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ON A GENERALIZATION OF SEMICOMMUTATIVE RINGS

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Abstract. We introduce weakly semicommutative rings which are a generalization of semicommutative rings, and give some examples which show that weakly semicommutative rings need not be semicommutative. Also we give some relations between semicommutative rings and weakly semicommutative rings.

1. Introduction

Throughout this paper, all rings are associative with identity. A ring R is called semicommutative if for any $a, b \in R, ab = 0$ implies $aRb = 0$ (this ring is also called ZI ring in [2,8]). R is semicommutative if and only if any right (left) annihilator over R is an ideal of R by [4, Lemma 1] or [7, Lemma 1.2]. A ring R is called reduced if it has no nonzero nilpotent elements. By [4], reduced rings are semicommutative, and semicommutative rings are Abelian (i.e., all idempotents are central). In this paper, we call a ring R a weakly semicommutative ring if for any $a, b \in R$, $ab = 0$ implies arb is a nilpotent element for any $r \in R$. Clearly semicommutative rings are weakly semicommutative. Examples will be given to show that the converse is not true.

In [3], N.K.Kim and Y.Lee show that if R is a reduced ring, then

$$R_3 = \left\{ \left(\begin{array}{ccc} a & b & c \\ 0 & a & d \\ 0 & 0 & a \end{array} \right) \middle| a, b, c, d \in R \right\}$$

is a semicommutative ring. But

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$$R_n = \left\{ \left(\begin{array}{ccccc} a & a_{12} & a_{13} & \cdots & a_{1n} \\ 0 & a & a_{23} & \cdots & a_{2n} \\ 0 & 0 & a & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & a \end{array} \right) \middle| a, a_{ij} \in R \right\}$$

may not be semicommutative for $n \geq 4$. However, in section 2, we will see that R_n is weakly semicommutative. More generally, we show that a ring R is weakly semicommutative if and only if for any n, the $n \times n$ upper triangular matrix ring $T_n(R)$ is a weakly semicommutative ring.

In section 3, we study the relationships between semicommutative rings and weakly semicommutative rings. Actually we show that (1) if R is semicommutative and satisfy α-condition then $R[x; \alpha]$ is weakly semicommutative. (2) for a ring R suppose that R/I is weakly semicommutative for some ideal I of R, if I is semicommutative then R is weakly semicommutative. These results also show that weakly semicommutative rings may not be semicommutative.

For a ring R, we denote by $nil(R)$ the set of all nilpotent elements of R.

2. Examples

Definition 2.1. A ring R is called weakly semicommutative if for any $a, b \in R$, $ab = 0$ implies $arb \in nil(R)$ for any $r \in R$.

Clearly any semicommutative ring is weakly semicommutative. In the following we will see the converse is not true.

Example 2.1. Let R be a reduced ring,

$$R_n = \left\{ \left(\begin{array}{ccccc} a & a_{12} & a_{13} & \cdots & a_{1n} \\ 0 & a & a_{23} & \cdots & a_{2n} \\ 0 & 0 & a & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & a \end{array} \right) \middle| a, a_{ij} \in R \right\}$$

By $[3, Example\ 1.3]$, R_n is not semicommutative for $n \geq 4$. But R_n is a weakly semicommutative ring.

Proof. First we will give some claims.

Claim 2.1. A ring R is a weakly semicommutative if and only if, for any n, the n-by-n upper triangular matrix ring $T_n(R)$ is a weakly semicommutative ring.

We note that any subring of weakly semicommutative rings is a weakly semicommutative ring. Thus if upper triangular matrix ring $T_n(R)$ is a weakly semicommutative ring, then so is R. Conversely, let

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ 0 & a_{22} & a_{23} & \cdots & a_{2n} \\ 0 & 0 & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & a_{nn} \end{pmatrix},$$

$$B = \begin{pmatrix} b_{11} & b_{12} & b_{13} & \cdots & b_{1n} \\ 0 & b_{22} & b_{23} & \cdots & b_{2n} \\ 0 & 0 & b_{33} & \cdots & b_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & b_{nn} \end{pmatrix} \in T_n(R)$$

with $AB = 0$, and let

$$C = \begin{pmatrix} c_{11} & c_{12} & c_{13} & \cdots & c_{1n} \\ 0 & c_{22} & c_{23} & \cdots & c_{2n} \\ 0 & 0 & c_{33} & \cdots & c_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & c_{nn} \end{pmatrix} \in T_n(R)$$

be any element. Then we have $a_{ii}b_{ii} = 0$ for any $1 \leq i \leq n$. Since R is a weakly semicommutative ring, there exists $k_i \in \mathbb{N}$ such that $(a_{ii}c_{ii}b_{ii})^{k_i} = 0$. Let $k = max\{k_1, k_2, \cdots, k_n\}$, then we have $(a_{ii}c_{ii}b_{ii})^k = 0$ for each i. Thus

$$(ACB)^k = \begin{pmatrix} a_{11}c_{11}b_{11} & * & * & \cdots & * \\ 0 & a_{22}c_{22}b_{22} & * & \cdots & * \\ 0 & 0 & a_{33}c_{33}b_{33} & \cdots & * \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & a_{nn}c_{nn}b_{nn} \end{pmatrix}^k$$

$$= \begin{pmatrix} 0 & * & * & \cdots & * \\ 0 & 0 & * & \cdots & * \\ 0 & 0 & 0 & \cdots & * \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix}$$

Hence $(ACB)^{kn} = 0$. This means that $T_n(R)$ is a weakly semicommutative ring.

By Claim 2.1, we can see the following result easily.

Claim 2.2. If R is a reduced ring, then, for any n, the n-by-n upper triangular matrix ring $T_n(R)$ is a weakly semicommutative ring.

Now it follows from Claim 2.2 that R_n is a weakly semicommutative ring since R_n is a subring of $T_n(R)$.

Example 2.2. Let F be a division ring and consider the 2×2 upper triangular matrix ring $R = \begin{pmatrix} F & F \\ 0 & F \end{pmatrix}$. Then R is not semicommutative by $[4, Example\ 5]$. But R is weakly semicommutative by Claim 2.2.

From [5], given a ring R and a bimodule ${}_RM_R$, the trivial extension of R by M is the ring $T(R, M) = R \oplus M$ with the usual addition and the following multiplication: $(r_1, m_1)(r_2, m_2) = (r_1r_2, r_1m_2 + m_1r_2)$. This is isomorphic to the ring of all matrices $\begin{pmatrix} r & m \\ 0 & r \end{pmatrix}$, where $r \in R$ and $m \in M$, and the usual matrix operations are used.

Corollary 2.1. *A ring R is weakly semicommutative if and only if the trivial extension $T(R, R)$ is weakly semicommutative.*

It is well-known that for a ring R and any positive integer n, if R is reduced then $R[x]/(x^n)$ is reversible, where (x^n) is the ideal generated by x^n [5, Theorem 2.5]. Based on it we may suspect that if R is reversible or semicommutative then $R[x]/(x^n)$ is semicommutative ($n \geq 2$). However the following example eliminates the possibility.

Example 2.3. Let $\mathbb{H}$ be the Hamilton quaternions over the real number field and R be the trivial extension of $\mathbb{H}$ by $\mathbb{H}$. Let S be the trivial extension of R by R. Then R is reversible, and hence is semicommutative. But $S = T(R, R) \cong \left\{ \begin{pmatrix} r & m \\ 0 & r \end{pmatrix} \middle| r, m \in R \right\}$ is not semicommutative by $[3, Example\ 1.7]$. Thus we have

$$(*) \quad R[x]/(x^n) \cong \left\{ \begin{pmatrix} a_0 & a_1 & a_2 & \cdots & a_{n-1} \\ 0 & a_0 & a_1 & \cdots & a_{n-2} \\ 0 & 0 & a_0 & \cdots & a_{n-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & a_0 \end{pmatrix} \middle| a_i \in R,\ i = 0, 1, \cdots, n-1 \right\}$$

is not semicommutative for any $n \geq 2$.

However, we will see that if R is semicommutative then $R[x]/(x^n)$ is weakly semicommutative. Taking into account $(*)$, we obtain the following result.

Corollary 2.2. *Let R be a ring and n any positive integer. Then R is weakly semicommutative if and only if $R[x]/(x^n)$ is weakly semicommutative, where (x^n)*

is the ideal generated by x^n.

From Claim 2.1, one may suspect that if R is weakly semicommutative, then every n-by-n full matrix ring $M_n(R)$ is weakly semicommutative, where $n \geq 2$. But the following example erases the possibility.

Example 2.4. Let $\mathbb{Z}$ be the ring of integers and $Mat_2(\mathbb{Z})$ the 2×2 full matrix ring over $\mathbb{Z}$, then $\mathbb{Z}$ is weakly semicommutative. Note that $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} = 0$, but we have $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$ is not nilpotent. So $Mat_2(\mathbb{Z})$ is not weakly semicommutative.

2. Semicommutative Rings and Weakly Semicommutative Rings

In this section, we let α be an endomorphism of R, unless especially noted. We call R satisfies α-condition if $ab = 0 \Leftrightarrow a\alpha(b) = 0$ for any $a, b \in R$.

In [4], the authors show that if R is semicommutative, then $R[x]$ may not be semicommutative. But we will show that if R is semicommutative, then $R[x]$ is weakly semicommutative. More generally, we can see that $R[x;\alpha]$ is weakly semicommutative if R is a semicommutative ring satisfing α-condition.

Lemma 3.1. *Let R satisfy α-condition. If $ab \in nil(R)$ for $a, b \in R$, then $a\alpha(b) \in nil(R)$.*

Proof. Let $(ab)^k = 0$, where $k \in \mathbb{N}$, i.e., $abab\cdots ab = 0$. So $a\alpha(b)\alpha(ab\cdots ab) = 0$, and thus $a\alpha(b)ab\cdots ab = 0$. Continuing this procedure yields that $(a\alpha(b))^k = 0$, this means $a\alpha(b) \in nil(R)$.

Lemma 3.2. (see [6, Lemma 3.1]) *Let R be a semicommutative ring. Then $nil(R)$ is an ideal of R.*

Lemma 3.3. *Let R be a semicommutative ring and satisfy α-condition. If $f(x) = a_0 + a_1x + \cdots + a_mx^m, g(x) = b_0 + b_1x + \cdots + b_nx^n \in R[x;\alpha]$ satisfies $f(x)g(x) = 0$, then $a_i\alpha^i(b_j) \in nil(R)$ for each i, j.*

Proof. Note that $f(x)g(x) = \sum_{k=0}^{m+n}(\sum_{i+j=k} a_i\alpha^i(b_j))x^k = 0$, then $\sum_{i+j=k} a_i\alpha^i(b_j) = 0$ for any $0 \leq k \leq m+n$. In the following we claim that $a_i\alpha^i(b_j) \in nil(R)$ for each i, j. We proceed by induction on $i+j$. Then we obtain $a_0b_0 = 0$. This proves for $i+j = 0$. Now suppose that our claim is true for $i+j < p$, where $1 \leq p \leq m+n$. Note that

$$(*) \qquad \sum_{i+j=p} a_i\alpha^i(b_j) = 0.$$

By induction hypothesis, we have $a_s\alpha^s(b_t) \in nil(R)$ for $s+t<p$. Thus $a_s\alpha^{s+w}(b_t) \in nil(R)$ for any $w \in \mathbb{N}$ by Lemma 3.1, and hence $\alpha^{s+w}(b_t)a_s \in nil(R)$. Now multiplying a_0 to Eq.($*$) from the right-hand side, we can get $a_0b_pa_0 \in nil(R)$ by Lemma 3.2, and hence $a_0b_p \in nil(R)$. Multiplying a_1 to Eq.($*$) from the right-hand side, we can get $a_0b_pa_1 + a_1\alpha(b_{p-1})a_1 \in nil(R)$. Thus $a_1\alpha(b_{p-1}) \in nil(R)$. Continuing this process, we can prove $a_i\alpha^i(b_j) \in nil(R)$ for $i+j=p$. Therefore $a_i\alpha^i(b_j) \in nil(R)$ for $0 \le i \le m, 0 \le j \le n$.

Lemma 3.4. *Let R be a semicommutative ring and satisfy α-condition, $f(x) = a_0 + a_1x + \cdots + a_nx^n \in R[x;\alpha]$. If $a_0, a_1, \cdots, a_n \in nil(R)$, then $f(x) \in nil(R[x;\alpha])$.*

Proof. Suppose that $a_i^{m_i} = 0$, $i = 0, 1, \cdots, n$. Let $k = m_0+m_1+\cdots+m_n+1$. Then

$$(f(x))^k = \sum_{s=0}^{kn}\Big(\sum_{i_1+i_2+\cdots+i_k=s} a_{i_1}\alpha^{i_1}(a_{i_2})\alpha^{i_1+i_2}(a_{i_3})\cdots\alpha^{i_1+i_2+\cdots+i_{k-1}}(a_{i_k})\Big)x^s,$$

where $a_{i_1}, a_{i_2}, \cdots, a_{i_k} \in \{a_0, a_1, \cdots, a_n\}$. Consider

$$a_{i_1}\alpha^{i_1}(a_{i_2})\alpha^{i_1+i_2}(a_{i_3})\cdots\alpha^{i_1+i_2+\cdots+i_{k-1}}(a_{i_k}). \qquad (*)$$

It can be easily checked that there exists $a_t \in \{a_0, a_1, \cdots, a_n\}$ such there are more than m_t a_t's in ($*$). We may assume that a_t appears $s > m_t$ times in ($*$). Rewrite ($*$) as

$$b_0\alpha^{j_1}(a_t)b_1\alpha^{j_1+j_2}(a_t)\cdots b_{s-1}\alpha^{j_1+j_2+\cdots+j_s}(a_t)b_s,$$

where $b_i \in R$, $j_1, j_2, \cdots, j_s \in \mathbb{N}$. Since $a_t^s = 0$, and R is a semicommutative ring satisfying α-condition, we can get

$$b_0\alpha^{j_1}(a_t)b_1\alpha^{j_1+j_2}(a_t)\cdots b_{s-1}\alpha^{j_1+j_2+\cdots+j_s}(a_t)b_s = 0,$$

and hence Eq.($*$)=0. Thus

$$\sum_{i_1+i_2+\cdots+i_k=s} a_{i_1}\alpha^{i_1}(a_{i_2})\alpha^{i_1+i_2}(a_{i_3})\cdots\alpha^{i_1+i_2+\cdots+i_{k-1}}(a_{i_k}) = 0,$$

which implies that $f(x) \in nil(R[x;\alpha])$.

Theorem 3.1. *Let R be a semicommutative ring and satisfy α-condition. Then $R[x;\alpha]$ is weakly semicommutative.*

Proof. Let $f(x) = \sum_{i=0}^m a_ix^i, g(x) = \sum_{j=0}^n b_jx^j \in R[x;\alpha]$ such that $f(x)g(x) = 0$, and let $h(x) = \sum_{s=0}^k c_sx^s \in R[x;\alpha]$ be any element. By Lemma

3.3, there exits $n_{ij} \in \mathbb{N}$ such that $(a_i\alpha^i(b_j))^{n_{ij}} = 0$ for any i and j, and hence $(a_i\alpha^{i+t}(b_j))^{n_{ij}} = 0$ for any $t \in \mathbb{N}$ by Lemma 3.1. Thus $a_i\alpha^i(c_s)\alpha^{i+s}(b_j) \in nil(R)$ for $0 \le i \le m, 0 \le j \le n, 0 \le s \le k$. Note that

$$f(x)h(x)g(x) = \sum_{t=0}^{m+n+k} (\sum_{i+j+s=t} a_i\alpha^i(c_s)\alpha^{i+s}(b_j))x^t.$$

We can see that $\sum_{i+j+s=t} a_i\alpha^i(c_s)\alpha^{i+s}(b_j) \in nil(R)$ for any t by Lemma 3.2. Thus $f(x)h(x)g(x) \in nil(R[x;\alpha])$ by Lemma 3.4. This means that $R[x;\alpha]$ is weakly semicommutative.

Corollary 3.1. *Let R be a semicommutative ring. Then $R[x]$ is weakly semicommutative ring.*

Let R be a ring and $\triangle$ be a multiplicatively closed subset of R consisting of central regular elements, and let $S_R = \{u^{-1}a | u \in \triangle, a \in R\}$, then S_R is a ring. For it, we have the following result.

Proposition 3.1. *Let R be a ring. Then the following statements are equivalent.*

(1) *R is weakly semicommutative.*

(2) *S_R is weakly semicommutative.*

Proof. (2)⇒(1) is obvious since R is a subring of S_R. (1)⇒(2): Let $\alpha\beta = 0$ with $\alpha = u^{-1}a, \beta = v^{-1}b$, $u, v \in \triangle$ and $a, b \in R$, and let $\gamma = w^{-1}c$ be any element of S_R, $w \in \triangle$, $c \in R$. Since $\triangle$ is contained in the center of R, we have $0 = \alpha\beta = u^{-1}av^{-1}b = (u^{-1}v^{-1})ab$, and hence $ab = 0$. But R is weakly semicommutative, so there exists $n \in \mathbb{N}$ such that $(acb)^n = 0$. Thus $(\alpha\gamma\beta)^n = (u^{-1}aw^{-1}cv^{-1}b)^n = ((vwu)^{-1}acb)^n = ((vwu)^{-1})^n(acb)^n = 0$. Therefore S_R is weakly semicommutative.

The ring of Laurent polynomials in x, with coefficients in a ring R, consists of all formal sum $\sum_{i=k}^{n} m_ix^i$ with obvious addition and multiplication, where $m_i \in R$ and k, n are (possibly negative) integers; denote it by $R[x;x^{-1}]$.

Corollary 3.2. *For a ring R, $R[x]$ is weakly semicommutative if and only if $R[x;x^{-1}]$ is weakly semicommutative.*

Proof. It suffices to establish necessity since $R[x]$ is a subring of $R[x;x^{-1}]$. Let $\triangle = \{1, x, x^2, \cdots\}$, then clearly $\triangle$ is a multiplicatively closed subset of $R[x]$. Since $R[x;x^{-1}] = S_{R[x]}$, it follows that $R[x;x^{-1}]$ is weakly semicommutative by Proposition 3.1.

Remark. The following is another direct proof of this corollary. Let $f(x), g(x) \in R[x; x^{-1}]$ with $f(x)g(x) = 0$, and let $h(x) \in R[x; x^{-1}]$ be any element. Then there exists $s \in \mathbb{N}$ such that $f_1(x) = f(x)x^s, g_1(x) = g(x)x^s, h_1(x) = h(x)x^s \in R[x]$, obtaining $f_1(x)g_1(x) = 0$; hence there exists $n \in \mathbb{N}$ such that $(f_1(x)h_1(x)g_1(x))^n = 0$ since $R[x]$ is weakly semicommutative, so we have $(f(x)h(x)g(x))^n = (x^{-3s} (f_1(x)h_1(x)g_1(x)))^n = (x^{-3s})^n(f_1(x)h_1(x)g_1(x))^n = 0$. This means $R[x; x^{-1}]$ is weakly semicommutative.

Corollary 3.3. *Let R be a semicommutative ring. Then $R[x; x^{-1}]$ is weakly semicommutative ring.*

It is well-known that for a ring R if I is a reduced ideal of R such that R/I is semicommutative, then R is semicommutative [4, Theorem 6]. Also they gave an example in [4, Example 5] to show that if for any nonzero proper ideal I of R, R/I and I are semicommutative, where I is considered as a semicommutative ring without identity, then R may not be semicommutative. Here we will show that R is weakly semicommutative if R/I and I are semicommutative. More generally we have the following result.

Theorem 3.2. *For a ring R suppose that R/I is weakly semicommutative for some ideal I of R. If I is semicommutative, then R is weakly semicommutative.*

Proof. Let $a, b \in R$ such that $ab = 0$, and let $r \in R$ be any element. Then $\overline{a}\overline{b} = 0$ in R/I. Since R/I is weakly semicommutative, there exists $n \in \mathbb{N}$ such that $(\overline{a}\overline{r}\overline{b})^n = 0$, and hence $(arb)^n \in I$. Note that $(ba)^2 = 0$, we have

$$((arb)^{n+1}ar)baba(rb(arb)^{n+1}) = 0.$$

Since $((arb)^{n+1}ar)ba \in I$, $ba(rb(arb)^{n+1}) \in I$, and I is semicommutative, it follows that

$$((arb)^{n+1}ar)ba(rb(arb)^{n}ar)ba(rb(arb)^{n+1}) = 0,$$

that is, $(arb)^{3n+6} = 0$, i.e., $arb \in nil(R)$. Thus R is weakly semicommutative.

Corollary 3.4. *For a ring R suppose that R/I is semicommutative for some ideal I of R. If I is semicommutative then R is weakly semicommutative.*

Corollary 3.5. *For a ring R if I is a reduced ideal of R such that R/I is weakly semicommutative, then R is weakly semicommutative.*

Corollary 3.6. (*see* $[4,\ Theorem\ 6]$) *For a ring R, if I is reduced ideal of R such that R/I is semicommutative, then R is semicommutative.*

Proof. Let $ab = 0$ with $a, b \in R$, and let $r \in R$ be any element. Then $arb \in nil(R)$ by Corollary 3.5. On the other hand, $arb \in I$ since R/I is semicommutative. Thus $arb = 0$ since I is reduced. This means that R is semicommutative.

Proposition 3.2. *Let R be a ring and I an ideal of R such that R/I is weakly semicommutative. If $I \subseteq nil(R)$, then R is weakly semicommutative.*

Proof. Let $a, b \in R$ with $ab = 0$, and let $r \in R$ be any element. Then $\overline{a}\overline{b} = 0$ in R/I. Since R/I is weakly semicommutative, there exists n such that $(\overline{a}\overline{r}\overline{b})^n = 0$. Thus $(arb)^n \in I$, and hence $(arb)^n \in nil(R)$. Therefore R is weakly semicommutative.

Remark. By Corollary 3.1 and [4, Example 2] or Corollary 3.4 and [4, Example 5], we can also see that weakly semicommutative rings may not be semicommutative.

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