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ERRATA

STOCHASTIC CALCULUS OVER SYMMETRIC MARKOV PROCESSES WITHOUT TIME REVERSAL

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1. Errata. The sentence "In view of Theorem 2.2 in [22], ... for q.e. $x \in E$." at page 1538 should be eliminated. The definitions of $\mathring{\mathcal{M}}^d$, $\mathring{\mathcal{M}}^j$ and $\mathring{\mathcal{M}}^\kappa$ are corrected to be like $\mathring{\mathcal{M}}^d := \{M \in \mathring{\mathcal{M}} \mid \langle M, N\rangle \equiv 0 \text{ for } N \in \mathring{\mathcal{M}}^c\}$.

The statements of Theorem 2.1 and Corollary 2.3 in [3] are incorrect, which come from the error in [1] (see [2]). The corrected statement of Theorem 2.1 in [3] can be found below. Its proof can be obtained in the same way as in [3]. The class $\widehat{\mathcal{J}}$ introduced in [3] is unnecessary for the corrected statement.

THEOREM 1.1 (Corrected statement of Theorem 2.1 in [3]). *There exists a one-to-one correspondence between $\mathcal{J}/\sim$ and $\mathcal{M}_{\text{loc}}^{d,[\![0,\zeta[\![}$ which is characterized by the relation that for $\phi \in \mathcal{J}$ (resp., $M \in \mathcal{M}_{\text{loc}}^{d,[\![0,\zeta[\![}$), there exists $M \in \mathcal{M}_{\text{loc}}^{d,[\![0,\zeta[\![}$ (resp., $\phi \in \mathcal{J}$) such that $\Delta M_t = \phi(X_{t-}, X_t)$, $t \in [0, \zeta[$, $\mathbb{P}_x$-a.s. for q.e. $x \in E$. Moreover, we have $\langle M\rangle_t = \int_0^t \int_{E_\partial} \phi^2(X_s, y)N(X_s, dy)\, dH_s$ for all $t \in [0, \infty[$ $\mathbb{P}_x$-a.s. for q.e. $x \in E$.*

We define subclasses of $\mathcal{M}_{\text{loc}}^{d,[\![0,\zeta[\![}$ as follows:

$$\mathcal{M}_{\text{loc}}^{j,[\![0,\zeta[\![} := \{M \in \mathcal{M}_{\text{loc}}^{d,[\![0,\zeta[\![} \mid \phi(\cdot, \partial) = 0, \kappa\text{-a.e. on } E\},$$
$$\mathcal{M}_{\text{loc}}^{\kappa,[\![0,\zeta[\![} := \{M \in \mathcal{M}_{\text{loc}}^{d,[\![0,\zeta[\![} \mid \phi = 0, J\text{-a.e. on } E \times E\}.$$

Then we have that $M \in \mathcal{M}_{\text{loc}}^{j,[\![0,\zeta[\![}$, $N \in \mathcal{M}_{\text{loc}}^{\kappa,[\![0,\zeta[\![}$ imply $\langle M, N\rangle \equiv 0$ $\mathbb{P}_x$-a.s. for q.e. $x \in E$, and every $M \in \mathcal{M}_{\text{loc}}^{[\![0,\zeta[\![}$ is decomposed to $M = M^c + M^j + M^\kappa$, where $M^c \in \mathcal{M}_{\text{loc}}^{c,[\![0,\zeta[\![}$, $M^j \in M_{\text{loc}}^{j,[\![0,\zeta[\![}$, $M^\kappa \in \mathcal{M}_{\text{loc}}^{\kappa,[\![0,\zeta[\![}$ have the properties $\langle M^c, M^j\rangle \equiv$

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$\langle M^j, M^\kappa\rangle \equiv \langle M^\kappa, M^c\rangle \equiv 0$ in view of Theorem 1.1. The statement of Remark 2.3 in [3] is changed to be the following:

REMARK 1.1. For each $i = c, d, j, \kappa$, we let

$$\mathring{\mathcal{M}}^{i+}_{\mathrm{loc}} := \big\{M \mid \text{ there exists } \{G_n\} \in \Theta \text{ and } M^{(n)} \in \mathring{\mathcal{M}}^i \text{ such that } M_t = M_t^{(n)} \text{ for all } t \le \tau_{G_n} \text{ and } n \in \mathbb{N}, \mathbb{P}_x\text{-a.s. for q.e. } x \in E\big\}.$$

Then $\mathcal{M}^{i,[\![0,\zeta[\![}_{\mathrm{loc}} = \mathring{\mathcal{M}}^{i+}_{\mathrm{loc}}$ $(i = c, d, j, \kappa)$. More strongly, we have $\mathcal{M}^{c,[\![0,\zeta[\![}_{\mathrm{loc}} = \mathring{\mathcal{M}}^{c}_{\mathrm{loc}}$.

All the statements of Corollary 4.1 and 4.3, Definition 4.3 and Theorem 4.1 in [3], the generalized Fukushima decomposition (Theorem 4.2 in [3]), the generalized Itô formula (Theorem 4.3 in [3]) and their corollaries (Corollaries 4.4 and 4.5 in [3]) hold only for $t \in [0, \zeta[$ $\mathbb{P}_x$-a.s. for q.e. $x \in E$. The proofs of them can be done in the same way as in [3]. The classes $\dot{\mathcal{F}}^{\ddagger}_{\mathrm{loc}}$ and $\mathcal{F}^{\ddagger}_{\mathrm{loc}}$ introduced in [3] are unnecessary for the corrected statements. We only expose the corrected statement of generalized Fukushima decomposition below for completeness.

THEOREM 1.2 (Corrected statement of Theorem 4.2 in [3]). *For $u \in \dot{\mathcal{F}}^{\dagger}_{\mathrm{loc}}$, the additive functional A^u defined by $A^u_t := u(X_t) - u(X_0)$ can be decomposed as*

$$A^u = M^u + N^u, \qquad M^u \in \mathcal{M}^{[\![0,\zeta[\![}_{\mathrm{loc}}, \qquad N^u \in \mathcal{N}_{c,\mathrm{loc}}$$

in the sense that $A^u_t = M^u_t + N^u_t$, $t \in [0, \zeta[$, $\mathbb{P}_x$-a.s. for q.e. $x \in E$. Such a decomposition is unique up to the equivalence of additive functionals on $[\![0, \zeta[\![$ (or of local additive functionals).

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