.

## REFERENCE

CHAMBERS, R. L., STEEL, D. G., WANG, S. and WELSH, A. (2012). *Maximum Likelihood Estimation for Sample Surveys*. *Monographs on Statistics and Applied Probability* **125**. CRC Press, Boca Raton, FL. MR2963765