

**CORRECTION ON**  
**MOMENTS OF MINORS OF WISHART MATRICES**

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Theorem 5.7 in [1] gives a formula for the variance of a minor (i.e., a subdeterminant) of a Wishart random matrix. The formula has to be corrected as follows.

**THEOREM 5.7.** *Let  $I, J \in \binom{[r]}{m}$  have intersection  $C := I \cap J$  of cardinality  $c = |C| = |I \cap J|$ . Define  $\bar{I} = I \setminus (I \cap J)$ ,  $\bar{J} = J \setminus (I \cap J)$  and  $\bar{I}\bar{J} = \bar{I} \cup \bar{J}$ . Then the minor  $\det(S_{I \times J}) = \det(S_{IJ})$  of the Wishart matrix  $S \sim \mathcal{W}_r(n, \Sigma)$  has variance*

$$\begin{aligned} & \text{Var}[\det(S_{IJ})] \\ &= \det(\Sigma_{IJ})^2 \frac{n!}{(n-m)!} \left[ \frac{(n+2)!}{(n+2-m)!} - \frac{n!}{(n-m)!} \right] \\ & \quad + \det(\Sigma_{C \times C})^2 \det(\bar{\Sigma}_{\bar{I}\bar{J} \times \bar{I}\bar{J}}) \frac{(n+2)!}{(n+2-c)!} \cdot \frac{n!}{(n-m)!} \\ & \quad \times \left[ \sum_{k=0}^{m-c-1} (m-c-k)! \frac{(n+2-c)!}{(n+2-c-k)!} (-1)^k \text{tr}\{(\bar{\Sigma}_{\bar{I}\bar{J}} \bar{\Sigma}^{\bar{I}\bar{J}})^{(k)}\} \right], \end{aligned}$$

where  $\bar{\Sigma} = \Sigma_{([r] \setminus C) \times ([r] \setminus C)} - \Sigma_{([r] \setminus C) \times C} \Sigma_{C \times C}^{-1} \Sigma_{C \times ([r] \setminus C)}$ .

**PROOF.** Define  $\bar{S}$  in analogy to  $\bar{\Sigma}$ . Since  $\det(S_{IJ}) = \det(S_{C \times C}) \det(\bar{S}_{\bar{I} \times \bar{J}})$ , and  $S_{C \times C}$  and  $\bar{S}_{\bar{I} \times \bar{J}}$  are independent (Lemma 5.2),

$$\begin{aligned} \text{Var}[\det(S_{IJ})] &= (\text{Var}[\det(S_{CC})] + \text{E}[\det(S_{CC})]^2) \text{Var}[\det(\bar{S}_{\bar{I}\bar{J}})] \\ & \quad + \text{Var}[\det(S_{CC})] \text{E}[\det(\bar{S}_{\bar{I}\bar{J}})]^2. \end{aligned}$$

The claim now follows from Corollary 4.2 and Propositions 5.1 and 5.5.  $\square$

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## REFERENCE

- [1] DRTON, M., MASSAM, H. and OLKIN, I. (2008). Moments of minors of Wishart matrices. *Ann. Statist.* **36** 2261–2283. [MR2458187](#)

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