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Research Article

Geometric Properties of a New Integral Operator

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We obtain sufficient conditions for the univalence, starlikeness, and convexity of a new integral operator defined on the space of normalized analytic functions in the open unit disk. Some subordination results for the new integral operator are also given. Several corollaries follow as special cases.

1. Introduction

Let $U = \{z : |z| < 1\}$ be the open unit disk and $\mathscr{A}$ the class of all functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \quad z \in U, \tag{1}$$

which are analytic in U and satisfy the condition $f(0) = f'(0) - 1 = 0$. Consider S the class of functions $f \in \mathscr{A}$ which are univalent in U.

A domain $D \subset \mathbb{C}$ is convex if the line segment joining any two points in D lies entirely in D, while a domain is starlike with respect to a point $w_0 \in D$ if the line segment joining any point of D to w_0 lies inside D.

A function $f \in \mathscr{A}$ is starlike if $f(U)$ is a starlike domain with respect to origin and convex if $f(U)$ is convex.

Analytically, $f \in \mathscr{A}$ is starlike if and only if

$$\operatorname{Re}\left[\frac{zf'(z)}{f(z)}\right] > 0, \quad \forall z \in U \tag{2}$$

and $f \in \mathscr{A}$ is convex if and only if

$$\operatorname{Re}\left[1 + \frac{zf''(z)}{f'(z)}\right] > 0, \quad \forall z \in U. \tag{3}$$

The classes consisting of starlike and convex functions are denoted by S^* and K, respectively. We denote by $S^*(\delta)$ and $K(\delta)$ the classes consisting of starlike and convex functions of order δ, $0 \le \delta < 1$, characterized, respectively, by

$$\begin{gathered}\operatorname{Re}\left[\frac{zf'(z)}{f(z)}\right] > \delta, \\ \operatorname{Re}\left[1 + \frac{zf''(z)}{f'(z)}\right] > \delta.\end{gathered} \tag{4}$$

If f and g are analytic functions in U, we say that f is subordinate to g, written as $f \prec g$, if there is a function w analytic in U, with $w(0) = 0$, $|w(z)| < 1$, for all $z \in U$, such that $f(z) = g(w(z))$ for all $z \in U$. If g is univalent, then $f \prec g$ if and only if $f(0) = g(0)$ and $f(U) \subseteq g(U)$ (see Miller and Mocanu [1]).

Using subordinations, Owa et al. [2] have defined the following subclass $S_b(a)$ of $\mathscr{A}$.

A function $f \in \mathscr{A}$ is said to be in the class $S_b(a)$ if it satisfies

$$\left(f'(z)\right)^b \prec \frac{a(1-z)}{a-z}, \quad z \in U \tag{5}$$

for some real $a > 1$ and $b > 0$.

For the class $S_b(a)$, Owa et al. proved the following.

Theorem 1 (see [2]). *If $f \in \mathcal{A}$ satisfies*

$$\operatorname{Re}\left[\frac{zf''(z)}{f'(z)}\right] < \frac{a-1}{2b(a+1)}, \quad z \in U, \tag{6}$$

for some real $a > 1$ and $b > 0$, then $f \in S_b(a)$.

Breaz et al. [3] have introduced the following subclasses $S_b^*(a)$ and $C_b^*(a)$ of $\mathcal{A}$. A function $f \in \mathcal{A}$ is said to be in the class $S_b^*(a)$ if it satisfies inequality (6) for some real $a > 1$ and $b > 0$.

A function $f \in \mathcal{A}$ is said to be a member of the class $C_b^*(a)$ if it satisfies

$$\operatorname{Re}\left[\frac{zf'(z)}{f(z)}\right] > \frac{1-a}{2b(a+1)} + 1, \quad z \in U, \tag{7}$$

for some real $a > 1$ and $b > 0$.

Recently, Frasin and Jahangiri [4] defined the family $B(\mu, \lambda)$, $\mu \geq 0$, $0 \leq \lambda < 1$ consisting of functions $f \in \mathcal{A}$ satisfying the condition

$$\left| f'(z)\left[\frac{z}{f(z)}\right]^{\mu} - 1 \right| < 1 - \lambda, \quad z \in U. \tag{8}$$

We note that $B(1, \lambda) = S^*(\lambda)$ and $B(2, \lambda) = B(\lambda)$ (see [5]).

In 1975, Pfaltzgraff [6] introduced the operator

$$I_\alpha(f)(z) = \int_0^z \left[f'(t)\right]^\alpha dt \tag{9}$$

and proved that $I_\alpha(S) \subset S$ if α is a complex number and $|\alpha| \leq 1/4$.

During the last several years many authors have employed different methods to study generalizations of Pfaltzgraff [6] integral operator which maps subsets of $\mathcal{A}$ into S.

Pascu and Pescar [7] studied the operator

$$I_{\alpha,n}(f)(z) = \int_0^z \left[f'(t^u)\right]^\alpha dt, \tag{10}$$

where n is a positive integer.

The next operator was defined by Breaz et al. in [8] and has the form

$$I_\alpha(f_1, \ldots, f_n)(z) = \int_0^z \left[f_1'(t)\right]^{\alpha_1} \cdots \left[f_n'(t)\right]^{\alpha_n} dt, \tag{11}$$

where $f_i \in \mathcal{A}$ and $\alpha_i \in \mathbb{C}$ for all $i \in \{1, \ldots, n\}$.

Further extensions of these operators were obtained in [9–11].

Recently, starlikeness and convexity properties for certain general families of integral operators in the open unit disk were given in [12–14].

In the present paper, we introduce the new integral operator

$$I_\alpha : \mathcal{A} \times \mathcal{A} \longrightarrow \mathcal{A} \tag{12}$$

defined by

$$I_\alpha(f, g)(z) = \int_0^z \left[\frac{tf'(t)}{g(t)}\right]^\alpha dt, \tag{13}$$

for the functions $f, g \in \mathcal{A}$ and the complex number α.

We observe that if we take $g(z) = z$, we obtain Pfaltzgraff [6] integral operator.

In the first section of this paper, our purpose is to derive univalence conditions, starlikeness properties, and the order of convexity for the integral operator introduced in (13). The object of the second section of this paper is to discuss some properties for the integral operator $I_\alpha(f, g)$ with the above classes $S_b(a)$, $S_b^*(a)$ and $C_b^*(a)$.

The following results will be required in our investigation.

Lemma 2 (Mocanu and Şerb [15]). *Let $M_0 = 1{,}5936\ldots$, the positive solution of equation*

$$(2 - M)e^M = 2. \tag{14}$$

If $f \in \mathcal{A}$ and

$$\left|\frac{f''(z)}{f'(z)}\right| \leq M_0, \quad \forall z \in U, \tag{15}$$

then

$$\left|\frac{zf'(z)}{f(z)} - 1\right| < 1, \quad \forall z \in U. \tag{16}$$

The edge M_0 is sharp.

Theorem 3 (Becker [16]). *If the function f is regular in the unit disk U, $f(z) = z + a_2z^2 + \cdots$, and*

$$\left(1 - |z|^2\right) \cdot \left|\frac{zf''(z)}{f'(z)}\right| \leq 1, \tag{17}$$

for all $z \in U$, then the function f is univalent in U.

Lemma 4 (the general Schwarz Lemma [17]). *Let the function f be regular function in the disk $U_R = \{z \in \mathbb{C} : |z| < R\}$, with $|f(z)| < M$ for fixed M. If f has one zero with multiplicity order bigger than m for $z = 0$, then*

$$|f(z)| \leq \frac{M}{R^m}|z|^m, \quad z \in U_R. \tag{18}$$

The equality case holds only if $f(z) = e^{i\theta}(M/R^m)z^m$, where θ is constant.

Lemma 5 (see [18]). *Let the functions p and q be analytic in U with*

$$p(0) = q(0) = 0, \tag{19}$$

and let δ be a real number. If the function q maps the unit disk U onto a region which is starlike with respect to the origin, the inequality

$$\operatorname{Re}\left[\frac{p'(z)}{q'(z)}\right] > \delta, \quad \forall z \in U \tag{20}$$

implies that

$$\operatorname{Re}\left[\frac{p(z)}{q(z)}\right] > \delta, \quad \forall z \in U. \tag{21}$$

2. Univalence, Starlikeness, and Convexity Properties of the Integral Operator $I_\alpha(f,g)$

The univalence condition for the operator $I_\alpha(f,g)$ defined in (13) is proved in the next theorem, by using the Becker univalence criterion.

Theorem 6. *Let α be a complex number, M_0 the positive solution of (14), $M_0 = 1{,}5936\ldots$, and $f, g \in \mathcal{A}$. If*

$$\begin{gathered} \left|\frac{f''(z)}{f'(z)}\right| \le M_0, \\ \left|\frac{g''(z)}{g'(z)}\right| \le M_0, \\ z \in U, \end{gathered} \tag{22}$$

$$|\alpha| \le \frac{3\sqrt{3}}{2M_0 + 3\sqrt{3}}, \tag{23}$$

then the integral operator

$$I_\alpha(f,g)(z) = \int_0^z \left[\frac{tf'(t)}{g(t)}\right]^\alpha dt \tag{24}$$

is in the class S.

Proof. Let the function be as follows:

$$h(z) := I_\alpha(f,g)(z). \tag{25}$$

Then, we have

$$\frac{zh''(z)}{h'(z)} = \alpha\left[\frac{zf''(z)}{f'(z)} - \left(\frac{zg'(z)}{g(z)} - 1\right)\right]. \tag{26}$$

From (26) we get

$$\begin{aligned} \left(1-|z|^2\right)\cdot\left|\frac{zh''(z)}{h'(z)}\right| \le{}& \left(1-|z|^2\right)\cdot|z|\cdot|\alpha|\cdot\left|\frac{f''(z)}{f'(z)}\right| \\ &+ \left(1-|z|^2\right)\cdot|\alpha| \\ &\cdot\left|\frac{zg'(z)}{g(z)} - 1\right|. \end{aligned} \tag{27}$$

By using (22) and applying Lemma 2, we have

$$\left|\frac{zg'(z)}{g(z)} - 1\right| < 1, \quad \forall z \in U, \tag{28}$$

which implies that

$$\begin{aligned} \left(1-|z|^2\right)\cdot\left|\frac{zh''(z)}{h'(z)}\right| \le{}& \left(1-|z|^2\right)\cdot|z|\cdot|\alpha|\cdot M_0 \\ &+ \left(1-|z|^2\right)\cdot|\alpha|. \end{aligned} \tag{29}$$

Let us consider the function

$$\begin{gathered} T : [0,1) \longrightarrow R, \\ T(x) = x\left(1-x^2\right), \quad x = |z|. \end{gathered} \tag{30}$$

Then we have

$$T(x) \le \frac{2}{3\sqrt{3}}, \quad \forall x \in [0,1). \tag{31}$$

From (29), (31), and (23) we obtain

$$\left(1-|z|^2\right)\cdot\left|\frac{zh''(z)}{h'(z)}\right| \le |\alpha|\cdot\left(\frac{2}{3\sqrt{3}}M_0 + 1\right) \le 1. \tag{32}$$

From Theorem 3 and (32), we yield that the function $I_\alpha(f,g)(z)$ is in the class S. □

If we put $g(z) = z$ in Theorem 6, we obtain the following.

Example 7. Let α be a complex number, M_0 the positive solution of (14), $M_0 = 1{,}5936\ldots$, and $f, g \in \mathcal{A}$. If

$$\begin{gathered} \left|\frac{f''(z)}{f'(z)}\right| \le M_0, \quad z \in U, \\ |\alpha| \le \frac{3\sqrt{3}}{2M_0}, \end{gathered} \tag{33}$$

then the integral operator

$$I_\alpha(f)(z) = \int_0^z \left[f'(t)\right]^\alpha dt \tag{34}$$

is in the class S.

In the following theorem we give sufficient conditions such that the integral operator $I_\alpha(f,g)(z) \in S^*$.

Theorem 8. *Let $f \in \mathcal{A}$ with $|zf''(z)/f'(z)| < 1$, $z \in U$, and g in the class $B(\mu,\lambda)$. Let M be a positive real number such that*

$$|g(z)| < M, \quad M \ge 1,\ z \in U. \tag{35}$$

If

$$|\alpha| \le \frac{1}{2 + (2-\lambda)M^{\mu-1}}, \tag{36}$$

then the integral operator $I_\alpha(f,g)(z)$ is in the class S^*.

Proof. For the function h defined in (25) we have

$$\frac{zh'(z)}{h(z)} = \frac{z^{\alpha+1}\left[f'(z)/g(z)\right]^\alpha}{\int_0^z \left[tf'(t)/g(t)\right]^\alpha dt}. \tag{37}$$

Letting

$$\begin{gathered} p(z) = zh'(z), \\ q(z) = h(z), \end{gathered} \tag{38}$$

we find that

$$\frac{p'(z)}{q'(z)} = 1 + \frac{zh''(z)}{h'(z)} = 1 + \alpha\left[\frac{zf''(z)}{f'(z)} + 1 - \frac{zg'(z)}{g(z)}\right]. \tag{39}$$

Thus,

$$\left|\frac{p'(z)}{q'(z)} - 1\right| \le |\alpha| \cdot \left|\frac{zf''(z)}{f'(z)}\right| + |\alpha| + |\alpha| \cdot \left|g'(z)\left(\frac{z}{g(z)}\right)^{\mu}\right| \cdot \left|\left(\frac{g(z)}{z}\right)^{\mu-1}\right|. \tag{40}$$

Since $|g(z)| < M$, $z \in U$, by applying the Schwarz Lemma, we have

$$\left|\frac{g(z)}{z}\right| \le M, \quad \forall z \in U. \tag{41}$$

By using the hypothesis and (41), we obtain

$$\left|\frac{p'(z)}{q'(z)} - 1\right| \le 2|\alpha| + |\alpha| \cdot \left[\left|g'(z)\left(\frac{z}{g(z)}\right)^{\mu} - 1\right| + 1\right] \cdot M^{\mu-1} \le |\alpha|\left[2 + (2-\lambda) \cdot M^{\mu-1}\right] \le 1; \tag{42}$$

that is,

$$\operatorname{Re}\left[\frac{p'(z)}{q'(z)}\right] > 0, \quad z \in U. \tag{43}$$

Therefore, applying Lemma 5, we find that

$$\operatorname{Re}\left[\frac{p(z)}{q(z)}\right] > 0, \quad z \in U. \tag{44}$$

This completes the proof of the theorem. □

Letting $\mu = 1$ in Theorem 8, we have the following.

Corollary 9. *Let $f \in \mathcal{A}$ with $|zf''(z)/f'(z)| < 1$, $z \in U$, and g in the class $S^*(\lambda)$. Let M be a positive real number such that*

$$|g(z)| < M, \quad M \ge 1, \ z \in U. \tag{45}$$

If

$$|\alpha| \le \frac{1}{4-\lambda}, \tag{46}$$

then the integral operator $I_\alpha(f,g)(z)$ is in the class S^*.

Letting $\lambda = 0$ in Corollary 9, we obtain the following.

Corollary 10. *Let $f \in \mathcal{A}$ with $|zf''(z)/f'(z)| < 1$, $z \in U$, and $g \in S^*$. Let M be a positive real number such that*

$$|g(z)| < M, \quad M \ge 1, \ z \in U. \tag{47}$$

If

$$|\alpha| \le \frac{1}{4}, \tag{48}$$

then the integral operator $I_\alpha(f,g)(z)$ is in the class S^*.

Since $g(z) = z$ is a starlike function, from Corollary 10, we obtain the following.

Example 11. Let $f \in \mathcal{A}$ with $|zf''(z)/f'(z)| < 1$, $z \in U$. If

$$|\alpha| \le \frac{1}{4}, \tag{49}$$

then the integral operator $I_\alpha(f)(z)$ is in the class S^*.

Theorem 12. *Let α be a complex number, $f \in \mathcal{A}$, and $g \in B(\mu,\lambda)$. If*

$$|g(z)| < M, \qquad \left|\frac{f''(z)}{f'(z)}\right| < N, \tag{50}$$

for $M \ge 1$, $N \ge 1$, and $z \in U$, then the integral operator

$$I_\alpha(f,g)(z) = \int_0^z \left[\frac{tf'(t)}{g(t)}\right]^{\alpha} dt \tag{51}$$

is in the class $K(\delta)$, where

$$\delta = 1 - |\alpha|\left[(N+1) + (2-\lambda)M^{\mu-1}\right], \qquad 0 < |\alpha|\left[(N+1) + (2-\lambda)M^{\mu-1}\right] \le 1. \tag{52}$$

Proof. Let the function h be defined in (25). Then, we obtain

$$\frac{zh''(z)}{h'(z)} = \alpha\left[\frac{zf''(z)}{f'(z)} + 1 - \frac{zg'(z)}{g(z)}\right]. \tag{53}$$

Therefore,

$$\left|\frac{zh''(z)}{h'(z)}\right| \le |z| \cdot |\alpha| \cdot \left|\frac{f''(z)}{f'(z)}\right| + |\alpha| + |\alpha| \cdot \left|g'(z)\left(\frac{z}{g(z)}\right)^{\mu}\right| \cdot \left|\left(\frac{g(z)}{z}\right)\right|^{\mu-1}. \tag{54}$$

Since $|g(z)| < M$, $z \in U$, applying the Schwarz Lemma, we have

$$\left|\frac{g(z)}{z}\right| \le M, \quad \forall z \in U. \tag{55}$$

From (54) we obtain

$$\left|\frac{zh''(z)}{h'(z)}\right| \leq |\alpha|(N+1)+|\alpha| \cdot\left[\left|g'(z)\left(\frac{z}{g(z)}\right)^{\mu}-1\right|+1\right]\cdot M^{\mu-1} \leq |\alpha|\left[(N+1)+(2-\lambda)\cdot M^{\mu-1}\right]=1-\delta. \tag{56}$$

This evidently completes the proof. □

Letting $\mu = 1$ in Theorem 12, we have the following.

Corollary 13. *Let α be a complex number, $f, g \in \mathscr{A}$, with g in the class $S^*(\lambda)$, $0 \leq \lambda < 1$. If $|g(z)| \leq M$, for $M \geq 1$, $z \in U$, and*

$$\left|\frac{f''(z)}{f'(z)}\right| < N, \quad N \geq 1,\ z \in U, \tag{57}$$

then the integral operator is as follows:

$$I_\alpha(f,g)(z)=\int_0^z\left[\frac{tf'(t)}{g(t)}\right]^\alpha dt \in K(\delta), \tag{58}$$

where $\delta = 1-|\alpha|(3+N-\lambda)$ and $0 < |\alpha|(3+N-\lambda) \leq 1$.

Letting $\delta = \lambda = 0$ in Corollary 13, we obtain the following.

Corollary 14. *Let α be a complex number, $f, g \in \mathscr{A}$, with g a starlike function in U. If $|g(z)| \leq M$, for $M \geq 1$, $z \in U$, and*

$$\left|\frac{f''(z)}{f'(z)}\right| < N, \quad N \geq 1,\ z \in U, \tag{59}$$

then the integral operator

$$I_\alpha(f,g)(z)=\int_0^z\left[\frac{tf'(t)}{g(t)}\right]^\alpha dt \tag{60}$$

is convex in U, where $|\alpha| = 1/(3+N)$.

Taking $g(z) = z$ in Corollary 14, we obtain the following.

Example 15. Let α be a complex number and function $f \in \mathscr{A}$. If

$$\left|\frac{f''(z)}{f'(z)}\right| < N, \quad N \geq 1,\ z \in U, \tag{61}$$

then the integral operator $I_\alpha(f)(z)$ is convex in U, where $|\alpha| = 1/(3+N)$.

3. Subordination Results

In view of the results due to Breaz et al. [3], we obtain some subordination results of the above integral operator $I_\alpha(f,g)$.

Theorem 16. *If $f \in S_b^*(a)$ and $g \in C_b^*(a)$, $a > 1$, and $b > 0$, then for $\alpha > 0$*

$$\operatorname{Re}\left[\frac{zI''_\alpha(f,g)(z)}{I'_\alpha(f,g)(z)}\right] < \frac{\alpha(a-1)}{b(a+1)}, \quad z \in U. \tag{62}$$

This implies that $I_\alpha(f,g)(z) \in S_{b/2\alpha}(a)$.

Proof. We have

$$\frac{zI''_\alpha(f,g)(z)}{I'_\alpha(f,g)(z)} = \alpha\left[\frac{zf''(z)}{f'(z)}-\frac{zg'(z)}{g(z)}+1\right]. \tag{63}$$

Noting that $f \in S_b^*(a)$ and $g \in C_b^*(a)$, we see that

$$\begin{aligned}\operatorname{Re}\left[\frac{zI''_\alpha(f,g)(z)}{I'_\alpha(f,g)(z)}\right] &= \alpha\operatorname{Re}\left[\frac{zf''(z)}{f'(z)}-\frac{zg'(z)}{g(z)}+1\right]\\ &=\alpha\left\{\operatorname{Re}\left[\frac{zf''(z)}{f'(z)}\right]-\operatorname{Re}\left[\frac{zg'(z)}{g(z)}\right]+1\right\}\\ &<\alpha\left\{\frac{a-1}{2b(a+1)}+\frac{a-1}{2b(a+1)}\right\}\\ &=\frac{a-1}{2(b/2\alpha)(a+1)}.\end{aligned} \tag{64}$$

This means that $I_\alpha(f,g)(z) \in S_{b/2\alpha}(a)$. □

By using the definition of the class $S_b(a)$ (5), we obtain the following.

Corollary 17. *If $f \in S_b^*(a)$ and $g \in C_b^*(a)$, for $a > 1$, $b > 0$, and $\alpha > 0$,*

$$\left[\frac{tf'(t)}{g(t)}\right]^{b/2} \prec \frac{a(1-z)}{a-z}, \quad z \in U. \tag{65}$$

Example 18. Let us consider the functions f and g which satisfy

$$\left[\frac{tf'(t)}{g(t)}\right]^\alpha = (1-z)^{p-1}, \tag{66}$$

where $p = 2(1-a)\alpha/b(a+1)+1$. We have

$$I_\alpha(f,g)(z)=\frac{1}{p}\left[1-(1-z)^p\right], \quad z \in U. \tag{67}$$

Thus,

$$\operatorname{Re}\left[\frac{zI''_\alpha(f,g)(z)}{I'_\alpha(f,g)(z)}\right] = \operatorname{Re}\left[\frac{(1-p)z}{1-z}\right] < \frac{\alpha(a-1)}{b(a+1)}, \quad z \in U. \tag{68}$$

Example 19. Let us consider the functions f which satisfy

$$\left[f'(z)\right]^\alpha = (1-z)^{p-1}, \tag{69}$$

where $z \in U$ and $p = 2(1-a)\alpha/b(a+1)+1$. We have

$$I_\alpha(f)(z) = \frac{1}{p}\left[1-(1-z)^p\right], \quad z \in U. \tag{70}$$

Thus,

$$\operatorname{Re}\left[\frac{zI_\alpha''(f)(z)}{I_\alpha'(f)(z)}\right] = \operatorname{Re}\left[\frac{(1-p)z}{1-z}\right] < \frac{\alpha(a-1)}{b(a+1)}, \quad z \in U. \tag{71}$$

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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References

[1] S. S. Miller and P. T. Mocanu, *Differential Subordinations*, Marcel Dekker, New York, NY, USA, 2000.

[2] S. Owa, J. Nishiwaki, and N. Niwa, "Subordination for certain analytic functions," *International Journal of Open Problems in Computer Science*, vol. 1, pp. 1–7, 2008.

[3] D. Breaz, S. Owa, and N. Breaz, "Some properties for general integral operators," *Advances in Mathematics*, vol. 3, no. 1, pp. 9–14, 2014.

[4] B. A. Frasin and J. M. Jahangiri, "A new and comprehensive class of analytic functions," *Analele Universității din Oradea, Fascicola Matematică*, vol. 15, pp. 59–62, 2008.

[5] B. A. Frasin and M. Darus, "On certain analytic univalent functions," *International Journal of Mathematics and Mathematical Sciences*, vol. 25, no. 5, pp. 305–310, 2001.

[6] J. Pfaltzgraff, "Univalence of the integral of $(f'(z))^\lambda$," *Bulletin of the London Mathematical Society*, vol. 7, no. 3, pp. 254–256, 1975.

[7] N. N. Pascu and V. Pescar, "On the integral operators of Kim-Merkes and Pfaltzgraff," *Mathematica, Universitatis Babes-Bolyai Cluj-Napoca*, vol. 32, no. 2, pp. 185–192, 1990.

[8] D. Breaz, S. Owa, and N. Breaz, "A new integral univalent operator," *Acta Universitatis Apulensis. Mathematics—Informatics*, vol. 16, p. 1116, 2008.

[9] D. Breaz and N. Breaz, "Univalence conditions for certain integral operators," *Studia Universitatis Babeş-Bolyai. Mathematica*, vol. 47, no. 2, pp. 9–15, 2002.

[10] S. Bulut and D. Breaz, "Univalency and convexity conditions for a general integral operator," *Chinese Journal of Mathematics*, vol. 2014, Article ID 923984, 4 pages, 2014.

[11] L. Stanciu and D. Breaz, "The univalence conditions for two integral operators," *Annals of the University of Craiova, Mathematics and Computer Science Series*, vol. 40, no. 1, pp. 95–99, 2013.

[12] D. Breaz, "A convexity property for an integral operator on the class $S_p(\beta)$," *Journal of Inequalities and Applications*, vol. 2008, Article ID 143869, 4 pages, 2008.

[13] E. A. Eljamal, M. Darus, and D. Breaz, "Some results of univalent and starlike integral operator," *Journal of Complex Analysis*, vol. 2013, Article ID 502363, 3 pages, 2013.

[14] A. Mohammed, M. Darus, and D. Breaz, "New criterion for starlike integral operators," *Analysis in Theory and Applications*, vol. 29, no. 1, pp. 21–26, 2013.

[15] P. T. Mocanu and I. Şerb, "A sharp simple criterion for a subclass of starlike functions," *Complex Variables: Theory and Application*, vol. 32, no. 2, pp. 161–168, 1997.

[16] J. Becker, "Lownersche Differential gleichung und quasi-konform fortset-zbare schlichte funktionen," *Journal für die Reine und Angewandte Mathematik*, vol. 255, pp. 23–43, 1972.

[17] O. Mayer, *The Functions Theory of One Variable Complex*, 1981.

[18] G. L. Reddy and K. S. Padmanabhan, "On analytic functions with reference to the Bernardi integral operator," *Bulletin of the Australian Mathematical Society*, vol. 25, no. 3, pp. 387–396, 1982.