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Research Article

Common Fixed Point Theorems for a Pair of Self-Mappings in Fuzzy Cone Metric Spaces

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In this paper, we present some common fixed point theorems for a pair of self-mappings in fuzzy cone metric spaces under the generalized fuzzy cone contraction conditions. We extend and improve some recent results given in the literature.

1. Introduction

In 1965, Zadeh [1] came up with a fabulous idea. He introduced the theory of fuzzy set, which is the generalization of crisp sets. A mapping $\mathscr{G}$ is from $\mathscr{X}$ to $[0,1]$; then $\mathscr{G}$ is known as a fuzzy set. Later on, the fuzzy metric space concept was given by Kramosil and Michalek [2], which is performing the probabilistic metric space and would approach the fuzzy set. In [3], George and Veeramani were given the stronger form of the fuzz metric. Some more set-valued mapping results for fixed point on fuzzy metric spaces can be seen, for example, in [4–6] and the references therein.

In 2007, Som [7] proved some continuous self-mapping results for common fixed point in fuzzy metric spaces. He generalized the results of Pant [8], Som [9], and Vasuki [10]. Some other common fixed point results in the fuzzy metric space can be found in [11–16] and the references therein.

Huang and Zhang [17] introduced the concept of cone metric space. They proved the convergent sequences, Cauchy sequences, and some fixed point theorems for contractive-type mappings in cone metric spaces. Later on, Abbas and Jungck [18] proved some noncommuting mapping results in cone metric spaces. After that, a series of authors proved some fixed point and common fixed point results for different contractive-type mappings in cone metric spaces (see, e.g., [19–25]).

Recently, the concept of fuzzy cone metric space was introduced by Oner et al. [26]. They proved some basic properties and a Banach contraction theorem for fixed point with the assumption of Cauchy sequences. Rehman and Li [27] generalized the result of Oner et. al. [26] and proved some fixed point theorems in fuzzy cone metric spaces without the assumption of Cauchy sequences. Some more fixed point and common fixed point results in fuzzy cone metric spaces can be found in [27–31].

In the demonstration of this research work, we generalize the results of Oner [26] and Rehman [27] for a pair of self-mappings in fuzzy cone metric spaces and prove some unique common fixed theorems with illustrative examples.

2. Preliminaries

Definition 1 ([32]). An operation $* : [0,1]^2 \longrightarrow [0,1]$ is known as a continuous t-norm if it holds the following:

(1) $*$ is commutative, associative, and continuous.

(2) $c * 1 = c$, $\forall c \in [0,1]$.

(3) $c * c_0 \leq c_1 * c_2$, whenever $c \leq c_1$ and $c_0 \leq c_2$, for every $c, c_0, c_1, c_2 \in [0,1]$.

Meanwhile, the basic t-norm continuous conditions are as follows.

The minimum, product, and Lukasiewicz t-norms are defined, respectively, as (see [32])

$$c * c_1 = \min\{c, c_1\},$$
$$c * c_1 = cc_1 \quad (1)$$
$$\text{and } c * c_1 = \max\{c + c_1 - 1, 0\}.$$

Definition 2 ([17]). A subset $\mathscr{P}$ of a real Banach space E is called a cone if

(1) $\mathscr{P} \neq \emptyset$, closed and $\mathscr{P} \neq \{\vartheta\}$, where ϑ represents the zero element of E,

(2) $cx + c_1 w \in \mathscr{P}$, if $0 \leq c, c_1 < \infty$ and $w, x \in \mathscr{P}$,

(3) $x = \vartheta$, if both $-x, x \in \mathscr{P}$.

All the cones have nonempty interior and the natural numbers set is denoted by $\mathbf{N}$.

Definition 3 ([26]). A 3-tuple $(\mathscr{X}, \mathfrak{M}, *)$ is known as a fuzzy cone metric space, if $*$ is a continuous t-norm, $\mathscr{X}$ is an arbitrary set, $\mathscr{P}$ is a cone of E, and $\mathfrak{M}$ is a fuzzy set on $\mathscr{X} \times \mathscr{X} \times \operatorname{int}(\mathscr{P})$ if the following hold:

(i) $\mathfrak{M}(w, x, s) > 0$ and $\mathfrak{M}(w, x, s) = 1$ if $w = x$,

(ii) $\mathfrak{M}(w, x, s) = \mathfrak{M}(x, w, s)$,

(iii) $\mathfrak{M}(w, y, s + t) \geq \mathfrak{M}(w, x, ss) * \mathfrak{M}(x, y, t)$,

(iv) $\mathfrak{M}(w, x, .) : \operatorname{int}(\mathscr{P}) \longrightarrow [0, 1]$ is continuous,

for all $w, x, y \in \mathscr{X}$ and $s, t \in \operatorname{int}(\mathscr{P})$.

Remark 4 ([27]). If we suppose that $E = \mathbb{R}$, $\mathscr{P} = [0, \infty)$, and $c * c_1 = cc_1$, then every fuzzy metric space becomes a fuzzy cone metric space.

Definition 5 ([26]). Let $(\mathscr{X}, \mathfrak{M}, *)$ be a fuzzy cone metric space, $x \in \mathscr{X}$, and a sequence (x_j) in $\mathscr{X}$ is

(i) converging to x if $c \in (0, 1)$ and $s \gg \vartheta\ \exists j_1 \in \mathbf{N}$ such that $\mathfrak{M}(x_j, x, s) > 1 - c, \forall j \geq j_1$. We can write this as $\lim_{j \longrightarrow \infty} x_j = x$ or $x_j \longrightarrow x$ as $j \longrightarrow \infty$.

(ii) Cauchy sequence if $c \in (0, 1)$ and $s \gg \vartheta\ \exists j_1 \in \mathbf{N}$ such that $\mathfrak{M}(x_j, x_k, s) > 1 - c, \forall j, k \geq j_1$.

(iii) $(\mathscr{X}, \mathfrak{M}, *)$ is complete if every Cauchy sequence is convergent in $\mathscr{X}$.

(iv) fuzzy cone contractive if $\exists c \in (0, 1)$, satisfying

$$\frac{1}{\mathfrak{M}(x_j, x_{j+1}, s)} - 1 \leq c\left(\frac{1}{\mathfrak{M}(x_{j-1}, x_j, s)} - 1\right) \quad (2)$$

for all $s \gg \vartheta$, $j \geq 1$.

Definition 6 ([27]). Let $(\mathscr{X}, \mathfrak{M}, *)$ be a fuzzy cone metric space. A fuzzy cone metric $\mathfrak{M}$ is triangular if

$$\frac{1}{\mathfrak{M}(w, y, s)} - 1 \leq \left(\frac{1}{\mathfrak{M}(w, x, s)} - 1\right) + \left(\frac{1}{\mathfrak{M}(x, y, s)} - 1\right), \quad (3)$$

$\forall w, x, y \in \mathscr{X}$ and $s \gg \vartheta$.

Lemma 7 ([26]). *Let $x \in \mathscr{X}$ and let (x_j) be a sequence in $\mathscr{X}$. Then $x_j \longrightarrow x$ in a fuzzy cone metric space $(\mathscr{X}, \mathfrak{M}, *)$ if $\mathfrak{M}(x_j, x, s) \longrightarrow 1$ as $j \longrightarrow \infty$, for each $s \gg \vartheta$.*

For more properties of fuzzy cone metric spaces, see [26].

Definition 8 ([26]). A mapping $\mathscr{G} : \mathscr{X} \longrightarrow \mathscr{X}$ is known as fuzzy cone contractive in a fuzzy cone metric space $(\mathscr{X}, \mathfrak{M}, *)$, if $\exists c \in (0, 1)$ such that

$$\frac{1}{\mathfrak{M}(\mathscr{G}w, \mathscr{G}x, s)} - 1 \leq c\left(\frac{1}{\mathfrak{M}(w, x, s)} - 1\right), \quad (4)$$

for all $w, x \in \mathscr{X}$, $s \gg \vartheta$, and c is known as a contraction constant of $\mathscr{G}$.

Theorem 9 ([26]). *A self-mapping in a complete fuzzy cone metric space, in which the fuzzy cone contractive sequences are Cauchy, has a unique fixed point.*

Further, in this paper, we shall study some common fixed point results in $(\mathscr{X}, \mathfrak{M}, *)$. Let $\mathscr{F}, \mathscr{G} : \mathscr{X} \longrightarrow \mathscr{X}$ be two self-mappings satisfying the following more generalized fuzzy cone contraction condition:

$$\begin{aligned}\frac{1}{\mathfrak{M}(\mathscr{F}x, \mathscr{G}w, s)} - 1 \leq{}& c_1\left(\frac{1}{\mathfrak{M}(x, w, s)} - 1\right) \\ &+ c_2\left(\frac{1}{\mathfrak{M}(x, \mathscr{F}x, s)} - 1\right) \\ &+ c_3\left(\frac{1}{\mathfrak{M}(w, \mathscr{G}w, s)} - 1\right) \\ &+ c_4\left(\frac{1}{\mathfrak{M}(w, \mathscr{F}x, s)} - 1\right) \\ &+ c_5\left(\frac{1}{\mathfrak{M}(x, \mathscr{G}w, s)} - 1\right),\end{aligned} \quad (5)$$

where $s \gg \vartheta$ and the constants $c_1, c_2, c_3, c_4, c_5 \in [0, +\infty)$. It is noted that (5) is the same as (4) if $\mathscr{F} = \mathscr{G}$, $c_2 = c_3 = c_4 = c_5 = 0$, and $c_1 \in (0, 1)$. On the other hand, the mappings $\mathscr{F}$ and $\mathscr{G}$ may not hold the fuzzy cone contraction condition if (5) is satisfied, which is shown in Example 14. Thus, in this research work, we generalize some recent results given in the literature (see Remark 13 and Example 14).

3. Main Result

Theorem 10. *Let $\mathscr{F}$, $\mathscr{G} : \mathscr{X} \longrightarrow \mathscr{X}$ be two self mappings and $\mathfrak{M}$ is triangular in a complete fuzzy cone metric space $(\mathscr{X}, \mathfrak{M}, *)$ which satisfies (5) with $(c_1 + c_2 + c_3 + 2\max\{c_4, c_5\}) < 1$. Then $\mathscr{F}$ and $\mathscr{G}$ have a unique common fixed point in $\mathscr{X}$.*

Proof. Fix $x_0 \in \mathscr{X}$ and we define the iterative sequences in $\mathscr{X}$ as

$$x_{2j+1} = \mathscr{F}x_{2j}$$
$$\text{and } x_{2j+2} = \mathscr{G}x_{2j+1}, \quad (6)$$
$$j \geq 0.$$

By view of (5), for $s \gg \vartheta$,

$$\begin{aligned}
&\frac{1}{\mathfrak{M}\left(x_{2j+1}, x_{2j+2}, s\right)} - 1 = \frac{1}{\mathfrak{M}\left(\mathcal{F}x_{2j}, \mathcal{G}x_{2j+1}, s\right)} - 1 \\
&\leq c_1\left(\frac{1}{\mathfrak{M}\left(x_{2j}, x_{2j+1}, s\right)} - 1\right) \\
&+ c_2\left(\frac{1}{\mathfrak{M}\left(x_{2j}, \mathcal{F}x_{2j}, s\right)} - 1\right) \\
&+ c_3\left(\frac{1}{\mathfrak{M}\left(x_{2j+1}, \mathcal{G}x_{2j+1}, s\right)} - 1\right) \\
&+ c_4\left(\frac{1}{\mathfrak{M}\left(x_{2j+1}, \mathcal{F}x_{2j}, s\right)} - 1\right) \\
&+ c_5\left(\frac{1}{\mathfrak{M}\left(x_{2j}, \mathcal{G}x_{2j+1}, s\right)} - 1\right) \\
&\leq c_1\left(\frac{1}{\mathfrak{M}\left(x_{2j}, x_{2j+1}, s\right)} - 1\right) \\
&+ c_2\left(\frac{1}{\mathfrak{M}\left(x_{2j}, x_{2j+1}, s\right)} - 1\right) \\
&+ c_3\left(\frac{1}{\mathfrak{M}\left(x_{2j+1}, x_{2j+2}, s\right)} - 1\right) \\
&+ c_5\left(\frac{1}{\mathfrak{M}\left(x_{2j}, x_{2j+1}, s\right)} - 1 + \frac{1}{\mathfrak{M}\left(x_{2j+1}, x_{2j+2}, s\right)} \right.\\
&\left. - 1\right).
\end{aligned} \tag{7}$$

Then

$$\frac{1}{\mathfrak{M}\left(x_{2j+1}, x_{2j+2}, s\right)} - 1 \leq \alpha\left(\frac{1}{\mathfrak{M}\left(x_{2j}, x_{2j+1}, s\right)} - 1\right), \tag{8}$$

where $\alpha = (c_1 + c_2 + c_5)/(1 - c_3 - c_5) < 1$, since $(c_1 + c_2 + c_3 + 2\max\{c_4, c_5\}) < 1$.

Let us denote $(1/\mathfrak{M}(x_j, x_{j+1}, s) - 1)$ by $\mathscr{M}_j$; then, from (8), we have

$$\mathscr{M}_{2j+1} \leq \alpha \mathscr{M}_{2j}. \tag{9}$$

Similarly,

$$\begin{aligned}
&\frac{1}{\mathfrak{M}\left(x_{2j+2}, x_{2j+3}, s\right)} - 1 = \frac{1}{\mathfrak{M}\left(\mathcal{F}x_{2j+2}, \mathcal{G}x_{2j+1}, s\right)} - 1 \\
&\leq c_1\left(\frac{1}{\mathfrak{M}\left(x_{2j+1}, x_{2j+2}, s\right)} - 1\right) \\
&+ c_2\left(\frac{1}{\mathfrak{M}\left(x_{2j+2}, \mathcal{F}x_{2j+2}, s\right)} - 1\right) \\
&+ c_3\left(\frac{1}{\mathfrak{M}\left(x_{2j+1}, \mathcal{G}x_{2j+1}, s\right)} - 1\right) \\
&+ c_4\left(\frac{1}{\mathfrak{M}\left(x_{2j+1}, \mathcal{F}x_{2j+2}, s\right)} - 1\right) \\
&+ c_5\left(\frac{1}{\mathfrak{M}\left(x_{2j+2}, \mathcal{G}x_{2j+1}, s\right)} - 1\right) \\
&\leq c_1\left(\frac{1}{\mathfrak{M}\left(x_{2j+1}, x_{2j+2}, s\right)} - 1\right) \\
&+ c_2\left(\frac{1}{\mathfrak{M}\left(x_{2j+2}, x_{2j+3}, s\right)} - 1\right) \\
&+ c_3\left(\frac{1}{\mathfrak{M}\left(x_{2j+1}, x_{2j+2}, s\right)} - 1\right) \\
&+ c_4\left(\frac{1}{\mathfrak{M}\left(x_{2j+1}, x_{2j+2}, s\right)} - 1 \right.\\
&\left. + \frac{1}{\mathfrak{M}\left(x_{2j+2}, x_{2j+3}, s\right)} - 1\right).
\end{aligned} \tag{10}$$

Then

$$\begin{aligned}
&\frac{1}{\mathfrak{M}\left(x_{2j+2}, x_{2j+3}, s\right)} - 1 \\
&\quad \leq \beta\left(\frac{1}{\mathfrak{M}\left(x_{2j+1}, x_{2j+2}, s\right)} - 1\right),
\end{aligned} \tag{11}$$

where $\beta = (c_1 + c_3 + c_4)/(1 - c_2 - c_4) < 1$, since $(c_1 + c_2 + c_3 + 2\max\{c_4, c_5\}) < 1$. Then (11) can be written as

$$\mathscr{M}_{2j+2} \leq \beta \mathscr{M}_{2j+1}. \tag{12}$$

Now, from (9) and (12), we can get the following inequalities:

$$\begin{aligned}
\mathscr{M}_{2j} &\leq \beta\mathscr{M}_{2j-1} \leq \beta\alpha\mathscr{M}_{2j-2} \leq \cdots \leq (\alpha\beta)^{j}\mathscr{M}_0, \\
\mathscr{M}_{2j+1} &\leq \alpha\mathscr{M}_{2j} \leq \alpha\beta\mathscr{M}_{2j-1} \leq \alpha^2\beta\mathscr{M}_{2j-2} \leq \cdots \\
&\leq \alpha(\alpha\beta)^{j}\mathscr{M}_0, \\
\mathscr{M}_{2j+2} &\leq \beta\mathscr{M}_{2j+1} \leq \beta\alpha\mathscr{M}_{2j} \leq \beta^2\alpha\mathscr{M}_{2j-1} \leq \cdots \\
&\leq (\alpha\beta)^{j+1}\mathscr{M}_0, \\
\mathscr{M}_{2j+3} &\leq \alpha\mathscr{M}_{2j+2} \leq \alpha\beta\mathscr{M}_{2j+1} \leq \alpha^2\beta\mathscr{M}_{2j} \leq \cdots \\
&\leq \alpha(\alpha\beta)^{j+1}\mathscr{M}_0.
\end{aligned} \tag{13}$$

Thus, we have

$$\begin{aligned}
\mathscr{M}_{2j} + \mathscr{M}_{2j+1} &\leq (\alpha\beta)^{j}(1+\alpha)\mathscr{M}_0, \\
\mathscr{M}_{2j+1} + \mathscr{M}_{2j+2} &\leq \alpha(\alpha\beta)^{j}(1+\beta)\mathscr{M}_0, \\
\mathscr{M}_{2j+2} + \mathscr{M}_{2j+3} &\leq (\alpha\beta)^{j+1}(1+\alpha)\mathscr{M}_0, \\
\mathscr{M}_{2j+3} + \mathscr{M}_{2j+4} &\leq \alpha(\alpha\beta)^{j+1}(1+\beta)\mathscr{M}_0.
\end{aligned} \tag{14}$$

Hence, from the above, we conclude that a sequence (x_j) is fuzzy cone contractive in $\mathscr{X}$; that is,

$$\lim_{j\longrightarrow\infty}\mathfrak{M}(x_j, x_{j+1}, s) = 1, \quad \text{for } s \gg \vartheta. \tag{15}$$

Let $j, k \in \mathbf{N}$ and let (x_j) be the above sequence; we assume that $k > j$. Then, two cases arise.

Case (i). If j is an even number,

$$\begin{aligned}
&\frac{1}{\mathfrak{M}(x_j, x_k, s)} - 1 \\
&\leq \left(\frac{1}{\mathfrak{M}(x_j, x_{j+1}, s)} - 1\right) \\
&\quad + \left(\frac{1}{\mathfrak{M}(x_{j+1}, x_{j+2}, s)} - 1\right) \\
&\quad + \left(\frac{1}{\mathfrak{M}(x_{j+2}, x_{j+3}, s)} - 1\right) \\
&\quad + \left(\frac{1}{\mathfrak{M}(x_{j+3}, x_{j+4}, s)} - 1\right) + \cdots \\
&\quad + \left(\frac{1}{\mathfrak{M}(x_{k-1}, x_k, s)} - 1\right) \\
&= \mathscr{M}_j + \mathscr{M}_{j+1} + \mathscr{M}_{j+2} + \mathscr{M}_{j+3} + \cdots + \mathscr{M}_{k-2} + \mathscr{M}_{k-1} \\
&\leq \left((\alpha\beta)^{j/2} + (\alpha\beta)^{j/2+1} + \cdots + (\alpha\beta)^{k/2-1}\right)(1+\alpha)\mathscr{M}_0 \\
&\leq \frac{(\alpha\beta)^{j/2}}{1-\alpha\beta}(1+\alpha)\mathscr{M}_0.
\end{aligned} \tag{16}$$

Case (ii). If j is an odd number,

$$\begin{aligned}
&\frac{1}{\mathfrak{M}(x_j, x_k, s)} - 1 \leq \left(\frac{1}{\mathfrak{M}(x_j, x_{j+1}, s)} - 1\right) \\
&\quad + \left(\frac{1}{\mathfrak{M}(x_{j+1}, x_{j+2}, s)} - 1\right) \\
&\quad + \left(\frac{1}{\mathfrak{M}(x_{j+2}, x_{j+3}, s)} - 1\right) \\
&\quad + \left(\frac{1}{\mathfrak{M}(x_{j+3}, x_{j+4}, s)} - 1\right) + \cdots \\
&\quad + \left(\frac{1}{\mathfrak{M}(x_{k-1}, x_k, s)} - 1\right) = \mathscr{M}_j + \mathscr{M}_{j+1} + \mathscr{M}_{j+2} \\
&\quad + \mathscr{M}_{j+3} + \cdots + \mathscr{M}_{k-2} + \mathscr{M}_{k-1} \\
&\quad \leq \left((\alpha\beta)^{(j-1)/2} + (\alpha\beta)^{(j+1)/2} + \cdots + (\alpha\beta)^{(k-3)/2}\right) \\
&\quad \cdot \alpha(1+\beta)\mathscr{M}_0 \leq \frac{(\alpha\beta)^{(j-1)/2}\alpha(1+\beta)}{1-\alpha\beta}\mathscr{M}_0.
\end{aligned} \tag{17}$$

Thus, the right-hand sides of (16) and (17) converge to zero as $j \longrightarrow \infty$, which yields that (x_j) is a Cauchy sequence. Since $\mathscr{X}$ is complete, $\exists\, z \in \mathscr{X}$ such that

$$\lim_{j\longrightarrow\infty}\mathfrak{M}(z, x_j, s) = 1, \quad \text{for } s \gg \vartheta. \tag{18}$$

Since $\mathfrak{M}$ is triangular,

$$\begin{aligned}
\frac{1}{\mathfrak{M}(z, \mathscr{F}z, s)} - 1 &\leq \left(\frac{1}{\mathfrak{M}(z, x_{2j+2}, s)} - 1\right) \\
&\quad + \left(\frac{1}{\mathfrak{M}(x_{2j+2}, \mathscr{F}z, s)} - 1\right), \\
&\quad \text{for } s \gg \vartheta.
\end{aligned} \tag{19}$$

By using (5), (15), and (18), for $s \gg \vartheta$,

$$\begin{aligned}
\frac{1}{\mathfrak{M}\left(x_{2j+2}, \mathcal{F}z, s\right)} - 1 &= \frac{1}{\mathfrak{M}\left(\mathcal{G}x_{2j+1}, \mathcal{F}z, s\right)} - 1 \\
&\leq c_1\left(\frac{1}{\mathfrak{M}\left(z, x_{2j+1}, s\right)} - 1\right) \\
&\quad + c_2\left(\frac{1}{\mathfrak{M}\left(z, \mathcal{F}z, s\right)} - 1\right) \\
&\quad + c_3\left(\frac{1}{\mathfrak{M}\left(x_{2j+1}, \mathcal{G}x_{2j+1}, s\right)} - 1\right) \\
&\quad + c_4\left(\frac{1}{\mathfrak{M}\left(x_{2j+1}, \mathcal{F}z, s\right)} - 1\right) \\
&\quad + c_5\left(\frac{1}{\mathfrak{M}\left(z, \mathcal{G}x_{2j+1}, s\right)} - 1\right) \\
&\leq c_1\left(\frac{1}{\mathfrak{M}\left(z, x_{2j+1}, s\right)} - 1\right) \\
&\quad + c_2\left(\frac{1}{\mathfrak{M}\left(z, \mathcal{F}z, s\right)} - 1\right) \\
&\quad + c_3\left(\frac{1}{\mathfrak{M}\left(x_{2j+1}, x_{2j+2}, s\right)} - 1\right) \\
&\quad + c_4\left(\frac{1}{\mathfrak{M}\left(x_{2j+1}, z, s\right)} - 1 + \frac{1}{\mathfrak{M}\left(z, \mathcal{F}z, s\right)} - 1\right) \\
&\quad + c_5\left(\frac{1}{\mathfrak{M}\left(z, x_{2j+2}, s\right)} - 1\right) \\
&\longrightarrow \left(c_2 + c_4\right)\left(\frac{1}{\mathfrak{M}\left(z, \mathcal{F}z, s\right)} - 1\right), \quad \text{as } i \longrightarrow \infty.
\end{aligned} \tag{20}$$

Then

$$\begin{aligned}
\limsup_{j\longrightarrow\infty}&\left(\frac{1}{\mathfrak{M}\left(x_{2j+2}, \mathcal{F}z, s\right)} - 1\right) \\
&\leq \left(c_2 + c_4\right)\left(\frac{1}{\mathfrak{M}\left(z, \mathcal{F}z, s\right)} - 1\right), \quad \text{for } s \gg \vartheta.
\end{aligned} \tag{21}$$

The above (21) together with (18) and (19) implies that

$$\frac{1}{\mathfrak{M}\left(z, \mathcal{F}z, s\right)} - 1 \leq \left(c_2 + c_4\right)\left(\frac{1}{\mathfrak{M}\left(z, \mathcal{F}z, s\right)} - 1\right), \quad \text{for } s \gg \vartheta. \tag{22}$$

$(c_2 + c_4) < 1$, since $(c_1 + c_2 + c_3 + 2\max\{c_4, c_5\}) < 1$; then $\mathfrak{M}(z, \mathcal{F}z, s) = 1$; that is, $\mathcal{F}z = z$. Similarly, by $\mathfrak{M}$ triangular,

$$\begin{aligned}
\frac{1}{\mathfrak{M}\left(z, \mathcal{G}z, s\right)} - 1 &\leq \frac{1}{\mathfrak{M}\left(z, x_{2j+1}, s\right)} - 1 \\
&\quad + \frac{1}{\mathfrak{M}\left(x_{2j+1}, \mathcal{G}z, s\right)} - 1, \\
&\quad \text{for } s \gg \vartheta.
\end{aligned} \tag{23}$$

Again, by using (5), (15), and (18), similar to the above, after simplification, we can get

$$\begin{aligned}
\limsup_{j\longrightarrow\infty}&\left(\frac{1}{\mathfrak{M}\left(x_{2j+1}, \mathcal{G}z, s\right)} - 1\right) \\
&\leq \left(c_3 + c_5\right)\left(\frac{1}{\mathfrak{M}\left(z, \mathcal{G}z, s\right)} - 1\right), \quad \text{for } s \gg \vartheta.
\end{aligned} \tag{24}$$

The above (24) together with (18) and (23) implies that

$$\frac{1}{\mathfrak{M}\left(z, \mathcal{G}z, s\right)} - 1 \leq \left(c_3 + c_5\right)\left(\frac{1}{\mathfrak{M}\left(z, \mathcal{G}z, s\right)} - 1\right), \quad \text{for } s \gg \vartheta. \tag{25}$$

$(c_3 + c_5) < 1$, since $(c_1 + c_2 + c_3 + 2\max\{c_4, c_5\}) < 1$; then $\mathfrak{M}(z, \mathcal{G}z, s) = 1$; that is, $\mathcal{G}z = z$. Hence, the fact that z is the common fixed point of $\mathcal{F}$ and $\mathcal{G}$ in $\mathcal{X}$ is proven.

Uniqueness: let $z^* \in \mathcal{X}$ be the other common fixed point of $\mathcal{F}$ and $\mathcal{G}$ in $\mathcal{X}$. Then, again by view of (5), for $s \gg \vartheta$,

$$\begin{aligned}
\frac{1}{\mathfrak{M}\left(z^*, z, s\right)} - 1 &= \frac{1}{\mathfrak{M}\left(\mathcal{F}z^*, \mathcal{G}z, s\right)} - 1 \\
&\leq c_1\left(\frac{1}{\mathfrak{M}\left(z^*, z, s\right)} - 1\right) \\
&\quad + c_2\left(\frac{1}{\mathfrak{M}\left(z^*, \mathcal{F}z^*, s\right)} - 1\right) \\
&\quad + c_3\left(\frac{1}{\mathfrak{M}\left(z, \mathcal{G}z, s\right)} - 1\right) \\
&\quad + c_4\left(\frac{1}{\mathfrak{M}\left(z, \mathcal{F}z^*, s\right)} - 1\right) \\
&\quad + c_5\left(\frac{1}{\mathfrak{M}\left(z^*, \mathcal{G}z, s\right)} - 1\right) \\
&= \left(c_1 + c_4 + c_5\right)\left(\frac{1}{\mathfrak{M}\left(z^*, z, s\right)} - 1\right).
\end{aligned} \tag{26}$$

We note that $(c_1+c_4+c_5) < 1$, where $(c_1+c_2+c_3+2\max\{c_4,c_5\}) < 1$. Therefore $\mathfrak{M}(z^*,z,s) = 1$, implying that $z = z^*$. Hence the fact that the common fixed point of $\mathcal{F}$ and $\mathcal{G}$ is unique is proven. □

Corollary 11. *Let $\mathcal{F}, \mathcal{G} : \mathcal{X} \longrightarrow \mathcal{X}$ be two self-mappings and $\mathfrak{M}$ is triangular in the complete fuzzy cone metric space $(\mathcal{X},\mathfrak{M},*)$ which satisfies*

$$\frac{1}{\mathfrak{M}(\mathcal{F}x,\mathcal{G}w,s)} - 1 \le c_1\left(\frac{1}{\mathfrak{M}(x,w,s)} - 1\right) + c_2\left(\frac{1}{\mathfrak{M}(x,\mathcal{F}x,s)} - 1\right) + c_3\left(\frac{1}{\mathfrak{M}(w,\mathcal{G}w,s)} - 1\right), \quad (27)$$

for all $w,x \in \mathcal{X}$, $s \gg \vartheta$, and $c_1,c_2,c_3 \in [0,\infty)$ such that $(c_1+c_2+c_3) < 1$. Then $\mathcal{F}$ and $\mathcal{G}$ have a unique common fixed point in $\mathcal{X}$.

Corollary 12. *Let $\mathcal{F}, \mathcal{G} : \mathcal{X} \longrightarrow \mathcal{X}$ be two self-mappings and $\mathfrak{M}$ is triangular in the complete fuzzy cone metric space $(\mathcal{X},\mathfrak{M},*)$ which satisfies*

$$\frac{1}{\mathfrak{M}(\mathcal{F}x,\mathcal{G}w,s)} - 1 \le c_1\left(\frac{1}{\mathfrak{M}(x,w,s)} - 1\right) + c_4\left(\frac{1}{\mathfrak{M}(w,\mathcal{F}x,s)} - 1\right) + c_5\left(\frac{1}{\mathfrak{M}(x,\mathcal{G}w,s)} - 1\right), \quad (28)$$

for all $w,x \in \mathcal{X}$, $s \gg \vartheta$, and $c_1,c_4,c_5 \in [0,\infty)$ such that $(c_1+2\max\{c_4,c_5\}) < 1$. Then $\mathcal{F}$ and $\mathcal{G}$ have a unique common fixed point in $\mathcal{X}$.

Remark 13. (i) In special case, Theorem 10, Corollaries 11 and 12, and [26, Theorem 3.3] (i.e., Theorem 9) all have the same results. In fact, if $\mathcal{G} = \mathcal{F}$, $c_1 \in (0,1)$ and $c_2 = c_3 = c_4 = c_5 = 0$ in (5).

(ii) Theorem 10 and [27, Theorem 3.1] both have similar proof. If $\mathcal{G} = \mathcal{F}$, $c_1,c_2,c_3,c_4 \in [0,\infty)$ and $c_5 = 0$ in (5).

Example 14. Let $\mathcal{X} = [0,\infty)$; $*$ is a continuous t-norm and $\mathfrak{M} : \mathcal{X}\times\mathcal{X}\times(0,\infty) \longrightarrow [0,1]$ is defined as

$$\mathfrak{M}(x,w,s) = \frac{s}{s+|x-w|} \quad (29)$$

$\forall w,x \in \mathcal{X}$ and $s > 0$. Then, one can easily prove that $\mathfrak{M}$ is triangular and $(\mathcal{X},\mathfrak{M},*)$ is a complete fuzzy cone metric space. Now we define $\mathcal{F},\mathcal{G} : \mathcal{X} \longrightarrow \mathcal{X}$ as

$$\mathcal{F}x = \begin{cases} \dfrac{7}{6}x + 3, & \text{if } 0 \le x \le 1, \\ \dfrac{5}{6}x + \dfrac{3}{2}, & \text{if } 1 < x < \infty. \end{cases} \quad (30)$$

And

$$\mathcal{G}w = \begin{cases} \dfrac{7}{6}w + 3, & \text{if } 0 \le w \le 1, \\ \dfrac{3}{4}w + \dfrac{9}{4}, & \text{if } 1 < w < \infty. \end{cases} \quad (31)$$

Then $\mathcal{F}$ and $\mathcal{G}$ are not fuzzy cone contractive, since

$$\frac{1}{\mathfrak{M}(\mathcal{F}x,\mathcal{G}w,s)} - 1 = \frac{7}{6}\left(\frac{1}{\mathfrak{M}(x,w,s)} - 1\right). \quad (32)$$

In special case, if $\mathcal{G} = \mathcal{F}$, then Theorem 9 does not hold. But it can be easily proven that all the conditions of Theorem 10 hold with $c_1 = 1/6$, $c_2 = c_3 = 1/4$, and $c_4 = c_5 = 1/8$. Thus, $\mathcal{F}$ and $\mathcal{G}$ have a unique common fixed point in $[0,\infty)$, that is, 9.

Theorem 15. *Let $\mathcal{F}, \mathcal{G} : \mathcal{X} \longrightarrow \mathcal{X}$ be two self-mappings and $\mathfrak{M}$ is triangular in the complete fuzzy cone metric space $(\mathcal{X},\mathfrak{M},*)$ which satisfies*

$$\frac{1}{\mathfrak{M}(\mathcal{F}x,\mathcal{G}w,s)} - 1 \le \alpha\left(\frac{1}{\min\{\mathfrak{M}(x,\mathcal{F}x,s),\mathfrak{M}(w,\mathcal{G}w,s),\mathfrak{M}(w,\mathcal{F}x,s),\mathfrak{M}(x,\mathcal{G}w,s)\}} - 1\right), \quad (33)$$

for all $w,x \in \mathcal{X}$, $s \gg \vartheta$, and $\alpha \in (0,1)$. Then $\mathcal{F}$ and $\mathcal{G}$ have a unique common fixed point in $\mathcal{X}$.

Proof. Fix $x_0 \in \mathcal{X}$ and a point $x_1 \in \mathcal{X}$ such that $\mathcal{F}x_0 = x_1$ and $\exists\, x_2 \in \mathcal{X}$ such that $\mathcal{G}x_1 = x_2$. If $\alpha = 0$, then we have that

$$\frac{1}{\mathfrak{M}(x_1,x_2,s)} - 1 = \frac{1}{\mathfrak{M}(\mathcal{F}x_0,\mathcal{G}x_1,s)} - 1 = 0, \quad (34)$$

which implies that $\mathcal{F}x_0 = \mathcal{G}x_1$ if and only if $x_0 = x_1$. Then the proof is complete. Otherwise, we assume that $\alpha > 0$ and let us take $\beta = 1/\sqrt{\alpha} > 1$. Now we define the iterative sequence in $\mathcal{X}$ such as

$$x_{2j+1} = \mathcal{F}x_{2j} \quad \text{and } x_{2j+2} = \mathcal{G}x_{2j+1}, \quad j \ge 0. \quad (35)$$

By view of (33),

$$\begin{aligned}\frac{1}{\mathfrak{M}\left(x_{2j+1},x_{2j+2},s\right)}-1&\leq\beta\left(\frac{1}{\mathfrak{M}\left(\mathcal{F}x_{2j},\mathcal{G}x_{2j+1},s\right)}-1\right)\\&\leq\sqrt{\alpha}\left(\frac{1}{\min\left\{\mathfrak{M}\left(x_{2j},\mathcal{F}x_{2j},s\right),\mathfrak{M}\left(x_{2j+1},\mathcal{G}x_{2j+1},s\right),\mathfrak{M}\left(x_{2j+1},\mathcal{F}x_{2j},s\right),\mathfrak{M}\left(x_{2j},\mathcal{G}x_{2j+1},s\right)\right\}}-1\right)\\&=\sqrt{\alpha}\left(\frac{1}{\min\left\{\mathfrak{M}\left(x_{2j},x_{2j+1},s\right),\mathfrak{M}\left(x_{2j+1},x_{2j+2},s\right),\mathfrak{M}\left(x_{2j},x_{2j+2},s\right)\right\}}-1\right).\end{aligned}\tag{36}$$

Now there are three possibilities.

(i) If $\mathfrak{M}(x_{2j},x_{2j+1},s)$ is minimum, then $(1/\mathfrak{M}(x_{2j},x_{2j+1},s)-1)$ will be the maximum in the above (36). Then, we have

$$\begin{aligned}&\frac{1}{\mathfrak{M}\left(x_{2j+1},x_{2j+2},s\right)}-1\\&\quad\leq\sqrt{\alpha}\left(\frac{1}{\mathfrak{M}\left(x_{2j},x_{2j+1},s\right)}-1\right).\end{aligned}\tag{37}$$

(ii) If $\mathfrak{M}(x_{2j+1},x_{2j+2},s)$ is minimum, then $(1/\mathfrak{M}(x_{2j+1},x_{2j+2},s)-1)$ will be the maximum in the above (36). Then, we have

$$\begin{aligned}&\frac{1}{\mathfrak{M}\left(x_{2j+1},x_{2j+2},s\right)}-1\\&\quad\leq\sqrt{\alpha}\left(\frac{1}{\mathfrak{M}\left(x_{2j+1},x_{2j+2},s\right)}-1\right),\end{aligned}\tag{38}$$

which is not possible.

(iii) If $\mathfrak{M}(x_{2j},x_{2j+2},s)$ is minimum, then $(1/\mathfrak{M}(x_{2j},x_{2j+2},s)-1)$ will be the maximum in the above (36). Then, we have

$$\begin{aligned}&\frac{1}{\mathfrak{M}\left(x_{2j+1},x_{2j+2},s\right)}-1\leq\sqrt{\alpha}\left(\frac{1}{\mathfrak{M}\left(x_{2j},x_{2j+2},s\right)}\right.\\&\left.-1\right)\leq\sqrt{\alpha}\left(\frac{1}{\mathfrak{M}(x_{2j},x_{2j+1},s}-1\right.\\&\left.+\frac{1}{\mathfrak{M}\left(x_{2j+1},x_{2j+2},s\right)}-1\right),\end{aligned}\tag{39}$$

which implies that

$$\begin{aligned}&\frac{1}{\mathfrak{M}\left(x_{2j+1},x_{2j+2},s\right)}-1\\&\quad\leq\sqrt{\gamma}\left(\frac{1}{\mathfrak{M}\left(x_{2j},x_{2j+1},s\right)}-1\right),\end{aligned}\tag{40}$$

where $\sqrt{\gamma}=\sqrt{\alpha}/(1-\sqrt{\alpha})<1$, since $\alpha\in(0,1)$. Thus, $\sqrt{\delta}=\max\{\sqrt{\gamma},\sqrt{\alpha}\}<1$. Now, from (i), (ii), and (iii), for all $j\geq 0$ and $s\gg\vartheta$,

$$\begin{aligned}&\frac{1}{\mathfrak{M}\left(x_{2j+1},x_{2j+2},s\right)}-1\\&\quad\leq\sqrt{\delta}\left(\frac{1}{\mathfrak{M}\left(x_{2j},x_{2j+1},s\right)}-1\right)\leq\cdots\\&\quad\leq\left(\sqrt{\delta}\right)^{2j+1}\left(\frac{1}{\mathfrak{M}\left(x_0,x_1,s\right)}-1\right),\end{aligned}\tag{41}$$

which shows that a sequence (x_j) is fuzzy cone contractive. Thus,

$$\lim_{j\longrightarrow\infty}\mathfrak{M}\left(x_{2j+1},x_{2j+2},s\right)=1,\quad\text{for } s\gg\vartheta.\tag{42}$$

Since $\mathfrak{M}$ is triangular, for all $k>j\geq j_0$, we have

$$\begin{aligned}&\frac{1}{\mathfrak{M}\left(x_j,x_k,s\right)}-1\leq\left(\frac{1}{\mathfrak{M}\left(x_j,x_{j+1},s\right)}-1\right)\\&\quad+\left(\frac{1}{\mathfrak{M}\left(x_{j+1},x_{j+2},s\right)}-1\right)+\cdots\\&\quad+\left(\frac{1}{\mathfrak{M}\left(x_{k-1},x_k,s\right)}-1\right)\\&\quad\leq\left(\left(\sqrt{\delta}\right)^j+\left(\sqrt{\delta}\right)^{j+1}+\cdots+\left(\sqrt{\delta}\right)^{k-1}\right)\\&\quad\cdot\left(\frac{1}{\mathfrak{M}\left(x_0,x_1,s\right)}-1\right)\\&\quad\leq\frac{\left(\sqrt{\delta}\right)^j}{1-\sqrt{\delta}}\left(\frac{1}{\mathfrak{M}\left(x_0,x_1,s\right)}-1\right)\longrightarrow 0,\\&\qquad\qquad\text{as } j\longrightarrow\infty,\end{aligned}\tag{43}$$

which shows that (x_j) is a Cauchy sequence. Since $\mathcal{X}$ is complete and $\exists\, z\in\mathcal{X}$, we have

$$\lim_{j\longrightarrow\infty}\mathfrak{M}\left(z,x_j,s\right)=1,\quad\text{for } s\gg\vartheta. \tag{44}$$

Now we shall show that $\mathcal{F}z=z$. By the triangular property of $\mathfrak{M}$, we have

$$\frac{1}{\mathfrak{M}(z,\mathcal{F}z,s)}-1\le\frac{1}{\mathfrak{M}\left(z,x_{2j+2},s\right)}-1+\frac{1}{\mathfrak{M}\left(x_{2j+2},\mathcal{F}z,s\right)}-1,\quad\text{for } s\gg\vartheta. \tag{45}$$

Now, by using (33), (42), and (44), for $s\gg\vartheta$, we have

$$\begin{aligned}\frac{1}{\mathfrak{M}\left(x_{2j+2},\mathcal{F}z,s\right)}-1&\le\beta\left(\frac{1}{\mathfrak{M}\left(\mathcal{F}z,\mathcal{G}x_{2j+1},s\right)}-1\right)\\&\le\sqrt{\alpha}\left(\frac{1}{\min\left\{\mathfrak{M}(z,\mathcal{F}z,s),\mathfrak{M}\left(x_{2j+1},\mathcal{G}x_{2j+1},s\right),\mathfrak{M}\left(x_{2j+1},\mathcal{F}z,s\right),\mathfrak{M}\left(z,\mathcal{G}x_{2j+1},s\right)\right\}}-1\right)\\&\le\sqrt{\alpha}\left(\frac{1}{\min\left\{\mathfrak{M}(z,\mathcal{F}z,s),\mathfrak{M}\left(x_{2j+1},x_{2j+2},s\right),\mathfrak{M}\left(x_{2j+1},\mathcal{F}z,s\right),\mathfrak{M}\left(z,x_{2j+2},s\right)\right\}}-1\right)\\&\longrightarrow\sqrt{\alpha}\left(\frac{1}{\mathfrak{M}(z,\mathcal{F}z,s)}-1\right),\quad\text{as } j\longrightarrow\infty.\end{aligned} \tag{46}$$

Thus,

$$\begin{aligned}\limsup_{j\longrightarrow\infty}\left(\frac{1}{\mathfrak{M}\left(x_{2j+2},\mathcal{F}z,s\right)}-1\right)&\le\sqrt{\alpha}\left(\frac{1}{\mathfrak{M}(z,\mathcal{F}z,s)}-1\right),\quad\text{for } s\gg\vartheta.\end{aligned} \tag{47}$$

The above (47) together with (44) and (45) implies that

$$(1-\sqrt{\alpha})\left(\frac{1}{\mathfrak{M}(z,\mathcal{F}z,s)}-1\right)\le 0,\quad\text{for } s\gg\vartheta, \tag{48}$$

and $(1-\sqrt{\alpha})<1$, since $\alpha\in(0,1)$. This implies that $\mathfrak{M}(z,\mathcal{F}z,s)=1$; that is, $\mathcal{F}z=z$. Similarly, we can prove that $\mathcal{G}z=z$. Thus, $\mathcal{F}z=\mathcal{G}z=z$.

Uniqueness: let $z^*\in\mathcal{X}$ such that $\mathcal{F}z^*=\mathcal{G}z^*=z^*$. Then, by using (33), for every $s\gg\vartheta$, we have

$$\begin{aligned}\frac{1}{\mathfrak{M}(z,z^*,s)}-1&\le\beta\left(\frac{1}{\mathfrak{M}(\mathcal{F}z,\mathcal{G}z^*,s)}-1\right)\\&\le\sqrt{\alpha}\left(\frac{1}{\min\{\mathfrak{M}(z,\mathcal{F}z,s),\mathfrak{M}(z^*,\mathcal{G}z^*,s),\mathfrak{M}(z^*,\mathcal{F}z,s),\mathfrak{M}(z,\mathcal{G}z^*,s)\}}-1\right)\\&=\sqrt{\alpha}\left(\frac{1}{\min\{\mathfrak{M}(z,z,s),\mathfrak{M}(z^*,z^*,s),\mathfrak{M}(z^*,z,s),\mathfrak{M}(z,z^*,s)\}}-1\right)=\sqrt{\alpha}\left(\frac{1}{\mathfrak{M}(z,z^*,s)}-1\right).\end{aligned} \tag{49}$$

This implies that

$$(1-\sqrt{\alpha})\left(\frac{1}{\mathfrak{M}(z,z^*,s)}-1\right)\le 0. \tag{50}$$

$1-\sqrt{\alpha}\neq 0$, since $\alpha\in(0,1)$. This implies that $\mathfrak{M}(z,z^*,s)=1$; that is, $z^*=z$. Hence the fact that $\mathcal{F}$ and $\mathcal{G}$ have a unique common fixed point is proven. That is, $\mathcal{F}z=\mathcal{G}z=z\in\mathcal{X}$. □

Corollary 16. *Let $\mathcal{F},\mathcal{G}:\mathcal{X}\longrightarrow\mathcal{X}$ be two self-mappings and $\mathfrak{M}$ is triangular in the complete fuzzy cone metric space $(\mathcal{X},\mathfrak{M},*)$ which satisfies*

$$\begin{aligned}&\frac{1}{\mathfrak{M}(\mathcal{F}x,\mathcal{G}w,s)}-1\\&\quad\le\alpha\left(\frac{1}{\min\{\mathfrak{M}(x,\mathcal{F}x,s),\mathfrak{M}(w,\mathcal{G}w,s)\}}-1\right),\end{aligned} \tag{51}$$

for all $w,x\in\mathcal{X}$, $s\gg\vartheta$, and $\alpha\in(0,1)$. Then $\mathcal{F}$ and $\mathcal{G}$ have a unique common fixed point in $\mathcal{X}$.

Example 17. From Example 14, we define $\mathcal{F}, \mathcal{G} : \mathcal{X} \longrightarrow \mathcal{X}$ as

$$\mathcal{F}x = \mathcal{G}x = \begin{cases} \frac{3}{7}x - \frac{1}{7}, & \text{if } 0 \leq x \leq 1, \\ \frac{1}{2}x + 2, & \text{if } 1 < x < \infty. \end{cases} \tag{52}$$

Then, $\mathcal{F}$ and $\mathcal{G}$ are fuzzy cone contractive, since

$$\begin{aligned} \frac{1}{\mathfrak{M}(\mathcal{F}x, \mathcal{G}w, s)} - 1 &= \frac{3}{7}\left(\frac{1}{\mathfrak{M}(x, w, s)} - 1\right) \\ &\leq \frac{3}{7}\left(\frac{1}{\min\{\mathfrak{M}(x, \mathcal{F}x, s), \mathfrak{M}(w, \mathcal{G}w, s), \mathfrak{M}(w, \mathcal{F}x, s), \mathfrak{M}(x, \mathcal{G}w, s)\}} - 1\right), \end{aligned} \tag{53}$$

for all $x, w \in X$. Then, all the conditions of Theorem 15 easily hold with $\alpha = 3/7 \in (0, 1)$, as well as Theorem 9, if $\mathcal{G} = \mathcal{F}$. Thus, $\mathcal{F}$ and $\mathcal{G}$ have a unique common fixed point in $[0, \infty)$, that is, 4.

4. Conclusion

We gave the concept of common fixed point for a pair of self-mappings in fuzzy cone metric spaces and proved some unique common fixed point results in fuzzy cone metric spaces. We also proved that a pair of self-mappings may not be a fuzzy cone contraction if it satisfies (5), which is shown in Example 14. According to this concept, one can study some more common fixed point results for two or more self-mappings in fuzzy cone metric spaces for different contractive-type mappings.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

All the authors share equal contributions to the final manuscript.

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